

# Asymmetric dark matter semi-annihilation into long-lived particles

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The contribution is focused on an asymmetric dark matter model based on semi-annihilations into long-lived standard-model-sector particles. The dark matter is stabilized by discrete  $\mathbb{Z}(3)$  symmetry. It is shown that despite strong dark matter annihilation, the asymmetry obtained from the semi-annihilations may significantly affect the resulting relic density, even at very small coupling values.

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## 1. Introduction

The asymmetry between matter and antimatter is one of the strongest indications of physics beyond the Standard Model. Investigating possible mechanisms for its generation is crucial not only for understanding the visible sector but also because such asymmetry may play a key role in determining the dark matter relic density. Typical approaches to asymmetry generation primarily focus on the decays of heavy particles [1–8], while asymmetric annihilations of particle pairs, though less commonly considered, have also been explored in the literature [9–12].

In this contribution, based on Ref. [13], we examine the role of semi-annihilations in dark matter dynamics. While such processes are not typically considered for generating asymmetry or stabilizing the dark matter relic density, they have been studied previously. In the symmetric case, it has been shown that dark matter semi-annihilations can dominate over conventional annihilations, significantly influencing the relic density [14]. Additionally, there have been efforts to generate dark-matter asymmetry through semi-annihilations, thereby modifying the freeze-out scenario [15]. This study suggests that, from a cosmological perspective, such a mechanism is indeed feasible. However, as we demonstrate in Section 2, the theoretical model employed in the aforementioned work does not permit asymmetry at the given perturbative order. To address this, in Section 3, we introduce a modified scalar model that enables asymmetry emergence with minimal additional complexity. Furthermore, Section 4 extends this mechanism to an effective theory involving fermions. Finally, we summarize in Section 5.

## 2. CPT and unitarity constraints for semi-annihilations

As mentioned above, semi-annihilations have previously been studied as a mechanism to achieve asymmetric dark matter in Ref. [15], where their presence also affected the relic density. The relevant part of the Lagrangian of the considered theory is given by

$$\mathcal{L} \supset \frac{\mu}{3!}\chi^3 + \frac{\lambda}{3!}\phi\chi^3 + \text{H.c.} + \frac{\lambda_1}{4}|\chi|^4 + \frac{\lambda_2}{2}\phi^2|\chi|^2 + \mu_1\phi|\chi|^2 + \frac{\mu_2}{3!}\phi^3 + \frac{\lambda_3}{4!}\phi^4. \quad (1)$$

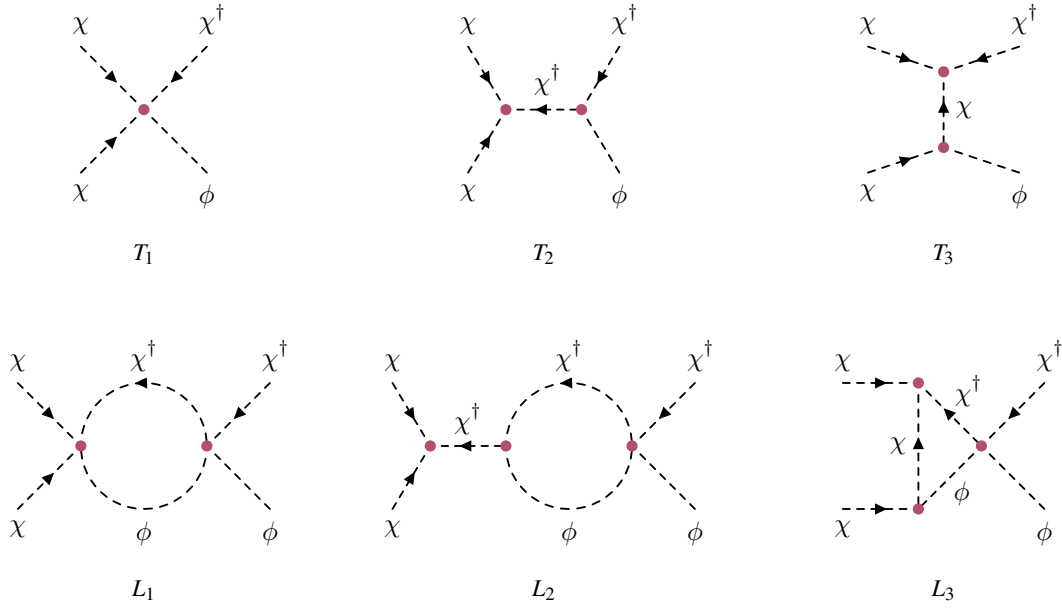
Here, the  $\chi$  particle serves as a scalar dark matter candidate that is  $\mathbb{Z}(3)$ -symmetric, while singlet  $\phi$  represents the visible sector. Only the first two terms in Eq. 1 allow for complex couplings, while one of them can be made real through a redefinition of fields. The leading order asymmetry for dark matter semi-annihilation  $2\chi \rightarrow \bar{\chi}\phi$  then comes from the interference between tree and 1-loop diagrams seen in Fig. 1. However, accounting for all contributions leads to complete asymmetry cancellation at the given perturbative order.

