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# Weak nonleptonic hyperon decays in relativistic $\chi$PT

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Following recent experimental updates on hyperon decay parameters, hyperon nonleptonic transitions are studied in relativistic chiral perturbation theory ($\chi$PT). Previous computations of the next-to-leading-order (NLO) corrections to the decay amplitude were carried out in non-relativistic frameworks. We attempt a (re)calculation of one-loop corrections in relativistic $\chi$PT in the EOMS renormalization scheme. We present our results for the combined fit to $S$- and $P$-wave amplitudes, as well as the relative importance of the considered contributions, such as the role of resonances and decuplet baryons as intermediate states. Our goal is to provide an updated theoretical description of weak nonleptonic hyperon decays in $\chi$PT up to one-loop corrections, based on the most recent data and theoretical framework.

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## 1. Introduction

Finding the right description for hyperon nonleptonic decays within the theory of Quantum Chromodynamics (QCD) represents a decades-long challenge. Chiral Perturbation Theory ($\chi$PT) is the tool often chosen to study these baryon decays, as the low-energy effective field theory of QCD. From angular momentum conservation, hyperon nonleptonic decays can be separated into two partial-wave amplitudes, the parity-violating $S$, and the parity-conserving $P$. Within $\chi$PT, several unsuccessful attempts have been made to find a simultaneous description of the leading order (LO) and higher-order corrections to $S$- and $P$-waves, ever since the first study of H. Bijnens and colleagues [1]. In that work, the NLO contribution consisted only of the non-analytic terms $M_K^2 \log\left(\frac{M_K^2}{\mu^2}\right)$, argued to be in principle larger than the analytic $M_K^2$ terms. It was followed a few years later by E. Jenkins [2], who produced a study based on the same non-relativistic premises – Heavy-Baryon (HB) $\chi$PT – with the important addition of decuplet baryons as explicit degrees of freedom. These assumptions produced a convergent chiral series describing the $S$-waves, but failed on the $P$-waves, a result which discouraged any attempt for a simultaneous fit to both partial waves. The degree of complexity of this problem also comes from the lack of data points needed to produce a reasonable fit: there is a limited number of decay variables, while several unknown low-energy constants (LECs) contribute to the analytic $O(M_K^2)$ term, with possible contributions from different orders of the LO and NLO Lagrangians. Additionally, isospin symmetry provides additional constraints, reducing further the number of available independent decay variables. The study of B. Borasoy and B. Holstein [3] and its continuation [4], proposes to approximate these parameters by including tree-level diagrams containing a resonance exchange. This constitutes yet another approach compared to the existing literature, due to the exclusion of the decuplet fields, with the final combined fit [4] limited to the tree-level contributions.
The recent update of the $\Lambda$ hyperon decay parameters from the BESIII collaboration [5, 6] prompted the recalculation of such decay amplitudes and their 1-loop corrections (see e.g. [7] for an overview of hyperon decay measurements). We do so in relativistic $\chi$PT, employing the Extended-On-Mass-Shell (EOMS) renormalization scheme [8, 9], in order to preserve the manifest covariance and analytic properties of the theory. To take the unknown weak-transition LECs into account, we follow Ref.[4] and approximate the NLO counterterms via the tree-level contribution from the $1/2^-$ and $1/2^+$ octets as resonant degrees of freedom. On the suggestion of Ref. [2], we include the decuplet baryons as explicit degrees of freedom exchanged in loop diagrams, as they are expected to have a significant impact.

