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The Unitarity-Limit Expansion for Two Nucleons with Perturbative Pions: Digest and Ideas

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Theorists love nontrivial fixed points. In the Unitarity Limit, the NN S-wave binding energies are zero, the scattering lengths infinite, Physics is universal, *i.e.* insensitive to details of the interactions, and observables display richer symmetries, namely invariance under both scaling and Wigner's combined SU(4) transformation of spin and isospin. In "Pionless" EFT, both are explicitly but weakly broken and hence perturbative in the Unitarity Window (phase shifts $45^\circ \lesssim \delta(k) \lesssim 135^\circ$, *i.e.* momenta $k \approx m_\pi$). This Unitarity Expansion provides strong hints that Nuclear Physics resides indeed in a sweet spot: bound weakly enough to be insensitive to the details of the nuclear interaction; and therefore interacting strongly enough that the NN scattering lengths are perturbatively close to the Unitarity Limit. In this paradigm change, NN details are less important than NNN interactions to explain the complexity and patterns of the nuclear chart.

This presentation is a digest of the first quantitative exploration of corrections to this picture when pions are included [1] (see there for a more comprehensive list of references). Since the pion mass and decay constant introduce dimensionful scales in the NN system, they explicitly break the symmetries of the Unitarity fixed point. In χEFT, these symmetries must therefore be hidden and instead be classified as emergent. This text focuses on the χEFT variant with Perturbative ("KSW") Pions at next-to-next-to leading order (N^{2}LO). In the ^{1}S$_0$ channel up to cm momenta $\lesssim 300$ MeV, the results are clearly converged order-by-order and agree very well with phase shift analyses. Apparent large discrepancies in the ^{3}S$_1$ channel even at $k \approx 100$ MeV are remedied by taking only the central part of the pion's N^{2}LO contribution. In contradistinction to the tensor part, it does not mix the different Wigner-SU(4) multiplets and hence is identical in ^{1}S$_0$ and ^{3}S$_1$. With this formulation, pionic effects are small in both channels even for $k \gtrsim m_\pi$, *i.e.* where both Unitarity and pion effects are expected to be relevant. This leads to the *Hypothesis* that both scale invariance and Wigner-SU(4) symmetry in the Unitarity Expansion show *persistence*, *i.e.* the footprint of both combined dominates even for $k \gtrsim m_\pi$ and is more relevant than chiral symmetry, so that the tensor/Wigner-SU(4) symmetry-breaking part of OPE is suppressed and does not enter before N^{3}LO. Included are also ideas about underlying mechanisms and LO results of χEFT with Nonperturbative Pions in the expansion about Unitarity.

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PoS(CD2024)085

This contribution remembers Thomas C. Mehen, who died unexpectedly a few months after this conference, in the waning days of 2024. His postdoctoral work on NN *systems with perturbative pions [2, 3] and with Wigner-*SU(4) *symmetry [4] are fundamental to this discussion, and his work on relations to entanglement [5, 6] is apt to provide further clues.*

1. Introduction

Effective Field Theories (EFTs) of Nuclear Physics do not offer an explanation why the observables of few-nucleon systems are dominated by anomalous scales: The $^1S_0$ and $^3S_1$ NN channels are very weakly bound, with associated momentum scales of −8 MeV and +45 MeV, respectively, set by the inverse scattering lengths. These scales are much smaller than the pion mass $m_\pi \approx 140$ MeV, the natural low-momentum QCD scale of Nuclear Physics. Rather than explain, EFTs simply impose an ordering scheme whose leading-order (LO) is found by iterating a NN interaction infinitely often to accommodate such anomalously shallow virtual and real bound states. Perfectly valid and consistent versions of χEFT exist in which LO is perturbative and the binding energy of light nuclei is set by the scale $\frac{m_\pi^2}{M}$, with M the nucleon mass. But these are not realised in Nature. A preference for highly-symmetric states could hold the key for understanding this.

So, consider the two-nucleon scattering amplitude at relative momentum k in the cm frame,

$$A(k) = \frac{4\pi}{M}\,\frac{1}{\mathrm{k}\cot\delta(k) - \mathrm{i}k} \quad , \tag{1.1}$$

where "$-\mathrm{i}k$" ensures Unitarity, *i.e.* probability conservation, while $\mathrm{k}\cot\delta(k)$ parametrises the part which encodes all information on the interactions. Depending on their relative sizes, any self-respecting theorist is tempted to set up two fundamentally different expansions

