

## Baryon number violation involving tauons

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Baryon number violation is our most sensitive probe of physics beyond the Standard Model, especially through the study of nucleon decays. Angular momentum conservation requires a lepton in the final state of such decays, kinematically restricted to electrons, muons, or neutrinos. We show that operators involving tauons, which are at first sight too heavy to play a role in nucleon decays, still lead to clean nucleon decay channels with tau neutrinos. While many of them are already constrained from existing two-body searches such as  $p \rightarrow \pi^+ \nu$ , other operators induce many-body decays such as  $p \rightarrow \eta \pi^+ \bar{\nu}_\tau$  and  $n \rightarrow K^+ \pi^- \nu_\tau$  that have never been searched for.

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## 1. Introduction

The Standard Model (SM) of particle physics predicts baryon number  $B$  and lepton number  $L$  to be conserved in all interactions at the perturbative level [1]. Physics beyond the SM can in principle violate both  $B$  and  $L$ , famous examples being grand unified theories and supersymmetric SM extensions [2]. The experimental signatures are spectacular: atomic matter would decay! Lower limits on proton and neutron decays are unfathomably long, in some cases exceeding the age of our universe by 24 orders of magnitude (i.e.  $10^{34}$  yr [3]). Any baryon-number-violating proton or neutron decay requires an odd number of leptons in the final state to conserve angular momentum, e.g. in the form  $p \rightarrow e^+ \pi^0$  or  $p \rightarrow e^- \mu^+ \mu^+$ . While electrons, muons, and neutrinos are kinematically allowed final states, tauons are roughly twice as heavy as protons and hence cannot be produced. At first sight, this makes tauon decays such as  $\tau^+ \rightarrow p \pi^0$  the better  $\Delta B$  signature to search for. Alas, tauons are rather difficult to produce and detect, rendering these searches [4, 5] far less sensitive than proton-decay searches. The crucial observation was already made by Marciano [6] almost three decades ago: any operator or new-physics model that would lead to  $\tau^+ \rightarrow p \pi^0$  would also induce  $n \rightarrow \pi^0 \bar{\nu}_\tau$  or  $p \rightarrow \pi^+ \bar{\nu}_\tau$ , which are much more sensitive [7] despite the unobservable final-state tau-neutrino. In the worst-case scenario, one can expect comparable decay rates,

$$\Gamma(n \rightarrow \bar{\nu}_\tau \pi^0) \sim \Gamma(\tau^+ \rightarrow p \pi^0) \simeq \frac{1}{10^{33} \text{ yr}} \frac{\text{BR}(\tau^+ \rightarrow p \pi^0)}{10^{-53}}, \quad (1)$$

which would force the  $\Delta B$  tauon branching ratios at least 40 orders of magnitude below any currently conceivable experimental limits [8]. In Ref. [9], we quantified this connection more carefully, identified scenarios that violate it and allow for faster tauon decays, and emphasized the importance of neutrino final states in nucleon decays to study tauon operators [10].

