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Chiral three-nucleon force in many-nucleon systems

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The three-nucleon force (3NF) plays a crucial role in many-nucleon systems. Specifically, it enhances the spin-orbit (SO) splitting, a key phenomenon in the manifestation of shell structure. However, its detailed mechanism has not been fully understood. Recently, we pointed out that a specific component, called the rank-1 component, originating from the two-pion exchange term of the 3NF, significantly contributes to the SO splitting. In this paper, we present shell-model results obtained by implementing the 3NF derived from chiral effective field theory, and discuss how the rank-1 component enhances the SO splitting in p-shell nuclei.

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1. Introduction

Recent progress in chiral effective field theory (EFT) [1–3] has advanced the microscopic understanding of nuclear phenomena from the perspective of the nuclear force. In particular, our knowledge of the chiral three-nucleon force (3NF) [4–6] has been significantly refined. The 3NF not only plays a role in the fundamental properties of three-nucleon systems [7, 8], but also makes essential contributions to many-nucleon systems, as demonstrated in examples such as the realistic reproduction of the spectroscopic properties of ^{10}B [9], the explanation of the neutron dripline for oxygen isotopes [10], and its crucial role in the saturation of nuclear matter [11].

Additionally, it is known that the chiral 3NF enhances the spin-orbit (SO) splitting [10, 12–16]. However, the detailed mechanism behind this enhancement remains elusive.

To clarify the mechanism, we performed an irreducible-tensor decomposition of the 3NF derived from chiral EFT at next-to-next-to-leading order (N^{2}LO). As a result of shell-model calculations, which were implemented with the decomposed 3NF contributions, we found that the rank-1 tensor component originating from the two-pion (2π) exchange term enlarges the SO splitting for p-shell nuclei. In particular, our calculations revealed the dominant role played by the c_3 term, which corresponds to the P-wave pion-nucleon scattering involved in the 2π-exchange process.

2. Theoretical framework

We briefly explain the irreducible-tensor decomposition of the chiral 3NF. For more details, see Ref. [17]. The three-nucleon potential in momentum space at N^{2}LO can be generally expressed as

$$v_{3N}^{(\alpha)} = \sum_{i\neq j\neq k} v^{(\alpha)}\left(\boldsymbol{\tau}_i, \boldsymbol{\tau}_j, \boldsymbol{\tau}_k\right) w^{(\alpha)}\left(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \boldsymbol{q}_i, \boldsymbol{q}_j\right), \tag{1}$$

where the superscript α denotes 2π, one-pion (1π), or contact (ct), distinguishing each contribution based on the number of pions exchanged. The indices i, j, and k specify the nucleons, and $v^{(\alpha)}$ represents the isospin part of the potential with the isospin operator $\boldsymbol{\tau}_i$. The spin and momentum-dependent parts are expressed by $w^{(\alpha)}$, which is a function of the spin operator $\boldsymbol{\sigma}_i$ and the momentum transfer $\boldsymbol{q}_i$. Note that due to momentum conservation, $\boldsymbol{q}_i + \boldsymbol{q}_j + \boldsymbol{q}_k = 0$, the $\boldsymbol{q}_k$ dependence in Eq. (1) is not explicitly shown.

The irreducible-tensor decomposition separates the coupling of the spin and momentum parts of $w^{(\alpha)}$ and classifies it in terms of the rank λ of the spherical tensors:

$$w^{(\alpha)}\left(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \boldsymbol{q}_i, \boldsymbol{q}_j\right) = w_{\mathrm{pro}}^{(\alpha)}(q_i, q_j) \sum_{\lambda} O_{\lambda}^{(\alpha)}\left(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j\right), \tag{2}$$

$$O_{\lambda}^{(\alpha)}\left(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j\right) = A_{\lambda}\left[\mathcal{M}_{\lambda}^{(\alpha)}\left(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k\right) \otimes \mathcal{N}_{\lambda}^{(\alpha)}\left(\hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j\right)\right]_{00}. \tag{3}$$

Here, $w_{\mathrm{pro}}^{(\alpha)}$ represents a function that depends only on the magnitude of the momentum transfer, and involves, for example, pion propagators, low-energy constants (LECs), etc. As shown by Eq. (3), $O_{\lambda}^{(\alpha)}$ represents the coupling of the spherical tensors, $\mathcal{M}_{\lambda\mu}^{(\alpha)}$ and $\mathcal{N}_{\lambda,-\mu}^{(\alpha)}$, each having rank λ, which together form the totally scalar operator. The coefficient A_{λ} contains the angular-momentum coupling coefficients.