Born Approximation in the Born Corridor: The phase shift is "small"$|\delta(k)| \lesssim 45°$, *i.e.* $|\mathrm{k}\cot\delta| \gtrsim |\mathrm{i}k|$, so that contributions from interactions are small and treated in perturbation,

$$A(k)\Big|_{\mathrm{Born}} = \frac{4\pi}{M}\,\frac{1}{\mathrm{k}\cot\delta(k)}\left[1 + \frac{\mathrm{i}}{\cot\delta(k)} + O(\cot^{-2}\delta)\right] \quad , \tag{1.2}$$

with the leading piece just given by the potential V: $\dfrac{1}{\mathrm{k}\cot\delta(k)} \propto \langle \mathrm{out}|V|\mathrm{in}\rangle$. In this expansion, interaction details are crucial to capture phase shifts well. The amplitude has no pole (bound state).

Unitarity Expansion in the Unitarity Window: This is the focus of this presentation. The phase shift is "large", $45° \lesssim |\delta(k)| \lesssim 135°$, *i.e.* $|\mathrm{k}\cot\delta| \lesssim |\mathrm{i}k|$ about the Unitarity Point $\cot\delta = 0$:

$$A(k)\Big|_{\mathrm{Uni}} = \frac{4\pi}{M}\,\frac{1}{-\mathrm{i}k}\left[1 + \frac{\cot\delta(k)}{\mathrm{i}} + \frac{\cot^2\delta(k)}{\mathrm{i}^2} + O(\cot^3\delta)\right] \quad . \tag{1.3}$$

In the Effective-Range Expansion, $\mathrm{k}\cot\delta = -\frac{1}{a} + \frac{r}{2}k^2 + \dots$ with scattering length a and effective range r, this converges for an expansion parameter $Q \sim \frac{1}{ak} \sim \frac{rk}{2} \ll 1$. Now, contributions from interactions are so strong that their details do not matter as much as that they are simply very strong – so strong indeed that probability conservation limits their impact on observables and Unitarity dominates. In the language of Information Theory, it is critical that a is large, but its value is less important and rather on par with other subdominant information (like from r). An anomalously shallow bound state emerges naturally at LO since the amplitude's pole is at zero.

PoS(CD2024)085

Since the Unitarity Limit has no intrinsic scale at LO, all dimensionless NN observables are zero or infinite, while all dimensionful quantities (like cross sections) are homogeneous functions of k whose dimensionless coefficients do not depend on the interaction. Since details of the interactions are washed out and can be treated in perturbation, differences between systems at Unitarity can then only come from different symmetry properties of the Unitarity Point itself but are universal otherwise. Therefore, Unitarity and Universality are closely related. This Unitarity Expansion has been proposed as key to the emergence of simple, unifying patterns in complex systems like the nuclear chart; see *e.g.* [7] and [8, 9] for reviews as well as [10].

How relevant is the Unitarity Window in the NN system? Figure 1 shows that only for two channels are phase shifts clearly inside it, and only at momenta 30 MeV $\lesssim k \lesssim$ 300 MeV (lab energies 2 MeV $\lesssim E_{\text{lab}} \lesssim$ 200 MeV) which are relevant for low-energy properties of nuclear systems: $^1S_0$ and $^3S_1$ (framed in blue). Therefore, the Unitarity Expansion is *only applicable* in these two channels, and in none of the higher partial waves. Thus, this presentation does not concern itself with the other partial waves – not because it wants to avoid describing them, but rather because the Unitarity Expansion is simply inapplicable there, meaning the present investigation into two-nucleon Unitarity is agnostic about them. Other systems can be a different matter.