## 2. Hyperon nonleptonic decays

The amplitude describing hyperon nonleptonic decays is defined as

$$i\mathcal{M}(B_i \to B_f\pi) = G_F m_{\pi^+}^2 \bar{u}_{B_f}\{A^{(S)} + A^{(P)}\gamma_5\}u_{B_i} \tag{1}$$

where the contributions to the transition amplitude are related to the dimensionless partial waves $s$ and $p$ via

$$s = A^{(S)} \quad ; \quad p = A^{(P)}\frac{|\vec{\mathbf{p}}_f|}{E_f + m_f} \,. \tag{2}$$



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The $s$ and $p$ amplitudes are the object of our theoretical calculations, and can be related to experimental data via the following relations to the decay parameters:

$$\Gamma = G_F^2 m_{\pi^+}^4 \frac{|\vec{\mathbf{p}}_f|}{4\pi m_i} \left(E_f + m_f\right) \left(|s|^2 + |p|^2\right) , \tag{3}$$

$$\alpha = \frac{2\Re(s^* p)}{|s|^2 + |s|^2} , \tag{4}$$

with $m_f$, $|\vec{\mathbf{p}}_f|$, $E_f$ the final-state mass, momentum and energy, respectively, in the rest frame of the mother baryon of mass $m_i$.

The "experimental" values for the partial-wave amplitudes $L_{\text{expt}}$ are extracted from the decay parameters using the relations in Eq.(3-4). For the purpose of a HB$\chi$PT analysis, one can assume the amplitudes assuming them to be real. We take into account final-state interaction effects by determining complex-valued amplitudes, parametrized as

$$L = \sum_j L_j \exp(i\delta_{2I}), \quad \text{with} j \in \{2\Delta I, 2I\} . \tag{5}$$

We used the most recent results on the hyperon decay parameters, and the final-state interaction phases, as they are listed in [7]. The real part of these complex-valued amplitudes, listed in Table 1 with their uncertainties, constitute the starting point of our fits.

| | $\Re(s_{\text{complex}})$ | $s_{\text{real}}$ | $s_{\text{Jenkins}}$ | $\Re(p_{\text{complex}})$ | $p_{\text{real}}$ | $p_{\text{Jenkins}}$ |
|---|---|---|---|---|---|---|
| $\Sigma^+ \to n\pi^+$ | 0.062(07) | 0.06 | 0.06 | 1.796(09) | 1.81 | 1.81 |
| $\Sigma^+ \to p\pi^0$ | −1.368(06) | −1.38 | −1.43 | 1.245(07) | 1.24 | 1.17 |
| $\Sigma^- \to n\pi^-$ | 1.848(07) | 1.88 | 1.88 | −0.064(08) | −0.06 | −0.06 |
| $\Lambda \to p\pi^-$ | 1.363(07) | 1.38 | 1.42 | 0.634(05) | 0.63 | 0.52 |
| $\Lambda \to n\pi^0$ | −1.023(10) | −1.03 | −1.04 | −0.419(13) | −0.41 | −0.39 |
| $\Xi^- \to \Lambda\pi^-$ | −1.994(09) | −1.99 | −1.98 | 0.393(05) | 0.39 | 0.48 |
| $\Xi^0 \to \Lambda\pi^0$ | 1.523(24) | 1.52 | 1.52 | −0.271(08) | −0.27 | −0.33 |

**Table 1:** Final estimate of "experimental" values for $s$-, $p$-waves, extracted from the most recent values of $\Gamma$, $\alpha$, final-state interaction phase shifts $\delta$, compared to the values extracted on the assumption of real amplitudes, and estimate from Ref. [2].