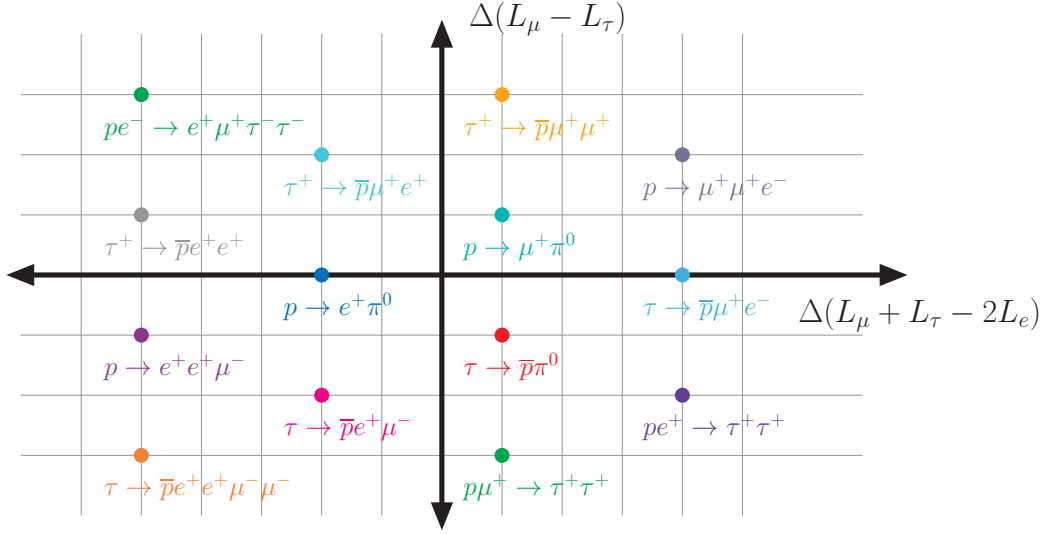
## 2. Dimension-six operators

In the Standard Model Effective Field Theory (SMEFT),  $\Delta B \neq 0$  operators start to appear at operator mass dimension  $d = 6$  [11]. These  $\Delta B = \Delta L = 1$  operators can be explicitly written as

$$\begin{aligned} \mathcal{L}_{d=6} = & y_{abcd}^1 \varepsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C u_{b,\beta}) (\bar{Q}_{i,c,\gamma}^C \varepsilon_{ij} L_{j,d}) + y_{abcd}^2 \varepsilon^{\alpha\beta\gamma} \varepsilon_{il} \varepsilon_{jk} (\bar{Q}_{i,a,\alpha}^C Q_{j,b,\beta}) (\bar{Q}_{k,c,\gamma}^C L_{l,d}) \\ & + y_{abcd}^3 \varepsilon^{\alpha\beta\gamma} (\bar{Q}_{i,a,\alpha}^C \varepsilon_{ij} Q_{j,b,\beta}) (\bar{u}_{c,\gamma}^C \ell_d) + y_{abcd}^4 \varepsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C u_{b,\beta}) (\bar{u}_{c,\gamma}^C \ell_d) + \text{h.c.}, \quad (2) \end{aligned}$$

where  $\alpha, \beta, \gamma$  denote the color,  $i, j, k, l$  the  $SU(2)_L$ , and  $a, b, c, d$  the family indices [11–15]. Operators involving the three lightest quarks can be converted to hadron operators using chiral effective field theory [2, 16] and lattice QCD [17], yielding mass-mixing terms  $\bar{p}^C \ell$  and  $\bar{n}^C \nu$  as well as interaction terms with mesons. The Wilson coefficients  $y^j$  have mass dimension  $-2$  and the first-generation entries are constrained to be  $< (\mathcal{O}(10^{15-16}) \text{ GeV})^{-2}$  due to the induced two-body nucleon decays such as  $p \rightarrow e^+ \pi^0$  [10, 18].

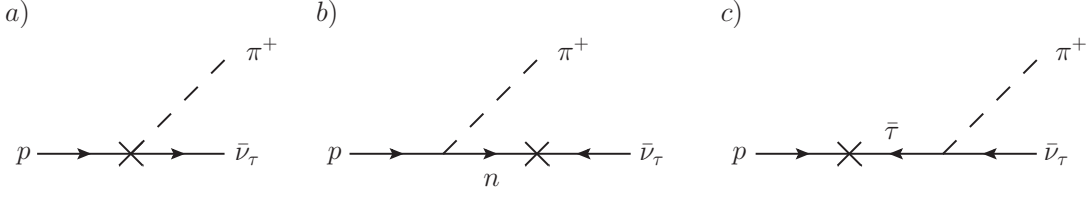
Since the three individual lepton numbers, electron, muon, and tauon, are conserved in the SM, operators with  $\Delta L_\tau = 1$  will never lead to  $\Delta L_\tau = 0$  processes. Even the observation of neutrino oscillations does not quantitatively change this conclusion: the continued absence of any charged-lepton flavor violation [19] can be taken as an indication that lepton flavor is only violated through