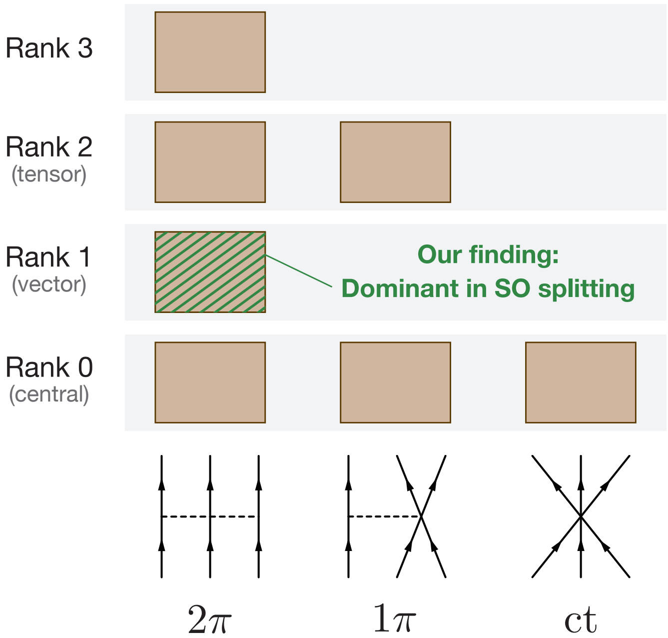

Figure 1: The classification of the chiral 3NF in terms of the rank of the spherical tensors. The contact (ct) term has only the central component as this is the leading contact interaction, while the 1π-exchange term consists of the central and tensor components, similar to the 1π-exchange two-nucleon force. The 2π-exchange term involves components with $\lambda = 0$ to 3, and one of them, the $\lambda = 1$ component, is the dominant source for enhancing the SO splitting.

The rank λ characterizes the interaction; we usually call the contributions with $\lambda = 0$, 1, and 2 the central, vector, and tensor forces, respectively. In contrast to two-nucleon forces, the 3NF can also include the component with $\lambda = 3$. For each α, the structure of the three-nucleon potential in terms of λ is shown in Fig. 1.

As emphasized in Fig. 1, the rank-1 component ($\lambda = 1$) plays an essential role in the SO splitting. In particular, the rank-1 component of the 2π-exchange potential, dependent on the LEC c_3, gives the dominant effect. The potential of the c_3 term for $\lambda = 1$ can be written as

$$v_{3N;\lambda=1}^{(c_3)} = \frac{g_A^2 c_3}{8 f_\pi^4} \sum_{i \neq j \neq k} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \frac{q_i^2 q_j^2}{\left(q_i^2 + m_\pi^2\right)\left(q_j^2 + m_\pi^2\right)} \left(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right) \cdot \left(\hat{\boldsymbol{q}}_i \times \hat{\boldsymbol{q}}_j\right) \left(\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j\right), \tag{4}$$

with the axial vector coupling constant g_A, the pion-decay constant f_π, and the average pion mass m_π. Below, we show the contributions of each λ to the SO splitting of p-shell nuclei.