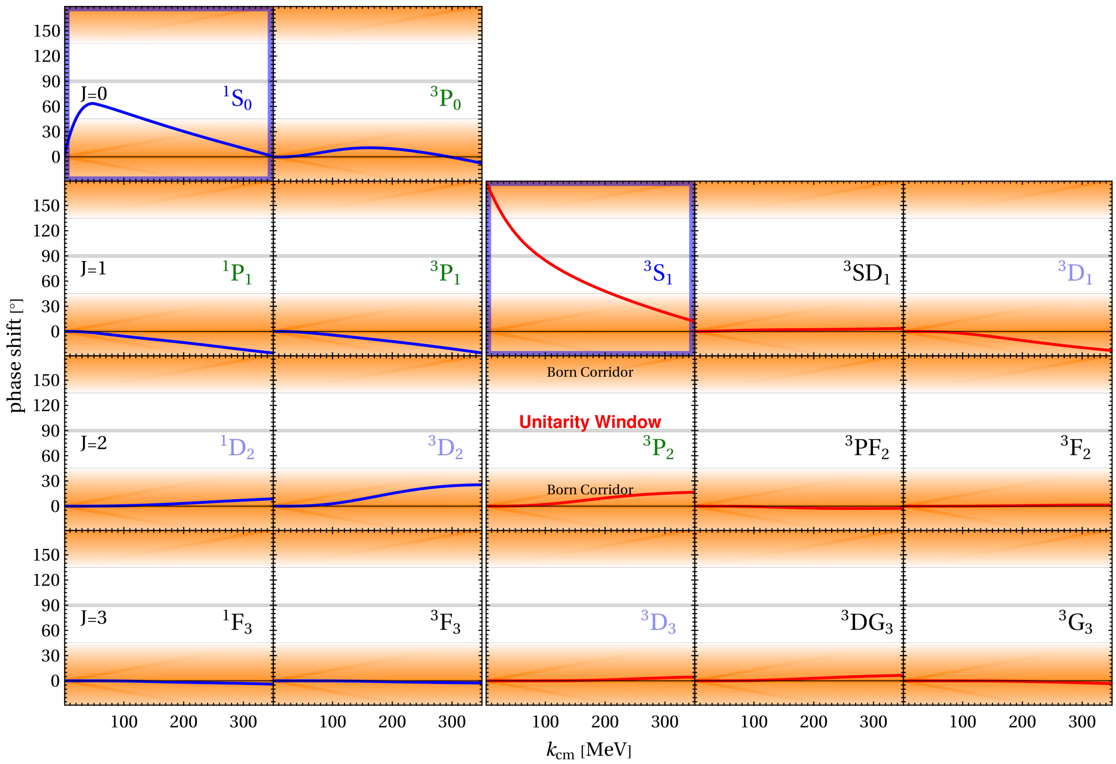

Figure 1: (Colour on-line) Born Corridor (shaded) and Unitarity Window (white) of NN phase shifts for the $J \leq 3$ channels in the Nijmegen PWA [11] (Stapp-Ypsilanti-Metropolis (SYM/"bar") parametrisation).

2. The Unitarity Expansion with Perturbative Pions

How is the Unitarity Window accommodated in χEFT? Let us investigate the transition between "pionless" and "pionic" EFT by employing χEFT "with Perturbative/KSW Pions", proposed by

Kaplan, Savage and Wise [12, 13]. This is the only χEFT which is generally accepted to be self-consistent and renormalisable order by order, with a well-understood power counting [14]. Its dimensionless expansion parameter Q is

$$Q = \frac{k, m_\pi}{\overline{\Lambda}_{\text{NN}}} \;\text{, with } \overline{\Lambda}_{\text{NN}} = \frac{16\pi f_\pi^2}{g_A^2 M} \approx 300 \text{ MeV} \tag{2.1}$$

the scale at which iterations of one-pion exchange (OPE) are not suppressed any more. Its contributions up to and including N^{2}LO are summarised in fig. 2. Traditionally, the scattering lengths a are set to the physical values already at LO. The analytic results of such amplitudes were derived by Rupak and Shoresh in the ^{1}S$_0$ channel [15], and by Fleming, Mehen and Stewart (FMS) in ^{3}SD$_1$ [2, 3]. The Unitarity Expansion simply demotes the contribution from the scattering lengths, so that LO contains no scale but the scattering momentum and knows only S-wave interactions; NLO consists of contributions from the scattering length, effective range r and non-iterated OPE; and N^{2}LO adds once-iterated OPE, plus parameters which cancel its contribution to a and r, as these are already reproduced at NLO. It thus adds an expansion parameter which can for practical purposes be taken to be numerically similar to that of eq. (2.1):

$$Q = \frac{1}{(k, m_\pi)a} \sim \frac{k, m_\pi}{\overline{\Lambda}_{\text{NN}}} \ll 1 \quad \text{in } \chi\text{EFT with KSW Pions about Unitarity.} \tag{2.2}$$

The Unitarity Expansion becomes hence inapplicable both as $k \searrow 0$ and $k \nearrow \overline{\Lambda}_{\text{NN}}$. This counting leads to quite a few simplifications. Analytic amplitudes are given in ref. [1], plus further details on their analytic structure, on parameter choices, on Bayesian and on-Bayesian estimates of theory uncertainties and on the extraction of phase shifts from amplitudes. The following compares results to both the phenomenological phase shifts and to the Unitarity Expansion of the EFT "Without Pions" (EFT($\not\pi$)) which in this case reduces to the effective-range expansion about $\frac{1}{a} = 0$.