**Figure 1:** Landscape of  $\Delta B = \Delta L = 1$  operators organized by their lepton-flavor structure. We only show one example process for each group, others are implied. Experimental limits for  $p \rightarrow e^+\pi^0, \mu^+\pi^0$  [3] and  $p \rightarrow e^+e^+\mu^-, \mu^+\mu^+e^-$  [23] come from Super-Kamiokande, for  $\tau \rightarrow p\ell\ell$  (except for the missing channel  $\tau^+ \rightarrow \bar{p}\mu^+e^+$ ) from Belle [5], and for  $\tau \rightarrow \bar{p}\pi^0$  there exist 25-year-old limits from CLEO [4]. From Ref. [9].

neutrino masses. If any and all lepton flavor violation is suppressed by neutrino masses, the effects are near impossible to observe and render lepton flavor an incredibly good *approximate* symmetry in the charged lepton sector. SMEFT operators can then be organized according to their quantum numbers under the global SM symmetry group  $U(1)_{B+L} \times U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$ , seeing as these symmetries are either extremely good approximate or even exact symmetries [20, 21]. An example is shown in Fig. 1, organizing all  $\Delta B = \Delta L = 1$  operators/processes by their lepton-flavor content. Only the three groups closest to the origin ( $p \rightarrow e^+\pi^0$ ,  $p \rightarrow \mu^+\pi^0$ , and  $\tau \rightarrow \bar{p}\pi^0$ ) arise at  $d = 6$ , the others require  $d \geq 10$ . It is easy to impose lepton numbers as global or even local  $U(1)$  symmetries, broken only in the neutrino sector [21], that forbid all but one group in Fig. 1. Similarly, we can easily construct models in which baryon number is only broken together with some linear combination of lepton flavor [10, 22]. This is sufficient motivation for a dedicated study of  $\Delta B$  operators involving tauons, which are usually ignored due to the kinematics but could well be the only baryon-number violating processes in nature [9]. For example, if we impose  $U(1)_{B-L_\tau}$  on the  $d = 6$  operators from Eq. (2), we are left with tauon operators.

Using the Chiral Perturbation Theory (ChPT) framework from Refs. [2, 16], we can calculate the dominant baryon-number-violating tauon decays induced by the four  $d = 6$  operators, which are  $\Gamma(\tau^+ \rightarrow p\pi^0) \simeq \frac{1}{2}\Gamma(\tau^+ \rightarrow n\pi^+)$  and  $\Gamma(\tau^+ \rightarrow p\eta)$ . These  $\Delta B = \Delta L_\tau = 1$  tauon decays are constrained by CLEO to rates  $\Gamma(\tau^+ \rightarrow p\pi^0) < 1.5 \times 10^{-5} \Gamma_\tau \simeq 3.4 \times 10^{-17} \text{ GeV} \simeq (2 \times 10^{-8} \text{ s})^{-1}$  [4], and similarly for the  $\eta$  mode. Judging by their results in Ref. [5], Belle could improve these bounds by three orders of magnitude with their large existing data set, and Belle II could eventually improve them by another two orders of magnitude [8, 24]. These limits probe viable SMEFT parameter space, but the limits are nowhere near typical proton-decay scales, making it crucial to evaluate nucleon-decay channels mediated by the same operators.



**Figure 2:** Proton decay into  $\pi^+\bar{\nu}_\tau$  through various dimension-six operators, indicated with a cross [9].

