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$SU(3)$ Flavor Symmetry Breaking in Baryon-Meson Scattering in the $1/N_c$ Expansion of QCD: A Numerical Approach

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The baryon-meson scattering amplitude is computed within the formalism of the $1/N_c$ expansion of QCD. The obtained results consider the effects of the decuplet-octet baryon mass difference and perturbative flavor $SU(3)$ symmetry breaking. Due to the complexity of the explicit algebraic computation of the symmetry breaking considering the $N_c = 3$ physical limit, a numerical method has been implemented.

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1. Introduction

Quantum Chromodynamics (QCD) is the theory of the strong interaction, with quarks and gluons as fundamental fields. It is a gauge theory characterized by the symmetry group $SU(N_c)$, whit $N_c = 3$ representing the number of color charges. At low energies, where QCD becomes strongly coupled, standard perturbative techniques are not applicable. To address this, various frameworks have been developed for describing hadronic physics in the low-energy regime. Among the most widely studied are the large-N_c limit of QCD, and chiral perturbation theory (ChPT).

The large-N_c limit generalizes QCD to $N_c \to \infty$, providing information on the structure of mesons and baryons, with physical quantities corrected by powers of $1/N_c$ [1–3]. Meanwhile, ChPT, and in particular Heavy Baryon Chiral Perturbation Theory (HBChPT) [4, 5], is an effective low-energy approximation aiming on meson-baryon interactions. HBChPT includes the spin-$1/2$ baryon octet and spin-$3/2$ decuplet in loop diagrams to match physical quark masses.

Combining $1/N_c$ and chiral expansions offers more precise constraints on baryon-pseudoscalar meson interactions [6]. For example, when baryon-meson scattering is described within both frameworks, Witten [3] and others [7, 8] derived consistency conditions for large-N_c.

In the formalisms described above, the baryon-meson scattering problem has been tackled using different approaches, and some important results have been obtained [9–12].

The present study is part of an ongoing research project aimed at analytically evaluating baryon-meson scattering amplitudes using the $1/N_c$ of leading order calculations, with a numerical approach for the symmetry breaking contribution.

2. Baryon-meson Scattering Amplitude at Tree Level

In this section, the analytical computation of the tree-level amplitude of baryon-meson scattering presented in Ref. [8] is explicitly carried out, specialized to the process

$$B(p) + \pi^a(k) \to B'(p') + \pi^b(k'). \tag{1}$$

The Feynman diagrams describing this process are depicted in Fig. 1.

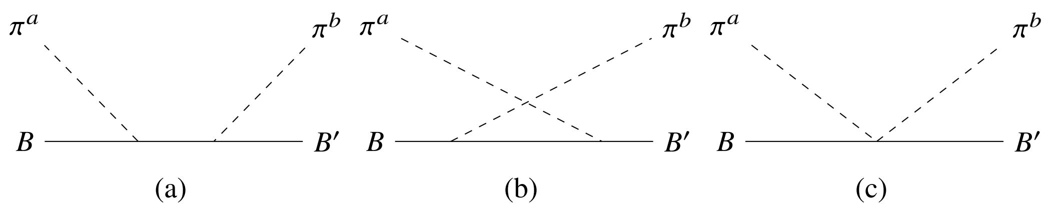

Figure 1: Feynman diagrams for the tree-level scattering process (1)

In Eq. (1), π denotes one of the nine pseudo scalar mesons π, K, η and η' of momenta $k = (k^0, k^1, k^2, k^3)$ and $k' = (k'^0, k'^1, k'^2, k'^3)$ and flavors a and b for the incoming and outgoing mesons, and B and B' denote the incoming and outgoing baryons of momenta p and p', respectively. Soft mesons with energies of order unity are considered in the process. The objective of this work is to explicitly calculate the corresponding scattering amplitude at tree level, while incorporating the effects of the baryon mass splitting Δ. Before tackling the problem, it is useful to introduce essential concepts from large-N_c QCD in order to establish the relevant notation and conventions. Further details on the formalism can be found in Refs. [13, 14].

In the large-N_c limit, the baryon sector has a contracted $SU(2N_f)$ spin-flavor symmetry, where N_f is the number of light quark flavors. For $N_f = 3$ the lowest lying baryon states fall into a representation of the spin-flavor group $SU(6)$. When $N_c = 3$, this corresponds to the **56** dimensional representation of $SU(6)$.