## 3. Relativistic Baryon $\chi$PT and resonance saturation

We compute the two contributions to the decay amplitudes up to $O(M_K^2)$ in relativistic $\chi$PT, where $M_K$ is the relevant scale for the hyperon $\Delta S = 1$ transitions. This study constitutes the first attempt at such calculations in a relativistic framework, and in particular using the EOMS scheme [8]. This scheme consists in the direct subtraction of the $\widetilde{\text{MS}}$ UV divergences and of the power-counting-breaking (PCB) terms caused by the presence of baryon masses in the loops. By doing so, manifest Lorentz invariance is preserved, and the analytic structure of the loops remains unaltered. The same cannot be said in the case of HB$\chi$PT, due to the effects of the $1/m_B$ expansion of the Lagrangian [10]. We also note that PCB terms are identified via an expansion in powers of $M_K$, as customary [11, 12], leaving the decuplet-octet baryon mass difference as it is. Finally, we choose an appropriate renormalization scale $\mu = 1$ GeV to carry out the standard dimensional regularization of the loops. As mentioned in 1, the $O(M_K^2)$ counterterms from the weak NLO Lagrangian are not included explicitly to avoid introducing additional unknown parameters to be fitted against the already few data points. Instead, they are approximated by tree-level diagrams including a resonance exchange.

We start from the following effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\phi + \mathcal{L}_{\phi BT} + \mathcal{L}_{\phi XB} + \mathcal{L}^{\rm W}_{\phi BT} + \mathcal{L}^{W}_{XB}\,, \tag{6}$$

including the strong meson-baryon ($\phi B$) interaction and the 'weak' baryon transitions, as well as the resonance ($X$) exchange. The mesonic LO Lagrangian $\mathcal{L}_\phi$ is standard and included in [13], while the LO relativistic chiral Lagrangian for octet and decuplet baryons is taken as the relativistic limit of Eq. (2.1) in [2]. The corresponding weak transition Lagrangian for baryons reads as:

$$\mathcal{L}^{W(1)}_{\phi BT} = h_D \;{\rm tr}\; \bar{B}\{h_+, B\} + h_F \;{\rm tr}\; \bar{B}[h_+, B] + h_C \;{\rm tr}\; \bar{T}^\mu h_+ T_\mu \quad , \tag{7}$$

where $h_+ = \xi^\dagger (h + h^\dagger)\xi$ selects the strangeness-changing transition acting on the exponential $\xi$ of the meson field matrix. Examples of the relevant diagrams stemming from Eq.(6) are depicted in Fig.1.

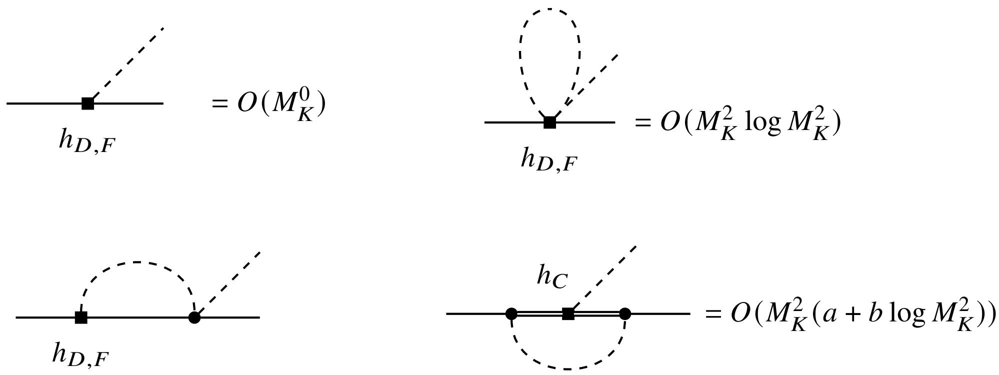



**Figure 1:** Examples of $s$-wave diagrams and their power-counting order. The box represents the weak transition mediated by the $h_{D,F,C}$ LECs.