$\mathcal{L}_{d=6}$  operators involving *left-handed* tausons unavoidably come with  $\nu_\tau$  operators that directly lead to  $p \rightarrow \bar{\nu}_\tau\pi^+$  [see Fig. 2a) and b)],  $n \rightarrow \pi^0\bar{\nu}_\tau$  and  $n \rightarrow \bar{\nu}_\tau\eta$ , and we generically expect the relationship of Eq. (1) to hold, confirmed by a more careful calculation [9]. This forces the Wilson coefficients at least 24 orders of magnitude down compared to the tauon limits, assuming one non-zero Wilson coefficient at a time. Operators involving *right-handed* tausons do not directly come with  $\nu_\tau$  operators and thus seemingly circumvent the dangerous nucleon decays into tau neutrinos; however, even these lead to  $p \rightarrow \pi^+\bar{\nu}_\tau$  through an off-shell tauon [see Fig. 2c)], as pointed out long ago by Marciano [6]. The off-shell tauon propagator and required mass flip do not cause any suppression since  $m_\tau \sim m_p$ , but the off-shell tauon *decay* comes with a  $G_F$  suppression, which gives roughly  $\Gamma(p \rightarrow \bar{\nu}_\tau\pi^+) \sim (G_F f_\pi)^2 \Gamma(\tau^+ \rightarrow p\pi^0) \simeq \text{BR}(\tau^+ \rightarrow p\pi^0)(10^{-6} \text{ yr})^{-1}$ , still forcing  $\text{BR}(\tau^+ \rightarrow p\pi^0)$  below  $10^{-40}$ .

The  $n \rightarrow \bar{\nu}_\tau$  channels give limits on the left-handed Wilson coefficients between  $(3 \times 10^{14} \text{ GeV})^{-2}$  and  $(2 \times 10^{15} \text{ GeV})^{-2}$ , probing scales at least eleven orders of magnitude above current tauon decay limits and ten orders of magnitude above future ones. Despite the relative suppression by  $G_F f_\pi^2 \sim 10^{-7}$  for the *right-handed* tauon operators, the resulting limits of order  $(5 \times 10^{11} \text{ GeV})^{-2}$  still far exceed any conceivable tauon-decay limits [25].

Importantly, all nucleon decays calculated so far only probe three linear combinations of the four  $d = 6$  Wilson coefficients. The remaining one corresponds to the operator  $(\bar{d}^C u)(\bar{u}^C P_R \tau) \propto \eta \bar{p}^C P_R \tau$ , which would then seemingly allow for a large  $\tau^+ \rightarrow p\eta$  decay rate without any competing nucleon decays.<sup>1</sup> Even the vector-meson final state  $p \rightarrow \rho^+\bar{\nu}_\tau$  vanishes in that limit [26]. Alas, the underlying operator of course still generates some form of nucleon decay, for example the three-body proton decay  $p \rightarrow \eta\pi^+\bar{\nu}_\tau$ . Compared to the two-body tauon decay  $\tau^+ \rightarrow p\eta$ , this proton decay rate is suppressed by a phase-space factor  $m_p^2/(32\pi^2 m_\tau^2)$  times partial phase-space closure from the large  $\eta$  mass, and of course still the  $G_F^2$  suppression from the off-shell tauon decay:

$$\Gamma(p \rightarrow \eta\pi^+\bar{\nu}_\tau) \simeq \frac{1}{200 \text{ yr}} \left( \frac{\text{BR}(\tau \rightarrow p\eta)}{8.9 \times 10^{-6}} \right). \quad (3)$$

Even though no dedicated exclusive search for the three-body decay  $p \rightarrow \eta\pi^+\bar{\nu}_\tau$  exists, old inclusive limits should exclude lifetimes below  $10^{30} \text{ yr}$  [10] and illustrate once again the disparity between  $\Delta B$  searches in nucleons and tausons, forcing the  $\Delta B$  tauon branching ratio into  $p\eta$  below  $10^{-30}$ .

This concludes our discussion of  $d = 6$  operators. Assuming one non-zero operator at a time, existing limits on two-body nucleon decays far outperform even optimistic  $\Delta B$  tausons decays. This

<sup>1</sup>The importance of  $\eta$  modes to probe this flat direction in the  $e$  and  $\mu$  cases has been pointed out recently in Ref. [18].

conclusion can be softened a bit by allowing for cancellations between operators, which can relegate nucleon decays to overlooked three- or four-body channels, but even they are indirectly constrained well enough to uphold the above conclusion.