The ^{1}S$_0$ phase shift, fig. 3, converges well order-by-order even as $k \to \overline{\Lambda}_{\text{NN}}$. Even well outside the Unitarity Window, N^{2}LO and PWA differ only as much as NLO and N^{2}LO – and still less than LO and NLO. That indicates good order-by-order convergence. Remarkably, explicit pionic degrees of freedom appear at $k \gtrsim 300$ MeV to have a minuscule impact since N^{2}LO is nearly indistinguishable from the N^{2}LO EFT($\not\pi$) result. In this channel, we have a self-consistent EFT with pions in all of the Unitarity Window, with a breakdown scale of about 300 MeV $\approx \overline{\Lambda}_{\text{NN}}$.

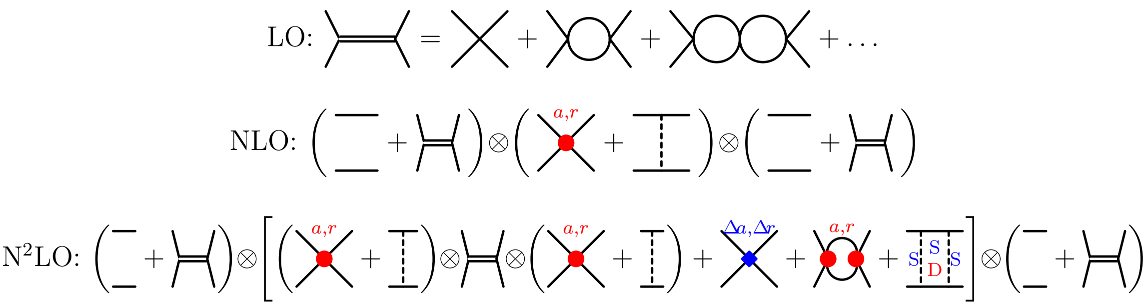

Figure 2: (Colour on-line) χEFT with Perturbative Pions in the Unitarity Expansion at LO (top); NLO (middle) with CTs (red circle) fixed to scattering length a and effective range r; N^{2}LO with CTs (blue diamonds) so that a and r do not change from the respective NLO values. The last term in square brackets at N^{2}LO is once-iterated OPE, with intermediate orbital angular momentum as indicated.

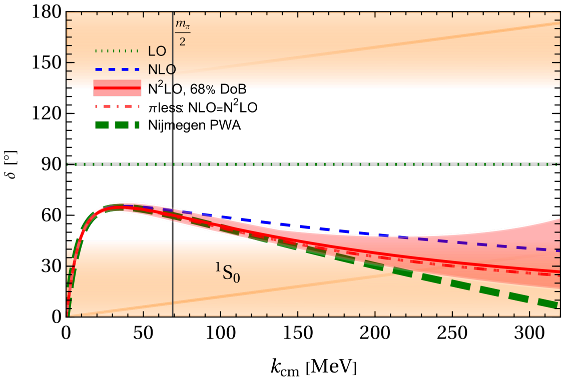

Figure 3: (Colour on-line) $^1S_0$ phase shift compared to the Nijmegen PWA [11] (thick green dashed). Green dotted: LO; blue dashed: NLO; red solid: N^{2}LO with 68% degree-of-belief (DoB) interval (Bayesian truncation uncertainty); red dash-dotted: NLO=N^{2}LO in EFT($\not\pi$). Shaded: "Born Corridors" of fig. 1.

In contradistinction, the result for the $^3S_1$ channel is catastrophic, as FMS already noticed; see left of fig. 4. While NLO looks reasonable by eye, N^{2}LO deviates dramatically from the PWA just above the OPE branch-point scale, $\frac{m_\pi}{2}$. This is the more puzzling as EFT($\not\pi$) agrees well with the PWA even at $k \gtrsim 100$ MeV. With pions, the deviation between NLO and N^{2}LO becomes around $k \approx m_\pi$ as large as the difference between LO and NLO, and larger than the deviation from the PWA. The breakdown is hardly gradual but sudden, with no hint at NLO of the unnaturally large N^{2}LO curvature around 100 MeV. This is the more concerning as phase shifts are there well inside the Unitarity Window. Pions at N^{2}LO appear to have an outsized and wrong impact for $k \gtrsim 100$ MeV, and most of the Unitarity Window lies outside the radius of convergence. That is unsatisfactory.

