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Light states in real multi-Higgs models with spontaneous CP violation

Carlos Miró[a], **Miguel Nebot**[b] **and Daniel Queiroz**[c,*]

[a=b=c] *Departament de Física Teòrica & Instituto de Física Corpuscular (IFIC), Universitat de València – CSIC, E-46100 Valencia, Spain*

E-mail: Carlos.Miro@uv.es, Miguel.Nebot@uv.es, Daniel.Queiroz@uv.es

In models with extended scalar sectors consisting of multiple Higgs doublets that trigger spontaneous electroweak symmetry breaking, it might be expected that the abundance of dimensionful quadratic couplings in the scalar potential could allow for a regime where, apart from the would-be Goldstone bosons and a neutral Higgs-like state, all new scalars have masses much larger than the electroweak scale. In the case of models where CP invariance holds at the lagrangian level but is broken by the vacuum, we show that such a reasonable expectation does not hold. When perturbativity requirements are placed on the dimensionless quartic couplings, the spectrum of the new scalars includes one charged and two additional neutral states whose masses cannot be much larger than the electroweak scale.

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1. Introduction

A better understanding of different open fundamental questions often involves new particles. Knowing their masses is a priority. As the decades-long history of the Higgs boson shows, short of the experimental grail of discovery [1] or indirect hints [2], a variety of theoretical arguments can be put forward to bring light on this respect [3]. An example of them is the use of perturbativity requirements in the classical works of B. Lee, Quigg and Thacker [4] (see also [5]) to obtain bounds on the Higgs mass. An important implication of perturbativity requirements on extended scalar sectors featuring spontaneous symmetry breaking (SSB), is the existence of a Higgs-like state whose mass cannot be (much) larger than the electroweak scale, as discussed in [6]. Two Higgs doublets models (2HDMs) were proposed by T.D. Lee [7], with the appealing possibility of a spontaneous origin of CP violation, that is, having a CP-invariant Lagrangian together with a CP-non-invariant vacuum; we simply refer to them as *real* 2HDMs with spontaneous CP violation (SCPV). Besides the Higgs-like state, perturbativity-based bounds on the masses of the additional scalars have attracted attention within 2HDMs endowed with some symmetry [8], typically a $\mathbb{Z}_2$. Complementarily, in multi-Higgs models [9], the analysis of scenarios with new *heavy* scalars, i.e. scalars with masses $\gg v$, was addressed generically in [10], and recently in [11] in connection with symmetries. Concerning real 2HDMs with SCPV, it was realized in [12], and analyzed in detail in [13], that those perturbativity-based bounds on the masses apply to *all* the new scalars in the model, one charged and two neutral ones, in addition to the neutral Higgs-like. With SSB, there are two types of mass terms in 2HDMs, either dimensionful quadratic couplings or dimensionless quartic couplings $\times$ (vevs)^2. If the quartic couplings are limited by perturbativity requirements, masses (much) larger than v can only arise through large quadratic couplings. The crucial particularity of the real 2HDM with SCPV is that there are only 3 quadratic couplings, which is also the number of stationarity conditions imposed on the scalar potential so that the vacuum is an extremum. These stationarity conditions can then be used to trade all 3 quadratic couplings for quartic couplings $\times$ (vevs)^2. Consequently, all mass terms are bounded through perturbativity requirements on the quartic couplings. This means that rather than the generic expectation of having at least one light scalar –the Higgs-like one–, perturbativity requirements bound *the whole spectrum*. Then, if instead of a real 2HDM with SCPV, one considers a real nHDM with SCPV, is there something similar at work? The number of quadratic couplings scales with n^2 while the number of stationarity conditions only scales with n. For $n > 2$, free quadratic couplings are necessarily present in the scalar potential. Does this imply that all the masses of the new scalars can be be much larger than the electroweak scale? The central result of this work [14] is that the answer to this question is in the negative: *for all n, the spectrum necessarily includes one charged and two neutral scalars (in addition to the neutral Higgs-like) that must be light, that is, whose masses cannot be much larger than the electroweak scale when perturbativity requirements are imposed on the quartic couplings.*