The $1/N_c$ expansion of a QCD operator can be written in terms of $1/N_c$-suppressed operators with well-defined spin-flavor transformation properties. A complete set of operators can be constructed using the 0-body operator $\mathbb{1}$ and the 1-body operators J^k, T^c and G^{kc}, which are the baryon spin, baryon flavor and baryon spin-flavor generators, respectively, and transform under $SU(2) \times SU(3)$ as (j, m), where j is the spin and m is the dimension of the $SU(3)$ flavor representation. The $SU(2N_f)$ spin-flavor generators satisfy well-defined commutation relations [14].

The Feynman diagrams depicted in Fig. 1 are analyzed separately since they contribute differently to the scattering process. The digram (c), won't be described in this work. Since the numerical method can be examined using just diagrams (a) and (b).

2.1 Scattering amplitude from Fig. 1(a,b)

The baryon operator that yields the tree level amplitude for the scattering process (1) represented in Fig. 1(a,b), in the rest frame of the initial baryon, is given by [8]

$$A_{\text{tree}}^{ab} = -\frac{1}{f^2} k^i k'^j \left(\frac{1}{k^0} \sum_{n=0}^{\infty} \frac{1}{k^{0n}} [A^{jb}, \underbrace{[\mathcal{M}, [\mathcal{M}, \ldots [\mathcal{M}}_{n \text{ insertions}}, A^{ia}] \ldots]]] \right), \tag{2}$$

where $f \approx 93\,\text{MeV}$ is the pion decay constant, A^{ia} is the baryon axial vector current and $\mathcal{M}$ is the baryon mass operator. Both operators have $1/N_c$ expansions [14].

The series expansion in (2) for the first three terms of the summation corresponds to,

$$A_{\text{tree}}^{ab} = -\frac{1}{f^2} k^i k'^j \left(\frac{1}{k^0} [A^{jb}, A^{ia}] + \frac{1}{k^{02}} [A^{jb}, [\mathcal{M}, A^{ia}]] + \frac{1}{k^{03}} [A^{jb}, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] + \ldots \right). \tag{3}$$

The constraint that A_{tree}^{ab} be at most $O(1)$ in the large-N_c limit, produces the following consistency conditions [7, 8]

$$[A^{jb}, A^{ia}] \leq O(N_c), \tag{4a}$$

$$[A^{jb}, [\mathcal{M}, A^{ia}]] \leq O(N_c), \tag{4b}$$

$$[A^{jb}, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] \leq O(N_c), \tag{4c}$$

$$\vdots$$

where k^0, f, and Δ are orders $O(1)$, $O(\sqrt{N_c})$, and $O(N_c^{-1})$ in that limit, respectively.

2.1.1 Spin-flavor transformation properties of A^{ab}_{tree}

The baryon operator A^{ab}_{tree} is a spin-zero entity and includes two adjoint (octet) indices. The tensor product of two adjoint representations, $\mathbf{8}\otimes\mathbf{8}$, can be divided into the symmetric product $(\mathbf{8}\otimes\mathbf{8})_S$ and the antisymmetric product $(\mathbf{8}\otimes\mathbf{8})_A$ [14], which can then be broken down into $SU(3)$ multiplets as follows:

$$(\mathbf{8}\otimes\mathbf{8})_S = \mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27}, \tag{5a}$$

$$(\mathbf{8}\otimes\mathbf{8})_A = \mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}. \tag{5b}$$

To utilize the transformation properties of A^{ab}_{tree} under the spin-flavor symmetry $SU(2)\times SU(3)$, the spin and flavor projectors described in Ref. [15] are useful. This method leverages the decomposition of the space formed by the n-th tensor product of adjoint spaces with itself, $\prod_{i=1}^{n} adj\otimes$, into subspaces that can be identified by a specific eigenvalue of the quadratic Casimir operator C of the Lie algebra of $SU(N)$. For each subspace, a corresponding projection operator $\mathcal{P}^{(m)}$ can be defined. In particular, for the product of two $SU(3)$ adjoints, the flavor projectors $[\mathcal{P}^{(m)}]^{abcd}$ for the irreducible representation of dimension m contained in (5) are given by