3. Results

To study the contribution of each rank of the 3NF, we applied the irreducible tensor decomposition described above to the shell model. As a test case, we chose p-shell nuclei with ^{4}He as the core state. The Hamiltonian necessary for the shell-model calculations was computed using many-body perturbation theory [18–20], where we adopted the two-nucleon force from chiral EFT

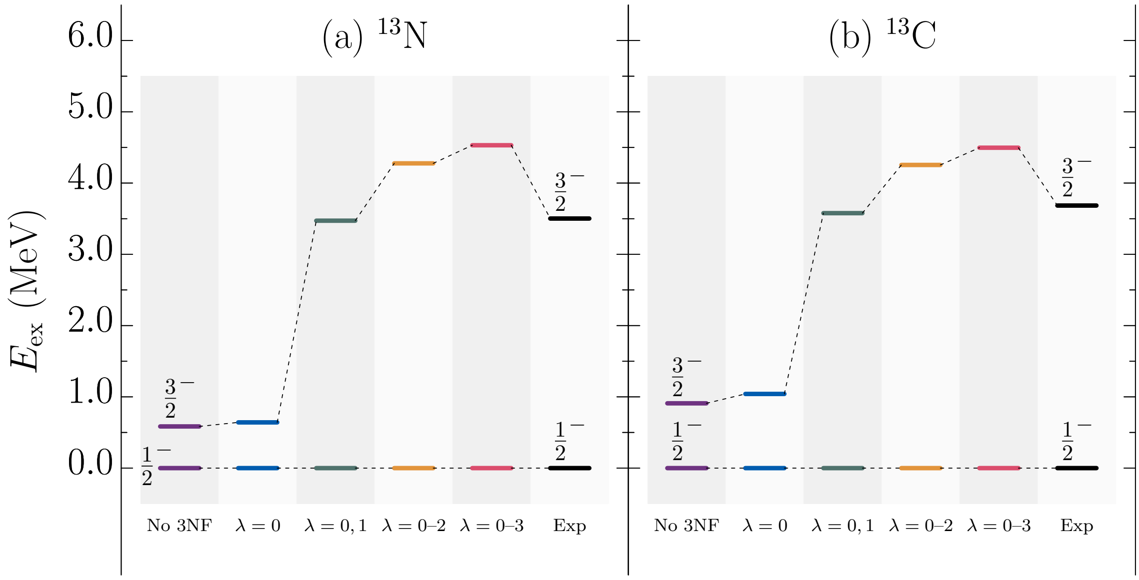

Figure 2: Excitation energy of the $3/2^-$ state measured from the ground state for (a) ^{13}N and (b) ^{13}C. The label "No 3NF" indicates the result without the 3NF, while the results including the 3NF are shown with λ taken into account. The experimental data ("Exp") are taken from Ref. [23].

at next-to-next-to-next-to-leading order [21], along with the 3NF in the normal-ordered form [22]. The numerical details, such as the LECs we used, can be found in Ref. [17].

Figure 2 shows the excitation energy of the $3/2^-$ state for (a) ^{13}N and (b) ^{13}C. When we ignore the 3NF, the $3/2^-$-state energy is below 1 MeV. In contrast, the results of the full contribution (λ = 0–3) are more than 3 MeV higher than the "No 3NF" results, indicating that the 3NF enhances the SO splitting relative to the $0p_{3/2}$ and $0p_{1/2}$ single-particle energies. This increase in energy is mainly due to the rank-1 component, as shown by the result for " $\lambda = 0, 1$". Although our calculated results do not perfectly match the measured data [23], we expect the conclusion to remain robust: the enhancement of the SO splitting by the 3NF primarily stems from its rank-1 component. This is because the rank-1 component arises exclusively from the 2π-exchange term, and its LECs, c_1, c_3, and c_4, which are constrained by the Roy–Steiner equation analysis of pion–nucleon scattering [24], are more precisely determined than the contact LECs, c_D and c_E.