2. Real nHDM with SCPV

For n Higgs doublets Φ_a, $a = 1, \ldots, n$, the most general scalar potential invariant under the CP transformation $\Phi_a \mapsto \Phi_a^*$ has the following form:

$$\mathcal{V}(\Phi_1, \ldots, \Phi_n) = \mathcal{V}_2(\Phi_1, \ldots, \Phi_n) + \mathcal{V}_4(\Phi_1, \ldots, \Phi_n), \tag{1}$$

with

$$\mathcal{V}_2(\Phi_1,\ldots,\Phi_n) = \sum_{a=1}^{n}\mu_a^2\Phi_a^\dagger\Phi_a + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\mu_{ab}^2\mathcal{H}_{ab}, \tag{2}$$

$$\begin{aligned}\mathcal{V}_4(\Phi_1,\ldots,\Phi_n) &= \sum_{a=1}^{n}\lambda_a(\Phi_a^\dagger\Phi_a)^2 + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\lambda_{a,b}(\Phi_a^\dagger\Phi_a)(\Phi_b^\dagger\Phi_b)\\ &+ \sum_{a=1}^{n}\sum_{b=1}^{n-1}\sum_{c=b+1}^{n}\lambda_{a,bc}(\Phi_a^\dagger\Phi_a)\mathcal{H}_{bc} + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)}\lambda_{ab,cd}\mathcal{H}_{ab}\mathcal{H}_{cd}\\ &+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)}\lambda_{ab,cd}^{\mathcal{A}}\mathcal{A}_{ab}\mathcal{A}_{cd}\end{aligned} \tag{3}$$

All quadratic μ_a^2, μ_{ab}^2 in $\mathcal{V}_2$, and quartic λ_a, $\lambda_{a,b}$, $\lambda_{a,bc}$, $\lambda_{ab,cd}$, $\lambda_{ab,cd}^{\mathcal{A}}$ in $\mathcal{V}_4$, parameters are real (hence *real* nHDM). We use hermitian and antihermitian bilinears

$$\mathcal{H}_{ab} \equiv \frac{1}{2}(\Phi_a^\dagger\Phi_b + \Phi_b^\dagger\Phi_a),\ \mathcal{A}_{ab} \equiv \frac{1}{2}(\Phi_a^\dagger\Phi_b - \Phi_b^\dagger\Phi_a),\ a < b. \tag{4}$$

Assuming an appropriate electroweak symmetry breaking vacuum, expansion of fields reads

$$\Phi_a = \frac{e^{i\theta_a}}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\mathrm{C}_a^+\\ v_a + \mathrm{R}_a + i\,\mathrm{I}_a\end{pmatrix},\quad \langle\Phi_a\rangle = \frac{v_a e^{i\theta_a}}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}, \tag{5}$$

where the vevs $\langle\Phi_a\rangle$ are parameterized by $v_a \in \mathbb{R}^+$, $v_1^2 + \ldots + v_n^2 = v^2 \simeq 246^2$ GeV^2, and $\theta_a \in [0; 2\pi[$, moduli and phases respectively. Individual phases have no physical meaning: all CP violation arising from the vacuum is encoded in the phase differences $\theta_a - \theta_b$. To obtain the stationarity conditions, one first computes

$$V(v_1,\ldots,v_n,\theta_1,\ldots,\theta_n) = \mathcal{V}(\langle\Phi_1\rangle,\ldots,\langle\Phi_n\rangle), \tag{6}$$

and then sets derivatives with respect to the vev parameters to zero,

$$\partial_{v_1}V = \ldots = \partial_{v_n}V = 0,\quad \partial_{\theta_1}V = \ldots = \partial_{\theta_n}V = 0, \tag{7}$$

with $\partial_x V \equiv \frac{\partial V}{\partial x}$. The lack of physical meaning of individual phases translates into $\partial_{\theta_1}V + \ldots + \partial_{\theta_n}V = 0$ *by construction*, irrespective of imposing that each term is zero: Eqs. (7) give $2n-1$ (independent) stationarity conditions. With Eqs. (2)-(3), the derivatives in Eqs. (7) read

$$\begin{aligned}\partial_{v_j}V &= \mu_j^2 v_j + \frac{1}{2}\sum_{a=1}^{j-1}\mu_{aj}^2 c_{aj}v_a + \frac{1}{2}\sum_{b=j+1}^{n}\mu_{jb}^2 c_{jb}v_b + [\lambda' s],\\ \partial_{\theta_j}V &= \frac{1}{2}\sum_{a=1}^{j-1}\mu_{aj}^2 s_{aj}v_a v_j - \frac{1}{2}\sum_{b=j+1}^{n}\mu_{jb}^2 s_{jb}v_j v_b + [\lambda' s],\end{aligned} \tag{8}$$