$$[\mathcal{P}^{(1)}]^{abcd} = \frac{1}{N_f^2-1}\delta^{ab}\delta^{cd}, \tag{6}$$

$$[\mathcal{P}^{(8)}]^{abcd} = \frac{N_f}{N_f^2-4}d^{abe}d^{cde}, \tag{7}$$

$$[\mathcal{P}^{(27)}]^{abcd} = \frac{1}{2}(\delta^{ac}\delta^{bd}+\delta^{bc}\delta^{ad}) - \frac{1}{N_f^2-1}\delta^{ab}\delta^{cd} - \frac{N_f}{N_f^2-4}d^{abe}d^{cde}, \tag{8}$$

$$[\mathcal{P}^{(8_A)}]^{abcd} = \frac{1}{N_f}f^{abe}f^{cde}, \tag{9}$$

and

$$[\mathcal{P}^{(10+\overline{10})}]^{abcd} = \frac{1}{2}(\delta^{ac}\delta^{bd}-\delta^{bc}\delta^{ad}) - \frac{1}{N_f}f^{abe}f^{cde}, \tag{10}$$

which fulfills the completeness relation

$$[\mathcal{P}^{(1)}+\mathcal{P}^{(8)}+\mathcal{P}^{(27)}+\mathcal{P}^{(8_A)}+\mathcal{P}^{(10+\overline{10})}]^{abcd} = \delta^{ac}\delta^{bd}. \tag{11}$$

Hence, $[\mathcal{P}^{(m)}A_{\text{tree}}]^{ab}$ effectively projects out the piece of A^{ab}_{tree} that transforms under the flavor representation of dimension m according to decompositions (5). However, for computational purposes, it is more convenient to group the operators $[\mathcal{P}^{(m)}A_{\text{tree}}]^{ab}$ based on their symmetry transformation properties under the interchange of a and b. Accordingly, $[\mathcal{P}^{(1)}+\mathcal{P}^{(8)}+\mathcal{P}^{(27)}]^{abcd}$ and $[\mathcal{P}^{(8_A)}+\mathcal{P}^{(10+\overline{10})}]^{abcd}$, acting on A^{cd}_{tree} will provide the symmetric [antisymmetric] part of A^{ab}_{tree} under the interchange of a and b.

2.1.2 Explicit form of the scattering amplitude at tree-level

A more detailed calculation, beyond the qualitative analyses of baryon-meson scattering presented in previous works [7, 8], can be performed by explicitly evaluating the first terms displayed in Eq. (3). The strategy implemented in this work follows the lines of Refs. [16, 17].

The matrix elements of A^{ab}_{tree} in Eq. (2), between $SU(6)$ baryon states for mesons of flavors a and b yield the corresponding scattering amplitude at tree level, namely,

$$\mathcal{A}_{\text{tree}}(B+\pi^a \longrightarrow B'+\pi^b) \equiv \langle B'\pi^b|A^{ab}_{\text{tree}}|B\pi^a\rangle. \tag{12}$$

The flavors associated to mesons are conventionally given by $\left\{\frac{1-i2}{\sqrt{2}}, 3, \frac{1+i2}{\sqrt{2}}, \frac{4-i5}{\sqrt{2}}, \frac{6-i7}{\sqrt{2}}, \frac{4+i5}{\sqrt{2}}, \frac{6+i7}{\sqrt{2}}, 8\right\}$ for $\{\pi^+, \pi^0, \pi^-, K^+, K^0, K^-, \overline{K}^0, \eta\}$, respectively.[1] For instance, an expressions such as $\mathcal{A}_{\text{tree}}(p+\pi^- \longrightarrow n+\pi^0)$ corresponds to $\langle n\pi^0|A^{13}_{\text{tree}}+iA^{23}_{\text{tree}}|p\pi^-\rangle/\sqrt{2}$.