On the suggestion of Ref. [4], we present the $1/2^\pm$ resonance multiplets via the weak effective LO Lagrangians

$$\mathcal{L}^{(1)}_{\phi RB} \propto i w_d \left[{\rm tr}(\bar{R}\{\xi^\dagger h \xi, B\}) - {\rm tr}(\bar{B}\{\xi^\dagger h\xi, R\})\right] + i w_f \left[{\rm tr}(\bar{R}[\xi^\dagger h\xi, B]) - {\rm tr}(\bar{B}[\xi^\dagger h\xi, R])\right] \tag{8}$$

$$\mathcal{L}^{W}_{\phi B^* B} \propto d^* \left[{\rm tr}(\bar{B}^*\{\xi^\dagger h\xi, B\}) + {\rm tr}(\bar{B}\{\xi^\dagger h\xi, B^*\})\right] + f^* \left[{\rm tr}(\bar{B}^*[\xi^\dagger h\xi, B]) + {\rm tr}(\bar{B}[\xi^\dagger h\xi, B^*])\right] \tag{9}$$

where $R$ stands for the $\frac{1}{2}^-$ multiplet, and $B^*$ denotes the Roper-like $\frac{1}{2}^+$ states, following the notation in Ref. [4]. A representative of the tree-level diagrams can be found in Fig.2. We note that the approach of Ref. [4] neglects possibly large contributions from the denominators of the intermediate fermion propagator in Fig.2. Degenerating the resonance masses to one representative mass per multiplet may offset very small mass differences between the resonances and the on-shell states. To avoid this, we evaluate each diagram at the mass of the exchanged resonance.

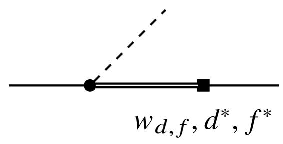
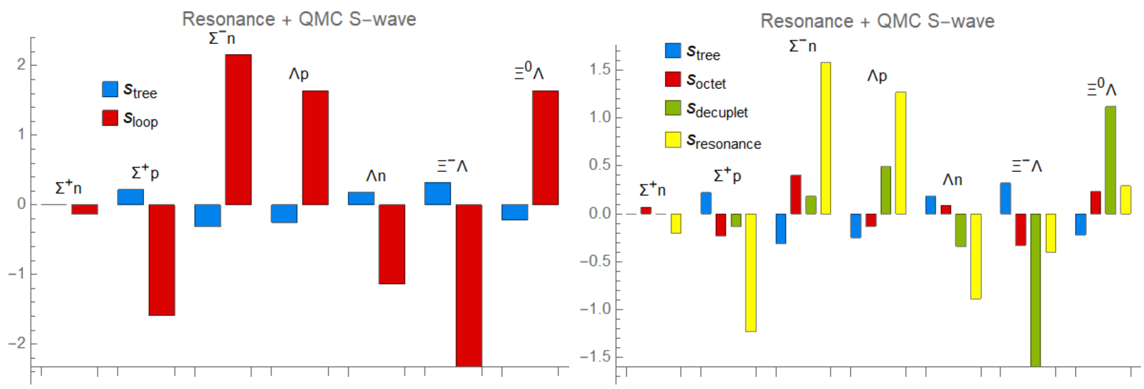


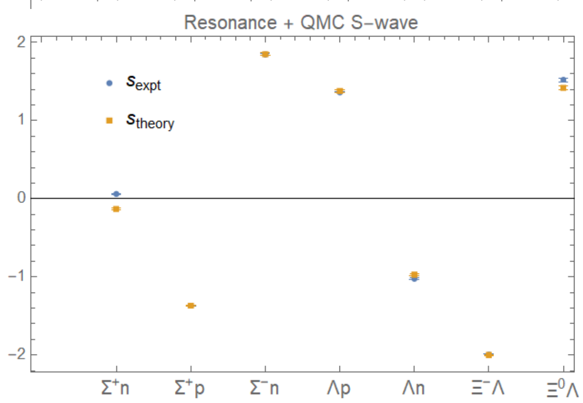

**Figure 2:** Resonance-exchange tree-level diagram, where the circle (square) denotes the strong (weak) interaction with the on-shell baryon states.

## 4. Fit to data

The first fit attempt is carried out including "true" 1-loop corrections to the theoretical amplitudes, i.e. neglecting resonances and the weak NLO Lagrangian counterterms. The result fails to describe simultaneously $s$- and $p$-waves. We do not include it in this report, as this result was expected based on the magnitude of the resonance contribution resulting from the tree-level fit from Ref. [4].