where the shorthand notation $\theta_{ab} = \theta_a - \theta_b$, $c_{ab} = \cos\theta_{ab}$, $s_{ab} = \sin\theta_{ab}$, is used, and $[\lambda' s]$ stand for terms involving only quartic couplings, not displayed for conciseness.
The mass terms within $\mathcal{V} \supset -\mathscr{L}_{\text{Mass}}$ are

$$-\mathscr{L}_{\text{Mass}} = \vec{\mathbf{C}}^\dagger M_\pm^2\vec{\mathbf{C}} + \frac{1}{2}\vec{\mathbf{N}}^T M_0^2\vec{\mathbf{N}},\ \text{with}\ \vec{\mathbf{C}}^\dagger = (\mathrm{C}_1^-,\ldots,\mathrm{C}_n^-),\ \vec{\mathbf{N}}^T = (\mathrm{R}_1,\ldots,\mathrm{R}_n,\mathrm{I}_a,\ldots,\mathrm{I}_n) \tag{9}$$

$M^2_\pm$ is the $n \times n$ charged mass matrix and M^2_0 the $2n \times 2n$ neutral mass matrix, whose elements are

$$(M^2_\pm)_{a,b} = \left[\frac{\partial^2 \mathcal{V}}{\partial \mathrm{C}^+_a \partial \mathrm{C}^-_b}\right], \qquad (M^2_0)_{a,b} = \left[\frac{\partial^2 \mathcal{V}}{\partial \mathrm{R}_a \partial \mathrm{R}_b}\right],$$
$$(M^2_0)_{n+a,n+b} = \left[\frac{\partial^2 \mathcal{V}}{\partial \mathrm{I}_a \partial \mathrm{I}_b}\right], \qquad (M^2_0)_{a,n+b} = (M^2_0)_{n+b,a} = \left[\frac{\partial^2 \mathcal{V}}{\partial \mathrm{R}_a \partial \mathrm{I}_b}\right] \tag{10}$$

where [ ] above are evaluated at $\mathrm{C}^\pm_a, \mathrm{R}_a, \mathrm{I}_a \to 0$. Focusing on the number of quadratic couplings, $\mathcal{V}_2$ in Eq. (2) has $n(n+1)/2$ of them (n μ^2_a and $n(n-1)/2$ μ^2_{ab} with $a < b$) to be compared with the $2n - 1$ (independent) stationarity conditions. For $n = 2$ both numbers match and the stationarity conditions can be used to trade all quadratic couplings for quartic couplings × vevs. For $n > 2$, one is quickly driven into an overabundance of quadratic couplings with respect to stationarity conditions. This could lead one to expect that the existing free quadratic couplings can drive arbitrarily large *new* scalar masses, *new* meaning neither the would-be Goldstone bosons (wbG) nor the light Higgs-like state. As we prove analytically, that reasonable expectation is surprisingly misled.

3. Analysis

If one were capable of carrying out analytically the obtention of the eigenvalues of the charged and neutral scalar mass matrices for an arbitrary number of doublets, one would read out that, surprisingly, apart from the massless wbG and the Higgs-like scalar, there are one charged and two neutral scalars that are light, i.e. with masses not exceeding $O(v)$. Without this capacity, one can gain some insights through a numerical analysis as discussed in detail in [14]. Analytically, we can nevertheless proceed as follows. First, driving the new scalar masses to a regime of large values requires large quadratic terms $\gg v^2$. In that case, it might be worth considering the stationarity conditions and the mass matrices without quartic couplings *at all* since our numerical exercise hints us in that the light states are possibly independent of those couplings. Then, considering the whole problem without quartic couplings, this can only make sense if these unexpected states appear as eigenstates with eigenvalues equal to zero, i.e. null eigenvectors. That is the case, and the path that we will follow is: *(i)* write down the mass matrices in the mentioned regime in which quartic couplings are dropped, *(ii)* notice a property that reduces the problem of dealing with both charged $n \times n$ and neutral $2n \times 2n$ mass matrices to just dealing with the charged mass matrix, *(iii)* analyze the case of the wbG for a better understanding or inspiration. One can then think of the complete problem including quartic couplings as a (degenerate) perturbation theory problem where the contributions to the entries of the mass matrices from the quartic couplings are the perturbation. The important point to anticipate is that, consequently, null eigenvectors of the quadratic-couplings-alone mass matrices cannot yield scalars with squared masses much larger than the $\lambda \times v^2$ perturbation. Without quartic couplings, the scalar potential in Eqs. (1)-(3) is simply $\mathcal{V} \to \mathcal{V}_2$. The stationarity conditions are as in Eqs. (8) with $[\lambda' s] \to 0$. Not using (yet) the stationarity conditions, one can read the mass terms $-\mathscr{L}_{\text{Mass}} \subset \mathcal{V}_2$:

$$[M^2_\pm]_{a,a} = \mu^2_a,\ [M^2_\pm]_{a,b} = [M^2_\pm]^*_{b,a} = \frac{1}{2} e^{i\theta_{ab}} \mu^2_{ab},\ a < b, \quad M^2_0 = \begin{pmatrix} \mathrm{Re}\,(M^2_\pm) & \mathrm{Im}\,(M^2_\pm) \\ -\mathrm{Im}\,(M^2_\pm) & \mathrm{Re}\,(M^2_\pm) \end{pmatrix} \tag{11}$$

with $\mathrm{Re}\left(M_\pm^2\right)^T = \mathrm{Re}\left(M_\pm^2\right)$, $\mathrm{Im}\left(M_\pm^2\right)^T = -\mathrm{Im}\left(M_\pm^2\right)$. Now, if $M_\pm^2$ has a null eigenvector $\vec{c} \in \mathbb{C}^n$, $M_\pm^2\,\vec{c} = \vec{0}_n$; expanding real and imaginary parts, one immediately obtains

$$\mathrm{Re}\left(M_\pm^2\right)\mathrm{Re}\left(\vec{c}\right) - \mathrm{Im}\left(M_\pm^2\right)\mathrm{Im}\left(\vec{c}\right) = \vec{0}_n, \quad \mathrm{Im}\left(M_\pm^2\right)\mathrm{Re}\left(\vec{c}\right) + \mathrm{Re}\left(M_\pm^2\right)\mathrm{Im}\left(\vec{c}\right) = \vec{0}_n. \tag{12}$$

In matrix form, Eqs. (12) give

$$M_0^2\vec{r}_1 = \vec{0}_{2n}, \quad M_0^2\vec{r}_2 = \vec{0}_{2n}, \text{ with } \vec{r}_1 = \left(\mathrm{Re}\left(\vec{c}\right), -\mathrm{Im}\left(\vec{c}\right)\right)^T, \quad \vec{r}_2 = \left(\mathrm{Im}\left(\vec{c}\right), \mathrm{Re}\left(\vec{c}\right)\right)^T. \tag{13}$$

From *one* null eigenvector $\vec{c}$ of $M_\pm^2$, we obtain the *two* null eigenvectors $\vec{r}_1$ and $\vec{r}_2$ of M_0^2 in Eq. (13). We already know one null eigenvector $\vec{c}_G^{\,T} = (v_1, \ldots, v_n)$ of $M_\pm^2$. In terms of the doubling of $\vec{c}_G$ in two null eigenvectors of M_0^2, we have

$$\vec{r}_G^{\,T} = \left(\vec{0}_n, \quad v_1, \quad \ldots, \quad v_n\right), \; \vec{r}_h^{\,T} = \left(v_1, \quad \ldots, \quad v_n, \quad \vec{0}_n\right), \text{ then } M_0^2\vec{r}_G = M_0^2\vec{r}_h = \vec{0}_{2n}. \tag{14}$$

The vector $\vec{r}_G$ corresponds to the neutral wbG and $\vec{r}_h$ to the Higgs-like scalar, *is there another null eigenvector of* $M_\pm^2$*?* If that was the case, it would account for the additional two neutral and one charged scalars with electroweak-scale masses encountered in the numerical explorations. Consider

$$\vec{c}_j = v_j e^{i2\theta_j}, \text{ then } M_\pm^2\vec{c} = \left(\frac{e^{i2\theta_1}}{v_1}(\partial_{v_1}V_2 - i\partial_{\theta_1}V_2), \ldots, \frac{e^{i2\theta_n}}{v_n}(\partial_{v_n}V_2 - i\partial_{\theta_n}V_2)\right)^T \tag{15}$$