Thus, with the operator reductions listed in [18], the scattering amplitude for process (1), arising from Fig. 1(a,b), can be organized as

$$\mathcal{A}_{\text{tree}}(B+\pi^a \longrightarrow B'+\pi^b) = -\frac{1}{f^2k^0}\sum_{m=1}^{139}(c_m^{(\text{s})}+c_m^{(\text{a})})k^ik'^j\langle B'\pi^b|S_m^{(ij)(ab)}|B\pi^a\rangle, \tag{13}$$

where $S_m^{(ij)(ab)}$ ($m=1,\ldots,139$) constitute a basis of linearly independent spin-2 baryon operators with two adjoint indices and $c_m^{(\text{s})}$ and $c_m^{(\text{a})}$ are well-defined coefficients which come along with the symmetric and antisymmetric pieces of A^{ab}. Notice that in Eq. (13) the sum over spin indices is implicit.

The operators $S_m^{(ij)(ab)}$ that constitute the basis, comprising up to 7-body operators are listed on [18], as well as the operator coefficients $c_m^{(\text{s})}$ and $c_m^{(\text{a})}$.

3. $SU(3)$ Flavor Symmetry Breaking in the Scattering Amplitude

First-order SB effects in the scattering amplitude are computed from the tensor product of the scattering amplitude itself, which transforms under $SU(2)\times SU(3)$ as $(2,\mathbf{8}\otimes\mathbf{8})$, and the perturbation, which transforms as $(0,\mathbf{8})$. The tensor product of three adjoint representations $\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}$ decomposes as

$$\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8} = 2(\mathbf{1})\oplus 8(\mathbf{8})\oplus 4(\mathbf{10}\oplus\overline{\mathbf{10}})\oplus 6(\mathbf{27})\oplus 2(\mathbf{35}\oplus\overline{\mathbf{35}})\oplus\mathbf{64}. \tag{14}$$

Thus, effects of SB can be evaluated by constructing the $1/N_c$ expansions of the pieces of the scattering amplitude transforming as $(2,\mathbf{1})$, $(2,\mathbf{8})$, $(2,\mathbf{10}\oplus\overline{\mathbf{10}})$, $(2,\mathbf{27})$, $(2,\mathbf{35}\oplus\overline{\mathbf{35}})$ and $(2,\mathbf{64})$ under $SU(2)\times SU(3)$. These $1/N_c$ expansions need be expressed in terms of a complete basis of linearly independent operators $\{R^{(ij)(a_1a_2a_3)}\}$, where a generic operator $R^{(ij)(a_1a_2a_3)}$ thus represents a spin-2 object with three adjoint indices. For $N_c=3$, up to 3-body operators should be retained in the series. Accordingly, first-order SB can be accounted for by setting one of the flavor indices to 8, *v.gr.*, $a_3=8$. For completeness, the set of up to 3-body operators used as a basis is listed in [18]. The set contains 170 linearly independent operators.

[1] For simplicity only the octet of pseudo scalar mesons is considered. Extending the analysis to include the η' is straightforward by using the baryon axial vector current $A^i \equiv A^{i9}$, which is written in terms of the 1-body operators $G^{i9}=\frac{1}{\sqrt{6}}J^i$ and $T^9=\frac{1}{\sqrt{6}}N_c\mathbb{1}$ [6].

In order to simplify the analysis of the SB effect, it is helpful again to apply the projection operator technique presented in Ref. [15]. In this case, for the decomposition (14), the projection operators are given in the form $\left[\mathcal{P}^{(m)}\right]^{c_1c_2c_3b_1b_2b_3}$.

Therefore, the product $\left[\mathcal{P}^{(m)}R^{(ij)}\right]^{c_1c_2c_3}$ will effectively provide the component of the operator $R^{(ij)(c_1c_2c_3)}$ transforming in the irreducible representation of dimension m, according to decomposition (14).

3.1 Numerical method for six flavor indices tensors

The direct analytical construction of $\left[\mathcal{P}^{(m)}\right]^{c_1c_2c_3b_1b_2b_3}$ presents several algebraic difficulties. A more practical approach is needed to address this issue, and it turns out that using a matrix-based numerical method is the most effective strategy.

To begin, observe that each projection operator (or quadratic Casimir operator) is an object with six adjoint indices, each having eight possible values. This means that such objects contain 8^6 elements in total. However, Casimir operators either fully contract all of their indices or contract half, and the projectors act on 3-body operators with three adjoint indices. As a result, half of the indices in the projectors will always be contracted. This allows us to combine the first three indices (c_1,c_2,c_3) and the last three indices (b_1,b_2,b_3) from both the Casimir operator and projectors into just two indices, one for each set. These new indices have $8^3 = 512$ possible values. In this way, the projectors can be represented as 512×512 matrices. Likewise, the 3-body operators with three adjoint indices can be treated as vectors with 512 components.