### 3. Dimension-seven operators

Restricting ourselves to non-derivative operators for simplicity [13], there are four independent baryon-number-violating operators at  $d = 7$  [27, 28]:

$$\begin{aligned} \mathcal{L}_{d=7} = & z_{abcd}^1 \varepsilon^{\alpha\beta\gamma} (\bar{Q}_{i,a,\alpha}^C Q_{j,b,\beta}) (\bar{L}_{i,c} d_{d,\gamma}) H_j^* + z_{abcd}^2 \varepsilon^{\alpha\beta\gamma} \varepsilon_{ij} (\bar{u}_{a,\alpha}^C d_{b,\beta}) (\bar{L}_{i,c} d_{d,\gamma}) H_j^* \\ & + z_{abcd}^3 \varepsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C d_{b,\beta}) (\bar{\ell}_c Q_{i,d,\gamma}) H_i^* + z_{abcd}^4 \varepsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C d_{b,\beta}) (\bar{L}_{c,i} d_{d,\gamma}) H_i + \text{h.c.} \end{aligned} \quad (4)$$

The Wilson coefficients  $z^j$  now have mass dimension  $-3$ . Upon electroweak symmetry breaking,  $H \rightarrow (0, v/\sqrt{2})$ , with  $v \simeq 246$  GeV, these give  $\Delta B = -\Delta L = 1$  [13] four-fermion operators that can be translated to hadronic operators using ChPT.

$z^1$  and  $z^2$  boil down to  $udd\bar{\nu}_\tau$ , and thus  $n \rightarrow \nu_\tau \pi^0$  just like the  $d = 6$  operators, which gives hopelessly suppressed  $\Delta B$  tauon decay rates. The other two operators give  $d_R d_R d_{L,R} \bar{\tau}$ , which vanish if all  $d$  quarks are from the same generation. The leading operator is then  $d_{RS} d_{L,R} \bar{\tau}$ , which leads to  $p \rightarrow \pi^+ K^+ \ell$  or  $n \rightarrow K^+ \ell$  [13] at tree level; however, for the charged tauon this is kinematically forbidden, forcing us to go through an off-shell tauon to  $p \rightarrow \pi^+ K^+ \pi^- \nu_\tau$  or  $n \rightarrow K^+ \pi^- \nu_\tau$ . This competes with the two-body tauon decay  $\tau \rightarrow \Lambda^0 \pi^-$  constrained by Belle [29]:

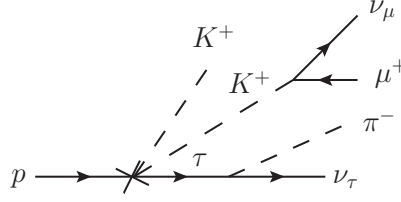
$$\Gamma(n \rightarrow K^+ \pi^- \nu_\tau) \simeq \left( \frac{\text{BR}(\tau \rightarrow \Lambda^0 \pi^-)}{7.2 \times 10^{-8}} \right) \begin{cases} \frac{1}{1.3 \times 10^5 \text{ yr}}, & z_3 = 0, \\ \frac{1}{7 \times 10^3 \text{ yr}}, & z_4 = 0. \end{cases} \quad (5)$$

No experimental constraints exist for the neutron decay, except for ancient inclusive limits [10], which push  $\text{BR}(\tau \rightarrow \Lambda^0 \pi^-)$  below  $10^{-30}$ , analogous to Eq. (3).

Let us give one more example of how far one would have to suppress nucleon decays to allow for testable  $\Delta B = 1$  tauon decays. We take  $z_{1232}^4$ , i.e. a  $dss$  operator, which induces a mixing of  $\tau^-$  with  $\Xi^-$ . This still allows for a kinematically allowed two-body tauon decay  $\tau \rightarrow \Xi \pi$ , but the double strangeness severely suppresses nucleon decays. Only at order  $f_\pi^{-2}$  do we find an operator  $\bar{\tau} P_R p K^- K^- \beta \nu z_{1232}^4 / (\sqrt{2} f_\pi^2)$  that allows for proton decay with emission of tauon and two kaons. The tauon is necessarily off-shell just like in previous examples, but now even one of the kaons needs to be off-shell too! This is then a doubly  $G_F$  suppressed five-body proton decay, illustrated in Fig. 3, estimated to