To understand the results shown in Fig. 2, we computed the effective single-particle energy (ESPE) [25], which is an effective quantity for studying the 3NF effect on single-particle properties. Figure 3 displays the gap between the $0p_{3/2}$ ESPE and that of $0p_{1/2}$ for (a) proton and (b) neutron with respect to ^{4}He, ^{8}Be, and ^{12}C. The gap, corresponding to the SO splitting, does not change significantly due to the rank-0 3NF, as indicated by the difference between the filled and open circles. By including the noncentral 3NF, a significant enhancement of the gap is observed. In particular, the results with the full contribution (λ = 0–3), shown by the triangles, are almost twice as large as the "No 3NF" result for ^{12}C. This doubling is consistent with the previous study on ^{15}N [26]. The rank-1 ($\lambda = 0, 1$) and rank-2 (λ = 0–2) components contribute about 75% and 20% of the full contributions, as shown by the filled and open squares, respectively. The rank-3 component (λ = 0–3) gives only a minor enhancement of the SO splitting. Thus, we conclude that

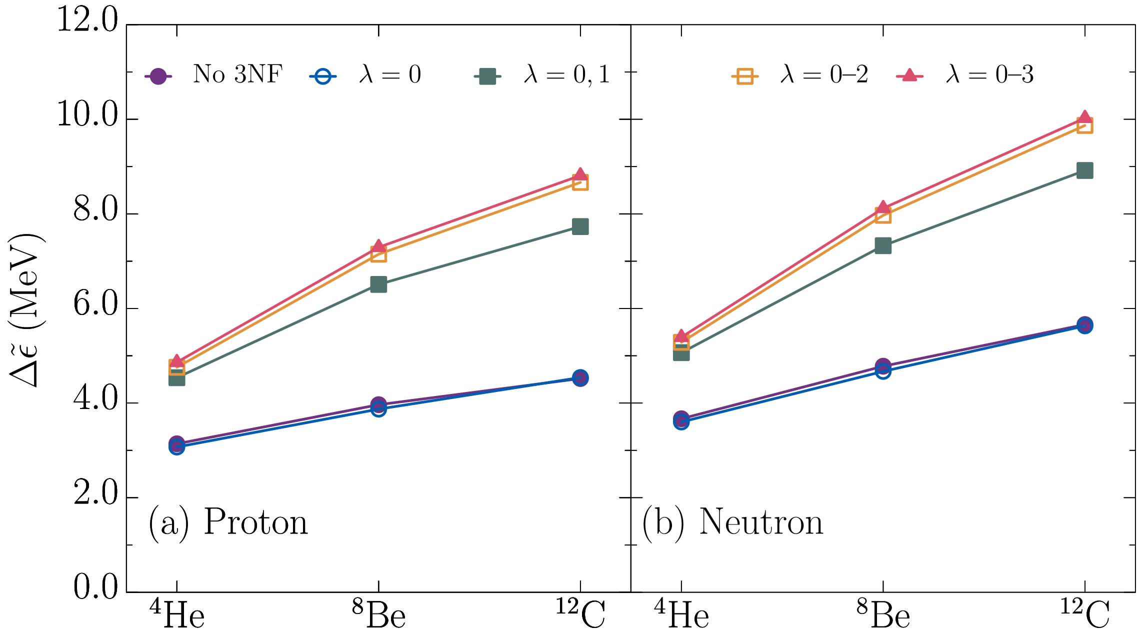

Figure 3: Gap of the ESPEs between the $0p_{3/2}$ and $0p_{1/2}$ states for (a) proton and (b) neutron with respect to ^{4}He, ^{8}Be, and ^{12}C. The filled circles correspond to the results without the 3NF, while the open circles, filled squares, open squares, and triangles represent the results with the 3NF contributions of rank $\lambda = 0$, $\lambda = 0, 1$, $\lambda = 0$–2, and $\lambda = 0$–3, respectively.

the enhancement of the SO splitting by the 3NF is primarily due to its rank-1 component.

