

## Non-vanilla Axion Solutions to the Strong CP Problem

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I review recent advances in model-building new solutions to the strong CP problem. The new models can either enhance the axion mass by small-size instanton effects or cancel the axion mass mechanism by soft-breaking terms of the Peccei-Quinn symmetry. Such models predict an axion discovery that lies outside the vanilla QCD axion band in the  $m_a$  vs.  $F_a$  plane relevant for a wide range of haloscope, helioscope, microwave cavity, and quantum sensor experiments.

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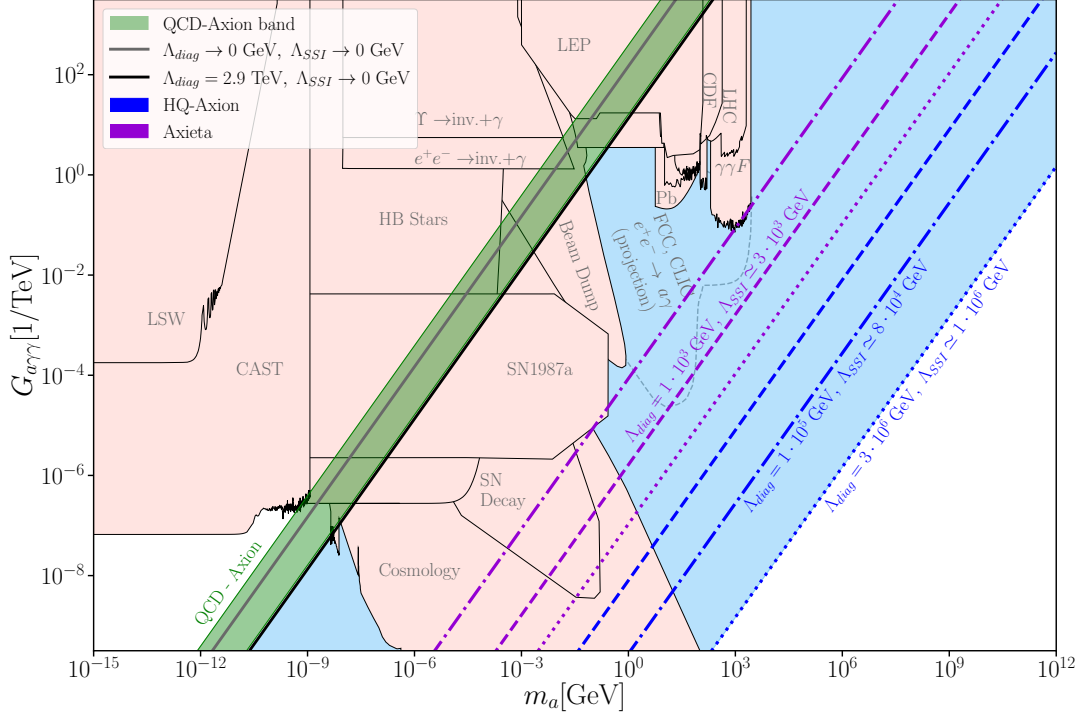
## 1. Introduction and Motivation

The longstanding strong CP problem asks why the neutron electric dipole moment,  $d_n$ , (nEDM) is consistent with being zero,  $|d_n| < 1.8 \times 10^{-26} e\text{cm}$  [1], even though the general expectation in the Standard Model (SM) is for a visible nEDM measurement 10 orders of magnitude larger. When considering the origin of the nEDM in terms of SM Lagrangian parameters, we can derive  $d_n = C_{\text{EDM}} e \bar{\theta}$ , where  $C_{\text{EDM}} = (2.4 \pm 1.0) \times 10^{-16}$  from running and matching quarks to the neutron energy scale [2] and  $\bar{\theta} = \theta + \arg \det Y_u Y_d$  [3] sums the quantum chromodynamics (QCD)  $\theta$  parameter for the dual Yang-Mills field strength and the overall phase of the quark Yukawa matrices responsible for the SM quark masses. Using the derived constraint from the nEDM collaboration, experiment dictates that  $|\bar{\theta}| < 7.5 \times 10^{-11}$ , while the two SM parameters controlling  $\bar{\theta}$  are each  $O(1)$  numbers and also have unrelated origins. Traditional classes of solutions to the strong CP problem have included the massless up quark solution (now excluded by lattice data [4], Nelson-Barr models [5, 6], and the Peccei-Quinn mechanism [7, 8] leading to DFSZ and KSVZ vanilla axion models [9–12]. Separately, recent work has revisited the technical foundation of the  $\theta$  vacuum in QCD and its role in the strong CP problem [13], but in these proceedings, I will focus on the canonical strong CP problem and the landscape of axion model solutions.