$$\Gamma(p \rightarrow K^+ \mu^+ \nu_\mu \pi^- \nu_\tau) \simeq \frac{1}{\mathcal{O}(10^{28}) \text{ yr}} \left( \frac{\text{BR}(\tau \rightarrow \Xi^- \pi^0)}{10^{-8}} \right), \quad (6)$$

where we approximated the amplitude – also including a factor  $m_\pi^2 m_\mu / (m_\tau^2 m_K^2)$  from the derivative  $K$  and  $\pi$  vertices and  $K$  and  $\tau$  propagators – as constant but included kaon, pion and muon mass in the phase space integration. Still, even with this immense suppression of nucleon decay due to  $G_F^2$  and the five-body phase space we are falling short of realistic viable lifetimes. Nevertheless, we urge our experimental colleagues to investigate  $\Delta B$  tauon decays into hyperons as they have the most suppressed associated nucleon decays.



**Figure 3:** Proton decay into  $K^+ \mu^+ \nu_\mu \pi^- \nu_\tau$  through the  $d = 7$  operator  $ds\bar{L}_\tau sH$ , indicated with a cross [9].

#### 4. Operators of higher mass dimension

Dimension-ten operators have been discussed in Refs. [10, 22], let us focus on one example operator here,  $QQul\bar{l}LH/\Lambda^6$ . Upon electroweak symmetry breaking, this can give  $u_L d_L u_R \bar{r}_R \ell_\alpha \ell_\beta \nu/\Lambda^6$ ; if  $\alpha$  and  $\beta$  correspond to electrons or muons, this operator generates the tauon decays  $\tau^+ \rightarrow \bar{p} e_\alpha^+ e_\beta^+$  shown in Fig. 1, recently searched for in Belle [5]. The same operator also generates  $p \rightarrow e_\alpha^+ e_\beta^+ \pi^- \nu_\tau$  through the off-shell tauon:

$$\Gamma(p \rightarrow e_\alpha^+ e_\beta^+ \pi^- \nu_\tau) \simeq \frac{1}{2 \times 10^4 \text{ yr}} \left( \frac{\text{BR}(\tau^- \rightarrow p e_\alpha^- e_\beta^-)}{3 \times 10^{-8}} \right). \quad (7)$$

Despite the four-body vs three-body phase space suppression and the fact that no exclusive limits on this channel exist, the proton decay will enforce a sufficient limit on the Wilson coefficient as to make the  $\Delta B$  tauon decay completely unobservable.

#### 5. Conclusions

Whether baryon number is conserved or not is an experimental question that will hopefully be answered eventually, for example by observing nucleon decays in detectors such as Super-Kamiokande or DUNE. No positive signal has been observed yet despite decade-long efforts, which could, however, simply mean that we are not looking in the right spots. From what little we know about the flavor structure of the SM, it could well be that baryon number is mainly or even only broken together with tauon number, which naively changes the expected signatures since tauons are heavier than nucleons. Indeed, searches for baryon-number-violating tauon decays such as  $\tau \rightarrow \bar{p} \pi^0$  have been performed. As pointed out long ago by Marciano, however, the underlying new physics will also generate nucleon decays such as  $p \rightarrow \pi^+ \bar{\nu}_\tau$ , which is far more sensitive. Here, we have studied the relationship between  $\Delta B$  nucleon and tauon decays quantitatively for a large number of new-physics operators to confirm Marciano's observation and scrutinize loopholes. As expected, we find that any operator that leads to  $\Delta B = 1$  tauon decays also leads to nucleon decays, the tauon flavor being carried away by tau neutrinos. However, it is not difficult to find examples in which the nucleon only decays in channels that have never been explicitly searched for, which significantly softens the relationship but does not practically change it, since even old weak inclusive limits are sufficient to beat tauon limits. We stress that this conclusion should in no way discourage anyone from searching for  $\Delta B$  tauon decays; if anything, this is meant to encourage broadening searches for nucleon decays, either by going beyond two-body final states, e.g.  $p \rightarrow \eta \pi^+ \bar{\nu}_\tau$  and  $n \rightarrow K^+ \pi^- \nu_\tau$ , or by improving inclusive searches [10].