The origin of this cancellation can be traced back to the  $S$ -matrix unitarity,

$$SS^\dagger = S^\dagger S = 1 \quad \rightarrow \quad S^\dagger = 1 - iT^\dagger = (1 + iT)^{-1}. \quad (2)$$

Expanding the transition matrix  $T^\dagger$  as a power series in terms of  $T$ -matrix elements then leads to [16–19]

$$|T_{fi}|^2 = -iT_{if}^\dagger iT_{fi} = -iT_{if} iT_{fi} + \sum_n iT_{in} iT_{nf} iT_{fi} - \dots \quad (3)$$



**Figure 1** All the tree and 1-loop diagrams contributing to asymmetric semi-annihilation  $\chi\chi \rightarrow \bar{\chi}\phi$  at the perturbative order considered in Ref. [15], where the  $T_3$  diagram was missing. Its inclusion, however, results in a vanishing asymmetry for this process.

Furthermore, assuming a  $CPT$ -symmetric theory, the  $CP$  asymmetry then contains only the terms of third and higher orders in  $T$ -matrix [19]

$$\begin{aligned} \Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 = & \sum_n (iT_{in}iT_{nf}iT_{fi} - iT_{if}iT_{fn}iT_{ni}) \\ & - \sum_{n,m} (iT_{in}iT_{nm}iT_{mf}iT_{fi} - iT_{if}iT_{fm}iT_{mn}iT_{ni}) \\ & + \dots \end{aligned} \quad (4)$$

Summing over all possible final states then yields the unitarity relation for  $CP$  asymmetries [20, 21]

$$\sum_f \Delta|T_{fi}|^2 = 0. \quad (5)$$

This means that for a fixed initial state, the asymmetry must vanish at the level of squared amplitudes when summed over all possible final states. In the model considered above, the reason for this cancellation becomes evident—at the given perturbative order, starting with  $\chi\chi$  in the initial state, only a single final state is allowed, which necessarily leads to a vanishing asymmetry.

There are multiple ways to address this issue, leading to  $CP$ -asymmetric semi-annihilations. The most straightforward path is to consider higher perturbative orders that can allow for multiple final states in addition to  $\bar{\chi}\phi$  in  $\chi\chi \rightarrow \bar{\chi}\phi$ . However, this leads to a very large number of diagrams to be considered, making the approach impractical despite its conceptual simplicity. A better option is to introduce an additional field into the Lagrangian—another copy of  $\phi$ , with different mass and coupling constants.

### 3. Asymmetric semi-annihilations of scalar dark matter

As outlined at the end of the previous section, we extend the model introduced in Ref. [15] with an additional particle. The relevant part of the modified Lagrangian density is given by

$$\mathcal{L} \supset -\frac{\lambda_1}{6}\chi^3\phi_1 - \frac{\lambda_2}{6}\chi^3\phi_2 + \text{H.c.} - \lambda_{12}|\chi|^2\phi_1\phi_2 - \frac{\lambda_3}{2}|\chi|^2\phi_3^2. \quad (6)$$