Consequently, instead of performing the index contractions $\left[\mathcal{P}^{(m)}R^{(ij)}\right]^{c_1c_2c_3}$, the problem simplifies to regular matrix multiplications. This entire approach preserves all relevant information while significantly simplifying the analysis.

Let $\mathsf{P}^{(m)}$ be the matrix corresponding to the projection operator $\left[\mathcal{P}^{(m)}\right]^{c_1c_2c_3b_1b_2b_3}$. With the method implemented, a series of consistency checks have been performed, namely,

$$\mathsf{P}^{(m)}\mathsf{P}^{(m)} = \mathsf{P}^{(m)}, \qquad \mathsf{P}^{(m)}\mathsf{P}^{(n)} = 0, \quad n \neq m, \tag{15}$$

along with

$$\mathsf{P}^{(1)} + \mathsf{P}^{(8)} + \mathsf{P}^{(10+\overline{10})} + \mathsf{P}^{(27)} + \mathsf{P}^{(35+\overline{35})} + \mathsf{P}^{(64)} = \mathsf{I}_{512}, \tag{16}$$

where I_{512} stands for the identity matrix of order 512. The above relations are the usual properties that projection operatators must satisfy. No further details on the method are needed here.

The way these flavor projection operators work can be better seen through a few examples. The operator $\{T^a,\{T^b,T^c\}\}$, for instance, contributes to the scattering amplitude of the process $n+\pi^+ \longrightarrow n+\pi^+$ through the components with flavor indices $a = (1-i2)/\sqrt{2}$, $b = (1-i2)/\sqrt{2}$, and $c = 8$.Using the matrix method, the (118) component of the flavor **8** piece becomes,

$$\begin{aligned}
&[\mathcal{P}^{(8)}]^{118cde}\{T^c,\{T^d,T^e\}\} \\
&= \frac{1}{15}T^1T^1T^8 + \frac{1}{30\sqrt{3}}T^1T^4T^6 + \frac{1}{30\sqrt{3}}T^1T^5T^7 + \frac{1}{30\sqrt{3}}T^1T^6T^4 + \frac{1}{30\sqrt{3}}T^1T^7T^5 + \frac{4}{15}T^1T^8T^1 \\
&\quad + \frac{1}{15}T^2T^2T^8 - \frac{1}{30\sqrt{3}}T^2T^4T^7 + \frac{1}{30\sqrt{3}}T^2T^5T^6 + \frac{1}{30\sqrt{3}}T^2T^6T^5 - \frac{1}{30\sqrt{3}}T^2T^7T^4 + \frac{4}{15}T^2T^8T^2
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{15}T^3T^3T^8+\frac{1}{30\sqrt{3}}T^3T^4T^4+\frac{1}{30\sqrt{3}}T^3T^5T^5-\frac{1}{30\sqrt{3}}T^3T^6T^6-\frac{1}{30\sqrt{3}}T^3T^7T^7+\frac{4}{15}T^3T^8T^3\\
&-\frac{1}{15\sqrt{3}}T^4T^1T^6+\frac{1}{15\sqrt{3}}T^4T^2T^7-\frac{1}{15\sqrt{3}}T^4T^3T^4+\frac{1}{30\sqrt{3}}T^4T^4T^3+\frac{1}{10}T^4T^4T^8+\frac{1}{30\sqrt{3}}T^4T^6T^1\\
&-\frac{1}{30\sqrt{3}}T^4T^7T^2+\frac{1}{5}T^4T^8T^4-\frac{1}{15\sqrt{3}}T^5T^1T^7-\frac{1}{15\sqrt{3}}T^5T^2T^6-\frac{1}{15\sqrt{3}}T^5T^3T^5+\frac{1}{30\sqrt{3}}T^5T^5T^3\\
&+\frac{1}{10}T^5T^5T^8+\frac{1}{30\sqrt{3}}T^5T^6T^2+\frac{1}{30\sqrt{3}}T^5T^7T^1+\frac{1}{5}T^5T^8T^5-\frac{1}{15\sqrt{3}}T^6T^1T^4-\frac{1}{15\sqrt{3}}T^6T^2T^5\\
&+\frac{1}{15\sqrt{3}}T^6T^3T^6+\frac{1}{30\sqrt{3}}T^6T^4T^1+\frac{1}{30\sqrt{3}}T^6T^5T^2-\frac{1}{30\sqrt{3}}T^6T^6T^3+\frac{1}{10}T^6T^6T^8+\frac{1}{5}T^6T^8T^6\\
&-\frac{1}{15\sqrt{3}}T^7T^1T^5+\frac{1}{15\sqrt{3}}T^7T^2T^4+\frac{1}{15\sqrt{3}}T^7T^3T^7-\frac{1}{30\sqrt{3}}T^7T^4T^2+\frac{1}{30\sqrt{3}}T^7T^5T^1\\
&-\frac{1}{30\sqrt{3}}T^7T^7T^3+\frac{1}{10}T^7T^7T^8+\frac{1}{5}T^7T^8T^7+\frac{1}{15}T^8T^1T^1+\frac{1}{15}T^8T^2T^2+\frac{1}{15}T^8T^3T^3\\
&+\frac{1}{10}T^8T^4T^4+\frac{1}{10}T^8T^5T^5+\frac{1}{10}T^8T^6T^6+\frac{1}{10}T^8T^7T^7+\frac{2}{5}T^8T^8T^8. \qquad (17)
\end{aligned}
$$