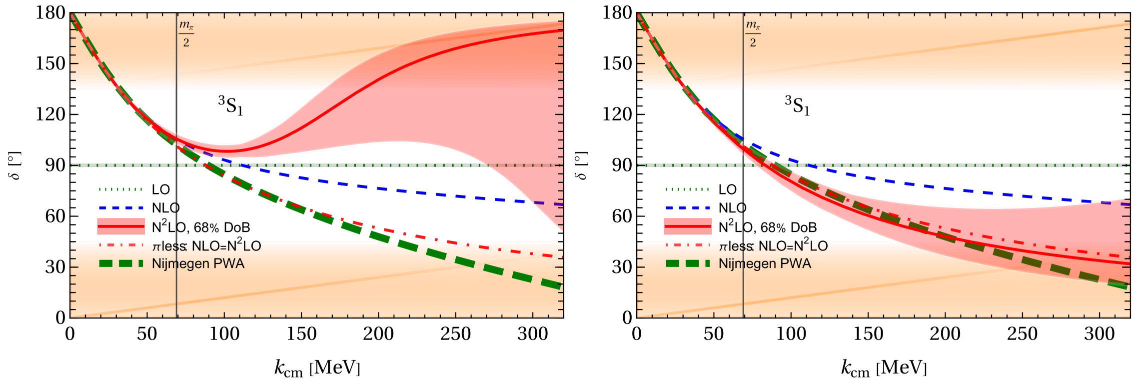

Figure 4: (Colour on-line) Phase shift in the $^3S_1$ channel compared to the Nijmegen PWA [11]. Left: full N^{2}LO amplitude. Right: Wigner-SU(4) symmetric part only. Details as in fig. 3.

What is the origin of the stark discrepancy between the $^1S_0$ and $^3S_1$ channels in both convergence and reasonable range of applicability? Recall that LO is independent of the scattering length, *i.e.* both amplitudes are identical and hence invariant under Wigner's combined-SU(4) spin and isospin rotations [4, 16, 17]. With scale invariance, this symmetry emerges therefore naturally in the Unitarity Limit. The long-range part of OPE, however, appears to break both:

$$\begin{aligned} V_{\text{OPE}} &= -\frac{g_A^2}{12f_\pi^2}\,\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\left[\,(\vec{\sigma}_1\cdot\vec{\sigma}_2)+[3\,(\vec{\sigma}_1\cdot\vec{e}_q)\,(\vec{\sigma}_2\cdot\vec{e}_q)-(\vec{\sigma}_1\cdot\vec{\sigma}_2)]\right](\vec{\tau}_1\cdot\vec{\tau}_2) \\ &=: (\vec{\sigma}_1\cdot\vec{\sigma}_2)\,(\vec{\tau}_1\cdot\vec{\tau}_2)\,V_C+[3\,(\vec{\sigma}_1\cdot\vec{e}_q)\,(\vec{\sigma}_2\cdot\vec{e}_q)-(\vec{\sigma}_1\cdot\vec{\sigma}_2)]\,(\vec{\tau}_1\cdot\vec{\tau}_2)\,V_T\quad. \end{aligned} \tag{2.3}$$

As f_π and m_π carry mass dimensions, they explicitly break scale invariance. On top of that, the spin-isospin structure of the tensor part, V_T, induces S $\leftrightarrow$ D and D $\to$ D transitions. These are only possible in $^3\text{S}_1$, and not in $^1\text{S}_0$ where the tensor piece is of course identically zero. Such partial-wave mixing therefore lifts the Wigner-SU(4) symmetry between the S waves – in a possibly slight abuse of language, this can be called “Wigner-SU(4) symmetry breaking”. On the other hand, the central part, V_C, is manifestly Wigner-SU(4) symmetric, *i.e.* it contributes only in S $\to$ S transitions and is hence identical in the $^1\text{S}_0$ and $^3\text{S}_1$ channels. Therefore, only amplitudes without V_T are Wigner-invariant, while those with at least one V_T automatically break the symmetry. Consequently, it is not self-understood how χEFT's explicit pionic degrees of freedom can be reconciled with the symmetries of the Unitarity Expansion which they break rather strongly, and how one can thus extend the Unitarity Expansion in χEFT to the whole Unitarity Window, including $k \gtrsim m_\pi$.

With perturbative pions, no Wigner-SU(4) symmetry-breaking OPE contribution enters at NLO, simply because a single OPE is sandwiched between the LO pure-S wave amplitudes (which are Wigner-SU(4) symmetric), disallowing S $\leftrightarrow$ D mixing. The only Wigner-SU(4) breaking terms at NLO come from scattering lengths and effective ranges. They are explicit but weak and, as EFT($\not\pi$) shows, can well be captured in perturbation about the Unitarity Limit $\frac{1}{a}=0$.