Here, the  $\phi_1$  and  $\phi_2$  particles are long-lived<sup>1</sup>. The last term leads to dark matter annihilation into visible sector particles represented by the  $\phi_3$  field. Its inclusion fixes the relic density via the freeze-out. The evolution of the asymmetry is then governed by the Boltzmann equation that can be split into two parts

$$\left(\frac{d\Delta_\chi}{dx}\right)_{\text{source}} = -\frac{3}{8}\frac{s}{Hx}Y_\chi^{\text{eq}}Y_\chi\left(\frac{Y_{\phi_1}}{Y_{\phi_1}^{\text{eq}}} - \frac{Y_{\phi_2}}{Y_{\phi_2}^{\text{eq}}}\right)\Delta\langle\sigma v\rangle_{\chi\chi\rightarrow\chi^\dagger\phi_1}, \quad (7)$$

$$\left(\frac{d\Delta_\chi}{dx}\right)_{\text{washout}} = -\frac{3}{2}\frac{s}{Hx}Y_\chi^{\text{eq}}\Delta_\chi\sum_{i=1,2}\left(\frac{Y_\chi}{Y_\chi^{\text{eq}}} + \frac{1}{2}\frac{Y_{\phi_i}}{Y_{\phi_i}^{\text{eq}}}\right)\langle\sigma v\rangle_{\chi\chi\rightarrow\chi^\dagger\phi_i}. \quad (8)$$

The  $Y$ s stand for the particle number densities divided by the entropy density. We emphasize that the source term would vanish if the equilibrium densities of  $\phi_1$  and  $\phi_2$  were equal. For the symmetric part as well as for  $Y_{\phi_i}$ , we have

$$\frac{dY_\chi}{dx} = -\frac{1}{2}\frac{s}{Hx}\left[\sum_{i=1,2}\frac{1}{2}\left(Y_\chi^2 + \Delta_\chi^2 - Y_\chi Y_\chi^{\text{eq}}\frac{Y_{\phi_i}}{Y_{\phi_i}^{\text{eq}}}\right)\langle\sigma v\rangle_{\chi\chi\rightarrow\chi^\dagger\phi_i} + \left(Y_\chi^2 - \Delta_\chi^2 - Y_\chi^{\text{eq}2}\frac{Y_{\phi_1}Y_{\phi_2}}{Y_{\phi_1}^{\text{eq}}Y_{\phi_2}^{\text{eq}}}\right)\langle\sigma v\rangle_{\chi\chi^\dagger\rightarrow\phi_1\phi_2} + \left(Y_\chi^2 - \Delta_\chi^2 - Y_\chi^{\text{eq}2}\right)\langle\sigma v\rangle_{\chi\chi^\dagger\rightarrow\phi_3\phi_3}\right] \quad (9)$$

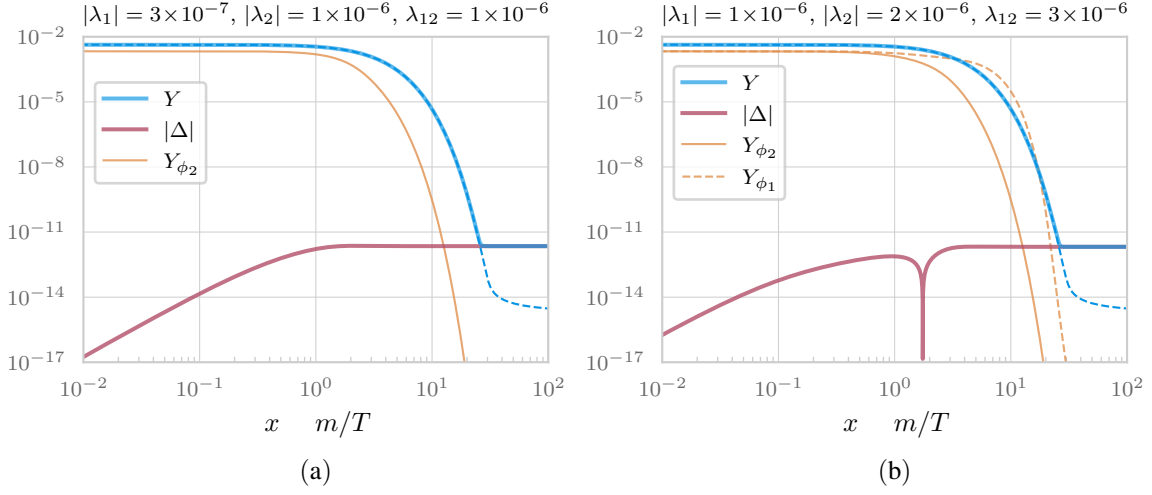
$$\frac{dY_{\phi_1}}{dx} = -\frac{\langle\Gamma_1\rangle}{Hx}\left(Y_{\phi_1} - Y_{\phi_1}^{\text{eq}}\right) + \frac{1}{4}\frac{s}{Hx}\left[\left(Y_\chi^2 + \Delta_\chi^2 - Y_\chi Y_\chi^{\text{eq}}\frac{Y_{\phi_1}}{Y_{\phi_1}^{\text{eq}}}\right)\langle\sigma v\rangle_{\chi\chi\rightarrow\chi^\dagger\phi_1} + \left(Y_\chi^2 - \Delta_\chi^2 - Y_\chi^{\text{eq}2}\frac{Y_{\phi_2}Y_{\phi_1}}{Y_{\phi_2}^{\text{eq}}Y_{\phi_1}^{\text{eq}}}\right)\langle\sigma v\rangle_{\chi\chi^\dagger\rightarrow\phi_1\phi_2} - 4Y_\chi Y_{\phi_1}^{\text{eq}}\left(\frac{Y_{\phi_1}}{Y_{\phi_1}^{\text{eq}}} - \frac{Y_{\phi_2}}{Y_{\phi_2}^{\text{eq}}}\right)\langle\sigma v\rangle_{\chi\phi_1\rightarrow\chi\phi_2}\right] \quad (10)$$