Resonances intermediate states are included first in separate fits to $s$ and $p$, detailed in Fig. 3 and 4, respectively. In all previous studies, the $s$-waves were described by a convergent chiral series, with the NLO contributions reasonably smaller than the tree-level terms. This characteristic is lost in the fit with resonances: nonetheless, good agreement with the experimental data is achieved, especially after enforcing the chiral Quark Model prediction $h_D = -\frac{1}{3}h_F$ [14].

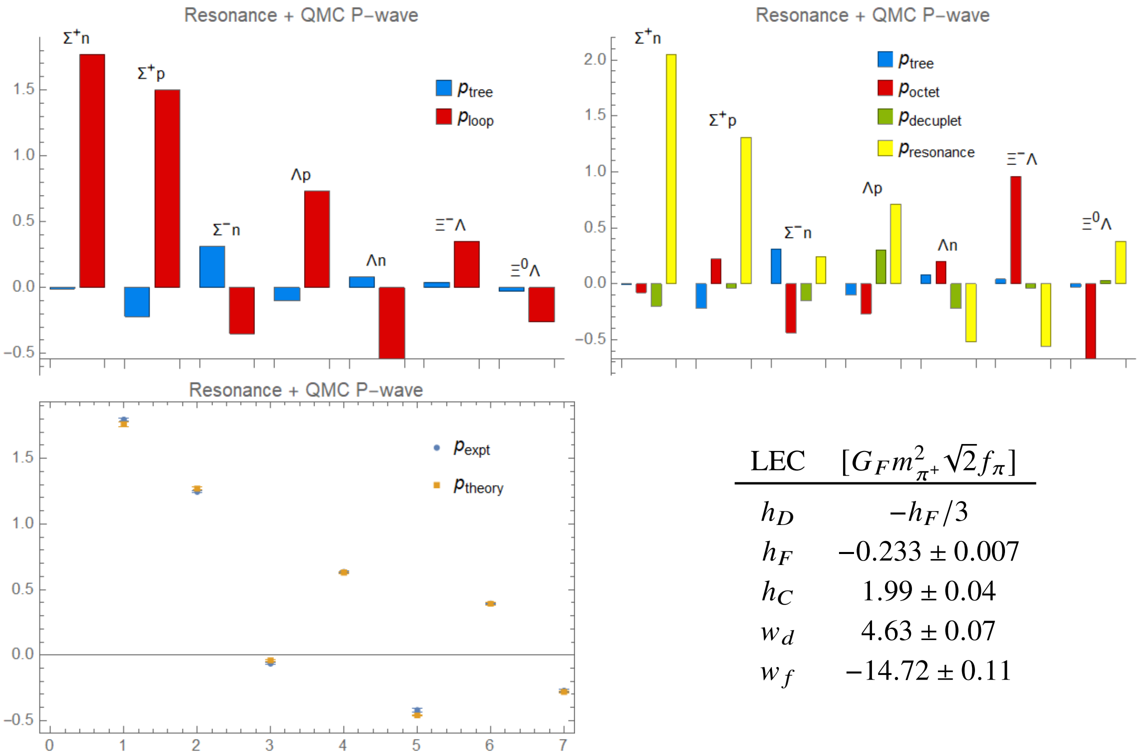



| LEC | $[G_F m_{\pi^+}^2 \sqrt{2} f_\pi]$ |
|---|---|
| $h_D$ | $-h_F/3$ |
| $h_F$ | $-0.233 \pm 0.007$ |
| $h_C$ | $1.99 \pm 0.04$ |
| $w_d$ | $4.63 \pm 0.07$ |
| $w_f$ | $-14.72 \pm 0.11$ |

**Figure 3:** Results from a separate fit to $s$-waves, including details on the relative sizes of the different contributions to the total 1-loop correction.