Computing the matrix elements of the operator (17) is straightforward; therefore,

$$\langle n\pi^+|[\mathcal{P}^{(8)}]^{118cde}\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle = \frac{1}{2}\sqrt{3}, \qquad (18)$$

and

$$\langle n\pi^+|[\mathcal{P}^{(r)}]^{118cde}\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle = 0, \qquad (19)$$

for $r \neq 8$.

The procedure can be repeated for each flavor combination so the different contributions of the operator $\{T^a,\{T^b,T^c\}\}$ to the scattering amplitude of the process $n+\pi^+ \to n+\pi^+$ can be available. For the canonical example, the final expression can be summarized as

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}k^i k'^j \delta^{ij}\langle n\pi^+|[\mathcal{P}^{(8)}]^{(1-i2)(1-i2)8cde}\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle = \frac{1}{2}\sqrt{3}\mathbf{k}\cdot\mathbf{k}', \qquad (20)$$

and

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}k^i k'^j \delta^{ij}\left[\langle n\pi^+|[P^{(m)}]^{(1-i2)(1-i2)8cde}\right]\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle = 0, \qquad (21)$$

for $r \neq 8$.

Using this procedure, after a long but otherwise standard calculation, first-order SB to the scattering amplitude $\mathcal{A}_{\text{tree}}$ Eq. (13), denoted hereafter by $\delta\mathcal{A}$, can be organized as

$$
\begin{aligned}
&f^2k^0\delta\mathcal{A}(B+\pi^a\to B'+\pi^b) = \\
&\sum_m\Big[N_c g_1^{(m)}k^ik'^j\langle B'\pi^b|[\mathcal{P}^{(m)}R_1^{(ij)}]^{(ab8)}|B\pi^a\rangle + N_c g_2^{(m)}k^ik'^j\langle B'\pi^b|[\mathcal{P}^{(m)}R_2^{(ij)}]^{(ab8)}|B\pi^a\rangle\\
&\quad+\sum_{r=3}^{16} g_r^{(m)}k^ik'^j\langle B'\pi^b|[\mathcal{P}^{(m)}R_r^{(ij)}]^{(ab8)}|B\pi^a\rangle + \frac{1}{N_c}\sum_{r=17}^{71} g_r^{(m)}k^ik'^j\langle B'\pi^b|[\mathcal{P}^{(m)}R_r^{(ij)}]^{(ab8)}|B\pi^a\rangle
\end{aligned}
$$

$$+ \frac{1}{N_c^2} \sum_{r=72}^{170} g_r^{(m)} k^i k'^j \langle B'\pi^b | [\mathcal{P}^{(m)} R_r^{(ij)}]^{(ab8)} | B\pi^a \rangle \Big], \tag{22}$$

where $g_r^{(m)}$, $r = 1, \ldots, 170$, are undetermined coefficients, which are expected to be of order one, the sum over m covers all six irreducible representations indicated in relation (14) and the sums over i and j are implicit. For details of the calculation see Ref. [18]