At N^{2}LO, however, sandwiching once-iterated OPE between the LO $^3\text{S}_1$ waves does allow for an $^3\text{S}_1 \to {}^3\text{D}_1 \to {}^3\text{S}_1$ transition, *i.e.* D-wave propagation between each OPE (last term in square brackets of the bottom of fig. 2). This piece is of course absent in the $^1\text{S}_0$ channel. OPE breaks thus Wigner-SU(4) symmetry in S-waves only when it is once-iterated, namely at N^{2}LO, but not earlier.

This led to the idea to simply *impose* Wigner-SU(4) symmetry on the pion at N^{2}LO: eliminate the V_T part! The qualitative and quantitative improvement is obvious; see left graph of fig. 4. Similar to $^1\text{S}_0$, the phase shift converges now order-by-order even as $k \to \overline{\Lambda}_{\text{NN}}$ just outside the Unitarity Window. The difference of N^{2}LO and PWA is wholly within the Bayesian 68% DoB band, even smaller than in the $^1\text{S}_0$ channel and quite a bit smaller than the shift from NLO to N^{2}LO. With only small differences to EFT($\not\pi$), pionic degrees of freedom have again a minuscule impact even at $k \gtrsim m_\pi$. The empirical breakdown scale is around $k \gtrsim 250$ MeV $\approx \overline{\Lambda}_{\text{NN}}$, as expected.

Thus, a Wigner-SU(4) invariant χEFT appears well-suited to describe perturbative-pionic Physics in both S waves up to a common breakdown scale of at least $k \gtrsim 250$ MeV $\approx \overline{\Lambda}_{\text{NN}}$, plus good agreement with PWAs even at these high momenta. The (non-analytic parts of) pionic effects appear then very small, as comparison to EFT($\not\pi$) shows.

Reference [1] includes detailed discussions of Bayesian order-by-order convergence, of other mutually consistent semi-quantitative theory uncertainty estimates, and of empirical determinations of the breakdown scale. All confirm the outline above.

Such an approach solves another puzzle described by FMS. As shown in fig. 5, both $^3\text{SD}_1$ mixing and $^3\text{D}_1$ phase shift are terrible with the full N^{2}LO amplitudes. The mixing angle may seem to agree well with the PWA, but the N^{2}LO correction is nearly as large as NLO itself for

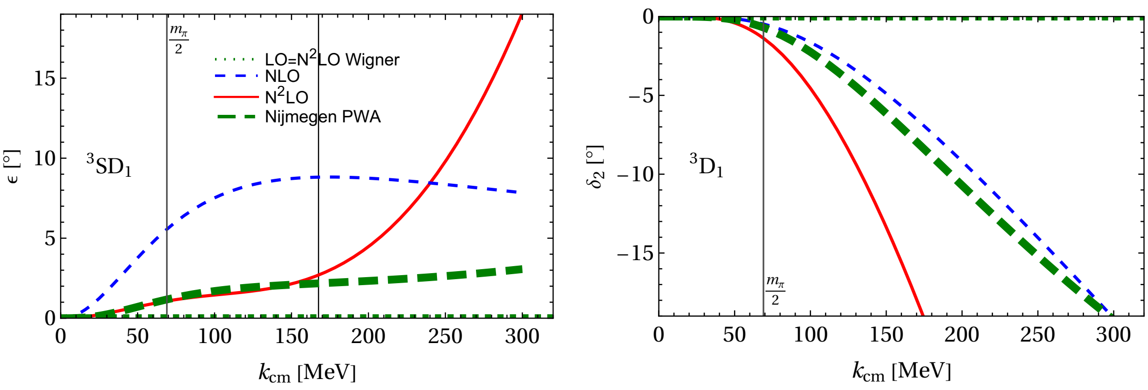

Figure 5: (Colour on-line) Mixing angle (left) and ^{3}D$_1$ phase shift (right) in the ^{3}SD$_1$ channel, compared to the Nijmegen PWA [11] (thick green dashed). Details as in fig. 3.

$k \gtrsim 50$ MeV, *i.e.* order-by-order convergence is extremely poor. It is even worse in ^{3}D$_1$. On the other hand, imposing Wigner-SU(4) symmetry eliminates V_T, so that the partial waves do not mix at all at N^{2}LO. Assuming that it only enters at N^{3}LO or higher, one can estimate the typical magnitude of phase shift and mixing angle at, say, $k \approx m_\pi$ as $Q^3 \approx \left(\frac{m_\pi}{\overline{\Lambda}_{\rm NN}}\right)^3$ of the LO phase shift $\delta_{\rm LO} = 90°$, namely about 10°. That is not inconsistent with the PWA values. Remember also that further contact interactions enter at N^{3}LO which contribute to SD mixing and DD transitions. These may help reproduce more-accurate partial wave mixing.