The rank-1 component is involved exclusively in the 2π-exchange term, which can be further decomposed into three pieces: the c_1, c_3, and c_4 terms. The c_1 and c_3 terms are responsible for the S- and P-wave pion-nucleon scattering, respectively, while the c_4 term corresponds to the process where a proton converts to a neutron and vice versa. Since pion-nucleon scattering is dominated by the P-wave, as indicated by the absence of the S-wave part in the Fujita–Miyazawa 3NF [27], the c_3 term is a crucial source for the SO splitting. This is demonstrated in Fig. 4, where the c_3-term contribution (open square) to the ESPE agrees well with the result obtained using the c_1, c_3, and c_4 terms (filled square): For example, the difference between them is only around 1% (2%) for the proton (neutron) ESPEs of ^{12}C.

The reason why the c_3-rank-1 3NF enhances the SO splitting can be understood by referring to Ref. [28], in which the one-body SO potential was derived from a three-body potential similar to Eq. (4), resulting in a density dependence that differs slightly from that of the standard one-body potential, i.e., the derivative of the square of the one-body density appears.

4. Conclusion and perspectives

We have elucidated the mechanism by which the chiral 3NF enhances the SO splitting by decomposing the three-body potential in terms of the rank of the irreducible tensors. The decomposed potential has been implemented in shell-model calculations for p-shell nuclei. Our calculations

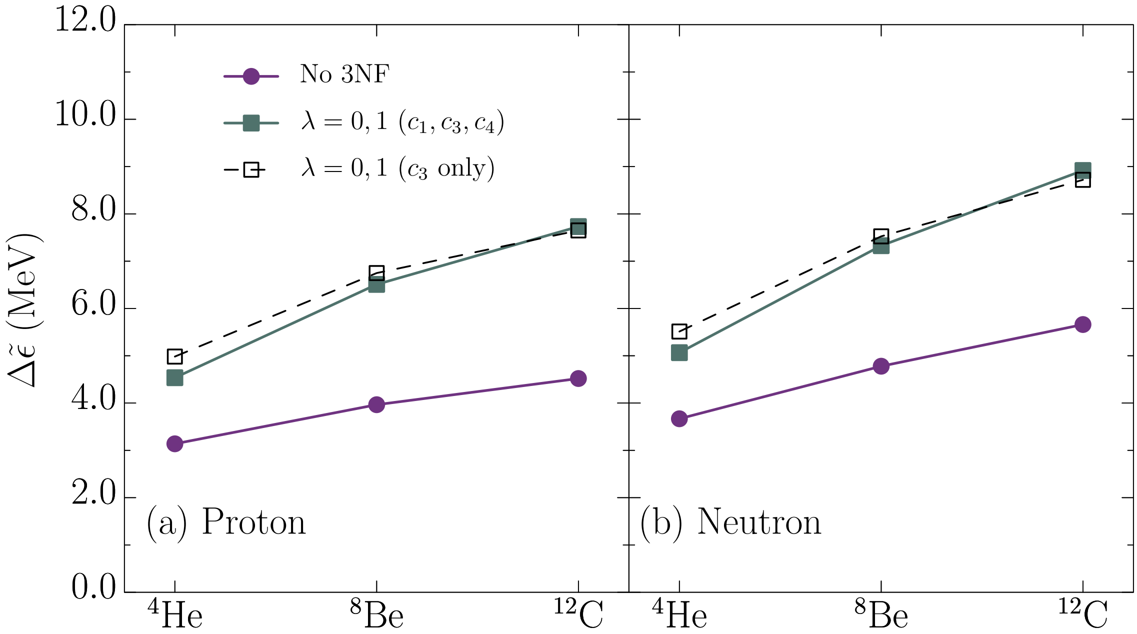

Figure 4: Same as Fig. 3, but the results for "No 3NF" (circle) and "$\lambda = 0, 1$" (filled square) are compared with those of $\lambda = 0, 1$ for the c_3 term. See text for details.

reveal that the rank-1 3NF, particularly the c_3 term in the 2π-exchange process, plays a crucial role in the SO splitting.

Since the rank-1 potential of the c_3 3NF acts as an antisymmetric SO interaction in spin-space, it would be interesting to investigate its role in connection with the single-triplet mixing of the two-body spin states, as well as its analogy to the Dzyaloshinsky–Moriya interaction [29–31] in the future.

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