The resonance fit to $p$-waves follows a similar development, and their good agreement to the experimental data constitutes a first in the study of these amplitudes, despite the lack of chiral convergence. As with $s$-waves, the 1-loop terms are decidedly bigger than the LO, with the resonance contributions carrying most of the total "correction" size (Fig. 4).




| LEC | $[G_F m_{\pi^+}^2 \sqrt{2} f_\pi]$ |
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**Figure 4:** Results from a separate fit to $p$-waves, on the same line of Fig.3.

Finally, a simultaneous fit is performed combining $s$- and $p$-waves, and the result is dramatically different between including resonances or not, detailed in Fig. 5. We note that the good agreement to the experimental data exhibited in the previous fits by $p$-waves is not foiled, while the $s$-waves are predicted to lay slightly farther away from the data points, compared to the $s$-wave-only fit. However, we also note that the estimation of the theoretical uncertainty needs to be updated after careful consideration of the relative sizes of the different contributions.

## 5. Conclusions and Outlook

This project studies hyperon nonleptonic decays in relativistic $\chi$PT, applying the EOMS renormalization scheme, and builds on several previous efforts to approach the topic. With the notable exception of Ref.[4] – which considered only tree-level resonance-exchange diagrams – all previous publications computed the partial-wave amplitudes in the non-relativistic framework of HB$\chi$PT, differing in the considered truncation order, or in the manner of inclusion of the counterterms from the weak NLO Lagrangian. As a first cross-check, we established a "non-relativistic" limit from our amplitudes to the ones in Refs. [2, 3, 15]. In particular, this solved the discrepancy regarding the three-meson-exchange vertex (mediated by the LEC $h_\pi$), between Refs. [2] and [15], in favor of