and similarly for the  $\phi_2$  field. The evolution of the number densities for all the relevant particles, as well as for the dark matter asymmetry, are plotted in Fig. 2. The figure shows that the presence of even very weak semi-annihilations can significantly influence the relic density compared to the typical annihilation freeze-out. Furthermore, depending on the masses of the  $\phi_i$  particles, the asymmetry may change sign during its evolution, as seen in Fig. 2b. However, achieving the behavior seen in Fig. 2 requires very small  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_{12}$  values.

### 4. Asymmetric semi-annihilations of fermionic dark matter

The necessity of unnaturally small couplings mentioned at the end of the previous section can be avoided within a fermionic model that, aside from a few differences, closely resembles the

<sup>1</sup>This holds as long as both particles are massive. We also consider a scenario with the  $\phi_1$  being massless, in which case it does not decay.



**Figure 2** The evolution of  $Y$ ,  $|\Delta|$ ,  $Y_{\phi_2}$ , and  $Y_{\phi_1}$  following from Eqs. (7)-(10). Values of  $Y$  computed solely from  $\chi\chi^\dagger \rightarrow \phi_3\phi_3$  with  $\lambda_3 = 6.0$  are plotted by the dashed blue lines and serve for comparison to what would be obtained if the effects of asymmetry were neglected. In both cases,  $m = 200$  GeV,  $m_2 = 400$  GeV,  $\Gamma_2/m_2 = 3 \times 10^{-16}$ , and  $\Im[\lambda_1\lambda_2^*]/|\lambda_1^*\lambda_2| = 0.8$ . We put  $m_1 = 0$  with  $\phi_1$  in thermal equilibrium (Fig. 2a), or  $m_1 = 260$  GeV and  $\Gamma_1/m_1 = 2 \times 10^{-17}$  (Fig. 2b).

scalar one discussed above. The key idea is to employ an effective theory, hiding the smallness of couplings in a high energy scale. Lagrangian with dimension-6 operators can be written as