3. Ideas

This evidence is not inconsistent with a **Hypothesis**: The symmetries of the Unitarity Limit are broken weakly in Nuclear Physics. They show *persistence*, *i.e.* the footprint of both combined in observables at $k \gtrsim m_\pi$ is more relevant than chiral symmetry. In particular, the tensor/Wigner-SU(4) symmetry-breaking part of one-pion exchange in the NN ^{3}S$_1$ channel of χEFT with Perturbative (KSW) Pions is super-perturbative, *i.e.* suppressed and does not enter before N^{3}LO.

If so, then the picture of fig. 6 emerges around the renormalisation-group fixed point (FP) of Unitarity in two-nucleon systems: The FP is of course non-Gaußian (LO is nonperturbative), and its universality class is all theories with Wigner-SU(4) symmetry (and scale invariance since one is at a FP). In its immediate vicinity, the weak breaking of scaling and Wigner-SU(4) symmetry dominate, while chiral symmetry is subdominant. EFT($\not\pi$) is the EFT of χEFT at low momenta but has no explicit chiral symmetry. If chiral symmetry were thus important right around the FP, then χEFT would lie in a different universality class than EFT($\not\pi$). In this scenario, the Unitarity FP protects Wigner-SU(4) symmetry to be only weakly broken, while chiral symmetry in the few-nucleon sector has no such strong protection, simply because it is not a characteristic symmetry of the FP. It *can* therefore be broken substantially, leading to suppression of the tensor part V_T. Further away from the FP, chiral symmetry becomes as important as Wigner-SU(4) symmetry, and will eventually dominate for large enough momenta – as will the OPE's tensor piece. The scale $\overline{\Lambda}_{\rm NN}$ at which OPE becomes nonperturbative is an obvious candidate for this inversion. Since the zero- and one-nucleon sector of χEFT are perturbative, *i.e.* the projection of the FP onto these is Gaußian, this does not affect their chiral counting.

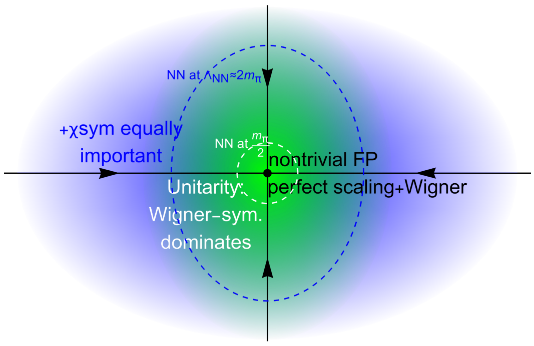

Figure 6: (Colour on-line) Sketch of an idea of symmetries around the Unitarity fixed point.

If this Hypothesis is to be more than a hunch, then a small, dimensionless (systematic) expansion parameter rooted in Wigner-SU(4) symmetry must be found to decide both *a priori* and semi-quantitatively at which order Wigner-SU(4) breaking pion contributions and correlated two-pion exchange enter. Ref. [1] discusses two candidates, both somewhat problematic: Large-N_C and entanglement. It also contains a shopping list of processes in which the Hypothesis can be tested, albeit the Goldstone mechanism converting global chiral symmetry into local, weakly interacting field excitations may eventually be too strong to overcome and lead to the Hypothesis' downfall. What is called for here is unusual since we are more used to promoting interactions in the chiral counting based on renormalisation-group arguments, than demoting them as in the Hypothesis.

Clearly, one should also explore how Wigner-SU(4) symmetry emerges inside the Unitarity