The Peccei-Quinn mechanism [7, 8] recognizes that an axial transformation of a vector-like set of QCD-charged fermions will redefine the  $\theta$  parameter, and hence, if this axial transformation maps out the vacuum manifold of a complex scalar field with such a global  $U(1)$  symmetry, the color anomaly of the global  $U(1)$  symmetry will force the pseudoscalar degree of freedom to acquire a tadpole and absorb the original  $\theta$  parameter. Notably, the continuous field definition of the pseudoscalar degree of freedom at tree-level is explicitly broken by the chiral anomaly with QCD to a periodic shift symmetry, and, after QCD confinement, the QCD topological susceptibility creates a tilt in the potential which gives the axion its mass. In DFSZ models, the  $U(1)_{PQ}$  anomaly is generated by coupling the PQ scalar to a two Higgs doublet extension of the SM Higgs sector, ensuring that the SM quarks generate a color anomaly for the tree-level Goldstone arising from the Higgs fields. In KSVZ models, the  $U(1)_{PQ}$  anomaly is generated directly by introducing a vector-like pair of new heavy quarks. In both cases, the global PQ symmetry is exact and accidental at the renormalizable level, leading to the fixed relationship  $m_a^2 F_a^2 \simeq \chi$ , where  $\chi \sim \Lambda_{\text{QCD}}^4$  is the QCD topological susceptibility.

## 2. Non-vanilla Models Solving Strong CP: Heavy Axions

It is important to consider non-minimal models that solve the strong CP problem for several reasons. First, vacuum angles are of central importance in CP studies, since the number of CP violating phases is typically a difficult counting problem. Second, non-decoupling effects in the vacuum structure of theories make the effective field theory descriptions highly non-trivial. Third, the chiral PQ anomaly connects to questions about the chiral nature of electroweak symmetry, which is a central aspect in addressing the hierarchy problem. Finally, non-vanilla axion models can have many phenomenological consequences, including addressing the axion quality problem, expanding the axion parameter space, and broadening the scope of axion cosmology.



**Figure 1:** Reproduced from Figure 5 of Ref. [15]. Axion bands of the color unified model variant M1 in the  $m_a$  vs.  $G_{a\gamma\gamma}$  plane. The band in green shows canonical DFSZ- and KSVZ-models, while the pink shaded regions show the current experimental bounds on axions and ALPs, and we include a contour from a future projection of CLIC sensitivity. The blue shaded region shows the possible parameter space for the new HQ-axion  $a$  and the axieta  $\eta_d$  enhanced by SSI effects, where lines in blue and violet show chosen models with the corresponding scales  $\Lambda_{\text{diag}}$  and  $\Lambda_{\text{SSI}}$ . A lower and upper bound on the  $\Lambda_{\text{diag}}$  and  $\Lambda_{\text{SSI}}$  scales arises because of experimental constraints on new color gauge group extensions discussed in the main text.

Based on the color-unified  $SU(6) \times SU(3)'$  model [14], we studied the effect of small-size instantons (SSI) on the axion potential to enhance the axion mass [15]. Here, by embedding the QCD gauge group into a larger non-Abelian gauge symmetry at higher scales, the topological susceptibility responsible for generating the axion potential gets new contributions, which we can illustrate by adopting the M1 variant of [14] and writing the following expression for the new  $m_a^2 F_a^2$  relationship:

$$\begin{aligned}
 m_a^2 F_a^2 &= 4(\Lambda_{\text{diag}}^4 + \Lambda_{\text{SSI}}^4) - 24\Lambda_{\text{diag}}^8 \\
 &\times \left| 2\Lambda_{\text{SSI}}^4 - \Lambda_{\text{diag}}^4 - 3(m_a^2 F_a^2)^{\text{KSVZ}} - \sqrt{\left(2\Lambda_{\text{SSI}}^4 - \Lambda_{\text{diag}}^4 - 3(m_a^2 F_a^2)^{\text{KSVZ}}\right)^2 + 24\Lambda_{\text{diag}}^8} \right|^{-1},
 \end{aligned} \tag{1}$$

$$\tag{2}$$

where  $(m_a^2 F_a^2)^{\text{KSVZ}}$  is the canonical KSVZ equation,  $\Lambda_{\text{diag}}$  is the confinement scale of the larger color gauge group, and  $\Lambda_{\text{SSI}}$  denotes the infrared SSI scale. Because of the new heavy quarks in the model, we also have a cousin of the SM  $\eta$  meson, denoted  $\eta_d$ , which plays an analogous role in its

mass generation:

$$m_{\eta_a}^2 F_a^2 = 2\Lambda_{\text{SSI}}^4 + 5\Lambda_{\text{diag}}^4 + 3(m_a^2 F_a^2)^{\text{KSVZ}} + \sqrt{\left(2\Lambda_{\text{SSI}}^4 - \Lambda_{\text{diag}}^4 - 3(m_a^2 F_a^2)^{\text{KSVZ}}\right)^2 + 24\Lambda_{\text{diag}}^8}. \quad (3)$$