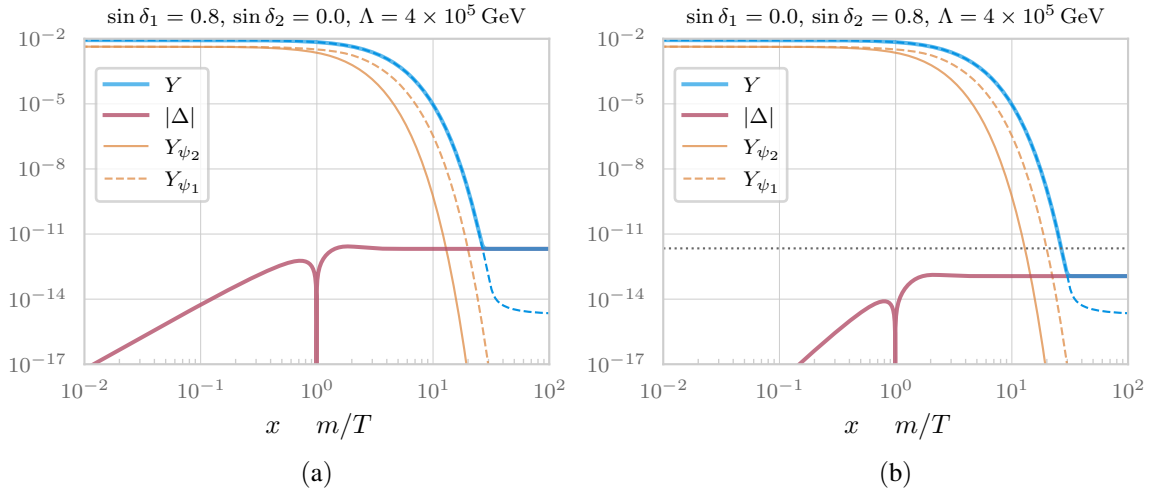
$$\begin{aligned} \mathcal{L} \supset & -\frac{\kappa_1}{6\Lambda^2} (\bar{\chi}^c P_R \chi) (\bar{\chi}^c P_L \psi_1) - \frac{\kappa_2}{6\Lambda^2} (\bar{\chi}^c P_R \chi) (\bar{\chi}^c P_L \psi_2) \\ & - \frac{\kappa_{12}}{2\Lambda^2} (\bar{\chi}^c P_L \psi_1)^\dagger (\bar{\chi}^c P_L \psi_2) + \text{H.c.} \end{aligned} \quad (11)$$

As in the scalar case, the observed relic density is achieved through semi-annihilations and with annihilations into visible scalars, facilitated by the additional Lagrangian term  $-\kappa_3 \bar{\chi} \chi \phi_3$ . Beyond the energy scale  $\Lambda$ , the fermionic model differs from the scalar one in another key aspect—the presence of an additional free parameter arising from the complex nature of  $\kappa_{12}$ . This introduces two independent complex phases, parametrized as  $\sin \delta_1 = \Im[\kappa_1^* \kappa_{12}^* \kappa_2] / |\kappa_1^* \kappa_{12}^* \kappa_2|$  and  $\sin \delta_2 = \Im[\kappa_1^* \kappa_{12} \kappa_2] / |\kappa_1^* \kappa_{12} \kappa_2|$ , in contrast to the single phase present in the scalar case.

The Boltzmann equations in this case are identical to those in the scalar model, and their solutions exhibit similar behavior. In Fig. 3, we present separate solutions for both complex phases, where it is evident that one phase dominates in generating asymmetry, assuming all other parameters remain equal. Another key observation is that the freedom associated with the large scale  $\Lambda$  allows us to set all semi-annihilation coupling strengths to unity. This stands in contrast to the scalar model, where these couplings had to be unnaturally small.

## 5. Summary

In this contribution, we highlighted the crucial role of the  $CPT$  and unitarity constraints in matter asymmetry generation—not only as a consistency check but also as a powerful tool for quickly assessing the viability of a given theoretical model. We analyzed semi-annihilations in both



**Figure 3** Particle density evolution within the fermionic model. Up to a complex phase,  $\kappa_1$ ,  $\kappa_2$  and  $\kappa_{12}$  were set to unity, while  $y = 3.0$ ,  $m = 200$  GeV,  $m_1 = 260$  GeV,  $m_2 = 400$  GeV,  $\Gamma_1/m_1 = 1 \times 10^{-15}$  and  $\Gamma_2/m_2 = 4 \times 10^{-15}$ . In Fig. 3a, we tuned the parameters to reproduce the observed dark matter density (the grey dotted line in Fig. 3b). Similarly to Fig. 2, the blue dashed line corresponds to density evolution calculated from annihilations only.

scalar and fermionic models, demonstrating how these processes can significantly impact the dark matter relic density, even when involving very weak interactions.

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