It is clear that $\vec{c}$ is indeed a null eigenvector of $M_\pm^2$ owing to the stationarity conditions. According to Eq. (13), $\vec{r}_1$ and $\vec{r}_2$ are null eigenvectors of M_0^2:

$$\vec{r}_1^{\,T} = \left(\mathrm{Re}\left(\vec{c}\right)^T, -\mathrm{Im}\left(\vec{c}\right)^T\right), \quad \vec{r}_2^{\,T} = \left(\mathrm{Im}\left(\vec{c}\right)^T, \mathrm{Re}\left(\vec{c}\right)^T\right) \text{ then } M_0^2\vec{r}_1 = M_0^2\vec{r}_2 = \vec{0}_{2n}. \tag{16}$$

Although $\vec{c}$ is not orthogonal to $\vec{c}_G$, and correspondingly $\vec{r}_1$ and $\vec{r}_2$ are not orthogonal to $\vec{r}_G$ and $\vec{r}_h$ in Eq. (14), since they are independent, one can always orthonormalize *à la* Gram-Schmidt.

Let us recap. Since masses much larger than the electroweak scale v can only be obtained with large quadratic couplings $\gg v^2$ when quartic couplings are bounded by perturbativity constraints, we have analyzed the mass matrices in the absence of quartic couplings. We have found that in this regime, besides the expected null eigenvectors of $M_\pm^2$ and M_0^2 associated to the wbG and the Higgs-like scalar, there are, unexpectedly, further null eigenvectors, one of $M_\pm^2$ and two of M_0^2. One can now think of the complete picture, including quartic couplings as a perturbation with respect to the previous analysis –it is indeed degenerate perturbation theory that must be considered–. Besides the wbG which remain massless, it is thus clear that out of the remaining null eigenvectors of the "no-quartics" mass matrices, $\vec{c}$ in Eq. (15), $\vec{r}_1$ and $\vec{r}_2$ in Eq. (16), together with $\vec{r}_h$ in Eq. (14), one charged and three neutral scalars get squared masses of order $(\lambda\text{'s})\times v^2$. They are *light*: considering perturbativity requirements on the quartic couplings, their masses cannot be much larger than the electroweak scale. More light states might be present when a regime other than "all quadratic couplings are much larger than v^2" is considered: the novel and relevant point is that with so few assumptions –a real nHDM with SCPV and bounded quartic couplings–, one can establish that, unexpectedly and no matter the number of doublets n, at least three new scalars *must* be light. Considering this result, phenomenological consequences command attention. The question is beyond the scope of the present work for several reasons. Three types of interactions of these scalars should be considered: *(i)* the ones among scalars arising from the quartic terms in Eq. (3), *(ii)* the ones arising from their Yukawa couplings to fermions, and *(iii)* the ones arising from the covariant derivatives in the kinetic terms $(D_\mu\Phi_a)^\dagger(D^\mu\Phi_a)$. Considering the minimality of our assumptions and the large freedom available in both the interactions among the scalars and their couplings to fermions, the third type of interaction, involving scalars and gauge bosons, appears a priori better suited on that respect if observables insensitive to the first two types of interactions were available. However, knowing that scalars with electroweak-scale masses might be present is not sufficient, a better understanding of the states is necessary. Even in the regime where, apart from the necessarily light states, all other scalars are much heavier, these eigenvectors –which define the mixings in the scalar sector– depend critically on the quartic couplings. If one is not in that regime and there are more light states, the situation is even more involved, precluding at this stage a generic approach to guaranteed or clearly promising discovery prospects.

4. Conclusions

Extended scalar sectors, in particular multi-Higgs doublets models, featuring spontaneous electroweak symmetry breaking, necessarily include a Higgs-like state with mass not larger than the electroweak scale if perturbativity requirements are imposed on the potential. Owing to unconstrained quadratic couplings, one naive expectation is that, generically, all new scalars could be made arbitrarily heavy, with masses much larger than the electroweak scale. We have analyzed the case of real nHDM with SCPV, where contrary to such expectations and despite the abundance of free quadratic couplings, at least one charged and two additional neutral states have masses that cannot be larger than the electroweak scale, also due to perturbativity requirements on the quartic couplings in the potential.

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