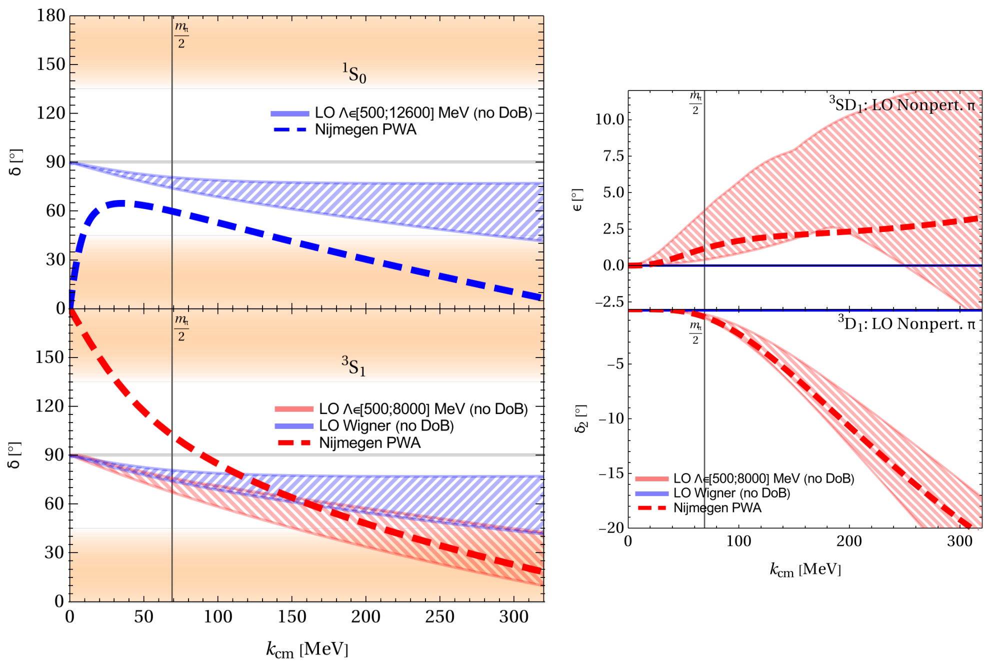

Figure 7: (Colour on-line) Phase shifts with nonperturbative pions at LO in the Unitarity Limit ($\frac{1}{a} = 0$), compared to the Nijmegen PWA [11]. Left: ^{1}S$_0$ (top) and ^{3}S$_1$ (bottom) channels. Right: ^{3}SD$_1$ mixing angle (top right) and ^{3}D$_1$ phase shift (bottom right), compared to the Nijmegen PWA [11]. Blue: Wigner-SU(4) symmetric part of OPE. Red: full OPE. The hatched areas are a rough estimate of higher-order corrections from varying the cutoff between 500 and 12,600 MeV, *not* Bayesian degree-of-belief intervals.

Window in χEFT with *non*-perturbative pions. LO results in fig. 7 indicate that the situation may not be hopeless. The ^{3}SD$_1$ results are again decomposed into full and Wigner-SU(4) symmetric pieces. The hatched corridors are not Bayesian degree-of-belief intervals but come from simply varying the momentum cutoff, so uncertainties are likely under-estimated. The one momentum-independent counter term per channel is determined by the Unitarity Limit, $\frac{1}{a} = 0$. Since OPE now enters already at LO, explicit scale breaking can already be seen, albeit it appears to be small. The predicted effective ranges are $[1 \dots 2]$ fm in the ^{1}S$_0$ channel and Wigner-SU(4) symmetric case, and $[1.5 \dots 2.5]$ fm in the full ^{3}SD$_1$ case. The PWA value of 2.77 fm for ^{1}S$_0$ appears superficially less compatible with that than the 1.85 fm for ^{3}SD$_1$. The ^{1}S$_0$ phase needs further study; the ^{3}S$_1$ phase shift might show a slight preference for the full version over the Wigner-SU(4) symmetric one. The full version is compatible with the phenomenological SD mixing and ^{3}D$_1$ phase, but especially the former comes with humongous uncertainties. More work is clearly needed, especially on more reliable theory uncertainties, and a study of higher orders is ongoing.

4. Concluding Questions

This contribution summarised and extemporated on a study in χEFT with Perturbative Pions [1]. Universality (insensitivity of amplitudes on details of interactions) and the associated Wigner-SU(4) symmetry emerge naturally in EFT($\not\pi$). Scattering lengths as fit parameters are simply set to infinity, and the dominant interactions in the Lagrangean are automatically scale and Wigner-SU(4) symmetric. But that is not manifest in χEFT. Rather, both are *a priori* hidden and only reconstructed as relevant by consulting data: Fitted to phase shifts, the coefficient of the Wigner-SU(4) symmetric two-nucleon interaction $C_S\ \mathbb{1}$ is much larger than that of $C_T\ \vec{\tau}_1 \cdot \vec{\tau}_2$ which splits the ^{1}S$_0$-^{3}S$_1$ Wigner-SU(4) multiplet. Therefore, both Wigner-SU(4) symmetry and Universality/Unitarity become *emergent* phenomena in χEFT– how is not yet clear.

Can imposing a preference for Unitarity as a highly symmetric state about which to expand provide a quantitative answer to fundamental questions: Why is fine-tuning preferred? How does varying m_π affect the conclusions? In-how-far is the Unitarity Limit compatible with the chiral limit? The Hypothesis and study of ref. [1] is merely a first proposal to merge two highly successful concepts of Nuclear Theory, namely the expansion about Unitarity and χEFT, for mutual benefit and a better understanding of why fine-tuning emerges in low-energy Nuclear Physics.

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