For the $N + \pi \longrightarrow N + \pi$ process, simpler expressions are obtained by defining alternative coefficients expressed in terms of linear combinations of the $g_r^{(m)}$ ones. After redefining the coefficients, the symmetry breaking scattering amplitude for our example can be addressed as

$$f^2 k^0 \delta\mathcal{A}^{(1)}(n + \pi^+ \longrightarrow n + \pi^+) = d_1^{(1)} \mathbf{k} \cdot \mathbf{k}' + e_1^{(1)} i (\mathbf{k} \times \mathbf{k}')^3. \tag{23}$$

where the $d_1^{(1)}$ and $e_1^{(1)}$ coefficients are easily read off from the full expansion of the contribution using (22).

4. Example: $p + \pi^+ + \longrightarrow p + \pi^+$ Scattering Amplitude

The formalism presented so far can be implemented to study scattering processes of the form $B + \pi^a \longrightarrow B' + \pi^b$ provided that the particles involved in the process have equal total strangeness on each side of the scattering, according to the Gell-Mann–Nishijima scheme. Since B and B' can be octet or decuplet baryons, from the theoretical point of view, the possibilities are numerous; examples are $\Lambda + K^+ \longrightarrow p + \pi^0$, $\Xi^{*-} + K^+ \longrightarrow \Sigma^{*0} + \pi^0$, $\Xi^{*-} + K^0 \longrightarrow \Sigma^- + \pi^0$, and so on. To examplify the developed method, the symmetry breaking contribution for the scattering amplitude of the process $p + \pi^+ \longrightarrow p + \pi^+$ will be computed.

Using the numerical method outlined in the previous section, the first-order SB effects to the scattering amplitudes for the $p + \pi^+ \longrightarrow p + \pi^+$ process corresponds to

$$\begin{aligned} f^2 k^0 \delta\mathbf{A}(p + \pi^+ \longrightarrow p + \pi^+) &= (d_1^{(1)} + d_1^{(8)} + d_1^{(10+\overline{10})} + d_1^{(27)}) \mathbf{k} \cdot \mathbf{k}' \\ &\quad + (e_1^{(1)} + e_1^{(8)} + e_1^{(10+\overline{10})} + e_1^{(27)}) i (\mathbf{k} \times \mathbf{k}')^3, \end{aligned} \tag{24}$$

which are written in terms of 11 unknown parameters. It should be highlighted that neither flavor **35**+$\overline{\mathbf{35}}$ nor flavor **64** representation participates in the final expressions, even when they appear in the decomposition (14). Those representation could emerge in higher order computations.

5. Concluding remarks

The material exposed in this work represents an innovative program for understanding the baryon-meson scattering processes in the context of the $1/N_c$ expansion. Specifically, the scattering amplitude for the process $B + \pi^a \longrightarrow B' + \pi^b$, including decuplet-octet baryon mass splitting and perturbative $SU(3)$ symmetry breaking, has been calculated. The formalism is general enough to cover the cases when B and B' are any baryon states and π^a and π^b are any pseudoscalar mesons, provided that the Gell-Mann–Nishijima scheme is fulfilled. In particular, expressions for $N\pi \longrightarrow N\pi$

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scattering amplitudes are given here. These amplitudes get simple forms once all the ingredients are put together, regardless of stunning original expressions, such as (13).

To achieve these simpler expressions, one can use the numerical method presented in this work. This method provides the formalism of a useful tool, which is helpful in computations that involve higher rank tensors.

The presented work is expected to be extended to loop computations of the baryon-meson scattering process, in the mixed formalism of Chiral Perturbation Theory and $1/N_c$ expansion. This extension includes higher rank tensors that could be analyzed using the numerical method. Moreover, the baryon-baryon interaction, which has been tackled in recent years using the Large N_c limit [19, 20], can be addressed in the mixed formalism. The projector numerical method would be useful to describe symmetry breaking contributions in these analyses as future work.

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