These new degrees of freedom are axion particles that have masses orders of magnitude heavier than the vanilla axion band, as shown in Figure 1. In particular, these axions can be targets for collider experiments operating currently or in the near future. It is also important to recognize that the effective theory of such SSI-enhanced axions can be significantly modified when new  $U(1)'$  gauge symmetries are also present in the ultraviolet completion of the Standard Model [16].

### 3. Non-vanilla Models Solving Strong CP: Unusually Light Axions

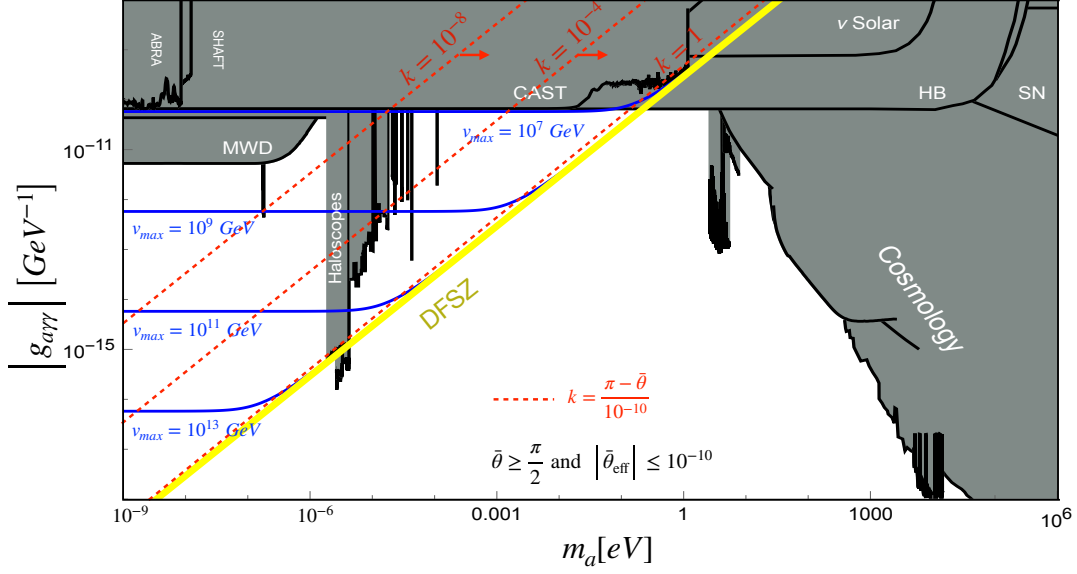
Instead of introducing new extensions of the color gauge symmetry to enhance the axion potential arising from an accidental tree-level PQ symmetry, we can also write explicit soft-breaking Lagrangian terms that control the quality of the tree-level PQ symmetry. In particular, such models help dictate the explicit parametric control between tree-level PQ symmetry and the effective description of the axion quality problem compared to the desired one-loop axion potential generated by the PQ color anomaly. As explored in [17], the soft-breaking PQ term can accidentally cancel the QCD anomaly-generated axion potential, leading to an interesting branch of parameter space where the axion becomes massless below the QCD scale. This is shown in Figure 2, where the effect of the soft-breaking of PQ symmetry cancels with the anomaly-induced axion potential, opening up the entire half-plane of unusually light axions to be discovered at current haloscopes, helioscopes, microwave cavity detectors, and future quantum sensors.

### 4. Conclusion

In this proceeding, I reviewed some recent progress in the field of axion model-building. I emphasized how new ideas about the treatment of Peccei-Quinn symmetry have led to new avenues for connecting ultraviolet physics to low-scale experiments via a deeper understanding of CP symmetry and chiral global symmetries. These examples demonstrate that the field of axion physics is rich and full of possibilities for future discovery, and that connections between low-energy discrete symmetries to ultraviolet physics will continue to play a vital role in resolving important theoretical puzzles such as the strong CP problem.

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**Figure 2:** Reproduced from Figure 2 of Ref. [17]. Parameter space for the axion-diphoton coupling in the Anarchic Axion model consistent with current nEDM constraint. Experimental limits are shown by the gray shaded regions. We show representative values of  $v_{\max}$  and  $k$  that highlight the accessible light axion parameter space probed by ongoing haloscope and microwave cavity experiments.

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