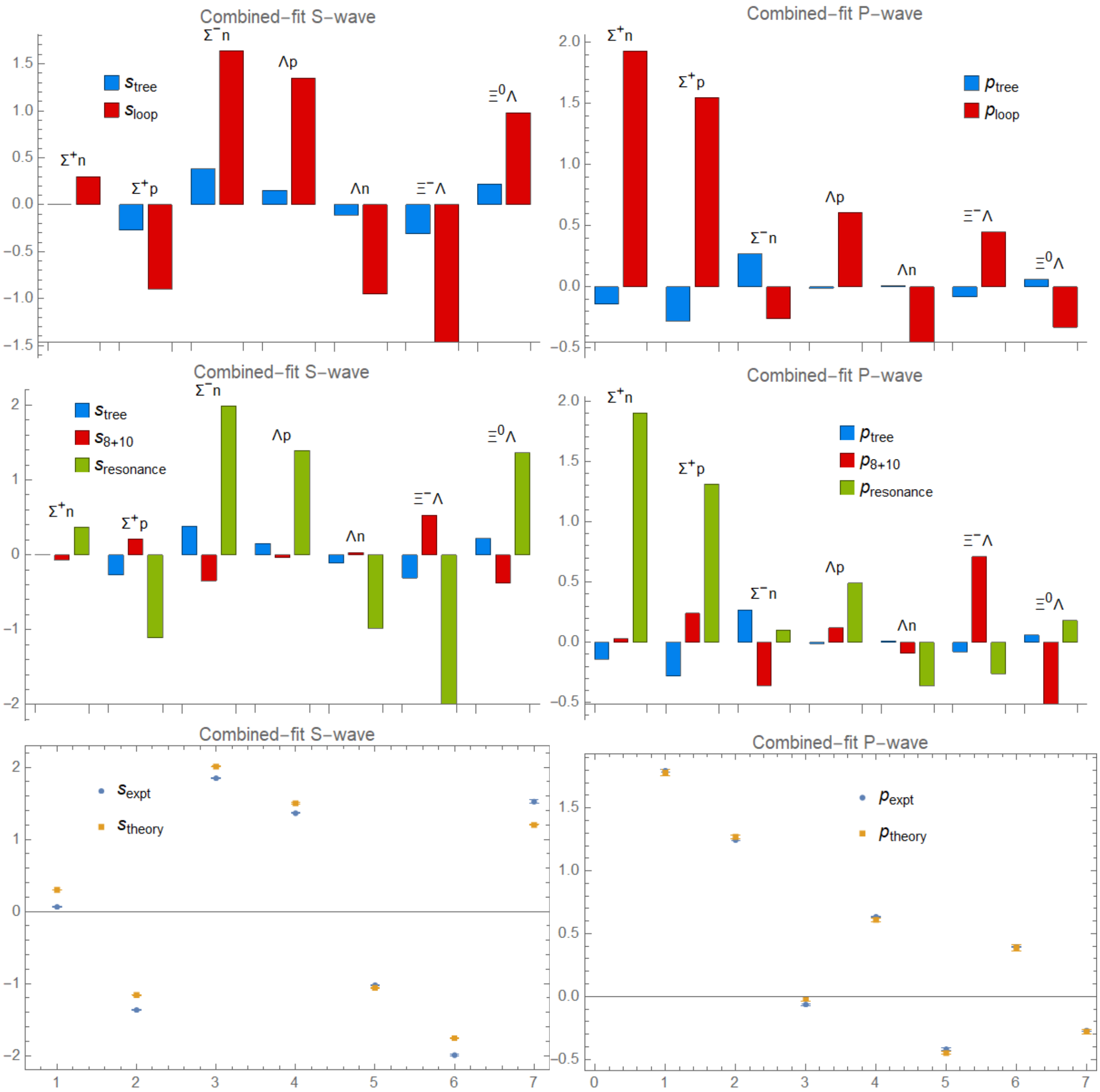


**Figure 5:** Results from the combined fit of the partial waves including resonance saturation, divided into different contributions to the 1-loop corrections. On the bottom line, a comparison of the theoretical amplitudes to the experimental data points.

the latter. This contribution is ultimately left out due to its negligible effect on the final relativistic amplitudes.

The main result of this study consists in the relativistic computation of the partial waves, and in a preliminary fit to the latest experimental data points, to produce a first estimate of the size and central value of the LECs from the weak LO Lagrangian ($h_D$, $h_F$, $h_D$). As remarked in Ref. [2], the decuplet spin-3/2 fields have a sizeable contribution also to the relativistic amplitudes, second only to the relative weight of the resonance-exchange diagrams – as foreseen in Ref. [4]. This results in the simultaneous inclusion of several new aspects, compared to the previous studies. For this reason, and for the limited availability of experimental data points, we do not strive for a high-precision determination of the weak LECs. Nonetheless, we note that there are two main sources for the theoretical error one should take into account for a more realistic estimates of the central values of these LECs. On one hand, there is the "truncation error", i.e. the relative size of the first chiral order left out from our theoretical calculation. On the other hand, a quick study on the error propagation of the partial-wave amplitudes reveals the heavier weight of the resonance contributions, as exemplified by the middle plots in Fig. 5. The finished study will include both aspects, to present a more realistic $\chi^2$ study and keeping track of the actual effects of the resonance exchange and the chiral truncation. Another source of uncertainty lies in the less well-established resonant states, e.g. the $\Lambda(1405)$ [16, 17]. We have included its contribution as a 3-quark state: to consider a (hypothetical) hadronic molecule structure would require careful consideration on the included $pK$ 1-loop diagram, to avoid double-counting.

It is therefore appropriate to say that the goal of this project is to pave the way by establishing the relative weight of the different previous approaches, applied in the most recent theoretical framework. Possible extensions would entail the careful determination of the weak LECs,, and by extension, of the partial-wave amplitudes, by considering 2-loop diagrams, and/or including resonances as degrees of freedom in the LO Lagrangian.

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