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## References

- [1] P. Fileviez Perez, A. Pocar, K. S. Babu, L. J. Broussard, V. Cirigliano, S. Gardner, J. Heeck, E. Kearns, A. J. Long and S. Raby, *et al.* [arXiv:2208.00010 [hep-ph]].
- [2] P. Nath and P. Fileviez Perez, *Phys. Rept.* **441** (2007), 191-317 [arXiv:hep-ph/0601023].
- [3] A. Takenaka *et al.* [Super-Kamiokande], *Phys. Rev. D* **102** (2020), 112011 [arXiv:2010.16098 [hep-ex]].
- [4] R. Godang *et al.* [CLEO], *Phys. Rev. D* **59** (1999), 091303 [arXiv:hep-ex/9902005].
- [5] D. Sahoo *et al.* [Belle], *Phys. Rev. D* **102** (2020), 111101 [arXiv:2010.15361 [hep-ex]].
- [6] W. J. Marciano, *Nucl. Phys. B Proc. Suppl.* **40** (1995), 3-15.
- [7] K. Abe *et al.* [Super-Kamiokande], *Phys. Rev. Lett.* **113** (2014), 121802 [arXiv:1305.4391 [hep-ex]].
- [8] S. Banerjee, *Universe* **8** (2022) no.9, 480 [arXiv:2209.11639 [hep-ex]].
- [9] J. Heeck and D. Watkins, *JHEP* **07** (2024), 170 [arXiv:2405.18478 [hep-ph]].
- [10] J. Heeck and V. Takhistov, *Phys. Rev. D* **101** (2020), 015005 [arXiv:1910.07647 [hep-ph]].
- [11] S. Weinberg, *Phys. Rev. Lett.* **43** (1979), 1566-1570.
- [12] F. Wilczek and A. Zee, *Phys. Rev. Lett.* **43** (1979), 1571-1573.
- [13] S. Weinberg, *Phys. Rev. D* **22** (1980), 1694.
- [14] H. A. Weldon and A. Zee, *Nucl. Phys. B* **173** (1980), 269-290.
- [15] L. F. Abbott and M. B. Wise, *Phys. Rev. D* **22** (1980), 2208.
- [16] M. Claudson, M. B. Wise and L. J. Hall, *Nucl. Phys. B* **195** (1982), 297-307.
- [17] J. S. Yoo, Y. Aoki, P. Boyle, T. Izubuchi, A. Soni and S. Syritsyn, *Phys. Rev. D* **105** (2022), 074501 [arXiv:2111.01608 [hep-lat]].
- [18] A. B. Beneito, I. J. Gargalionis, J. Herrero-Garcia, A. Santamaria and M. A. Schmidt, *JHEP* **07** (2024), 004 [arXiv:2312.13361 [hep-ph]].
- [19] S. Davidson, B. Echenard, R. H. Bernstein, J. Heeck and D. G. Hitlin, [arXiv:2209.00142 [hep-ex]].

- [20] J. Heeck, Phys. Lett. B **739** (2014), 256-262 [arXiv:1408.6845 [hep-ph]].
- [21] J. Heeck, Phys. Rev. D **95** (2017), 015022 [arXiv:1610.07623 [hep-ph]].
- [22] T. Hambye and J. Heeck, Phys. Rev. Lett. **120** (2018), 171801 [arXiv:1712.04871 [hep-ph]].
- [23] M. Tanaka *et al.* [Super-Kamiokande], Phys. Rev. D **101** (2020), 052011 [arXiv:2001.08011 [hep-ex]].
- [24] L. Aggarwal *et al.* [Belle-II], [arXiv:2207.06307 [hep-ex]].
- [25] W. S. Hou, M. Nagashima and A. Soddu, Phys. Rev. D **72** (2005), 095001 [arXiv:hep-ph/0509006].
- [26] O. Kaymakcalan, C. H. Lo and K. C. Wali, Phys. Rev. D **29** (1984), 1962.
- [27] L. Lehman, Phys. Rev. D **90** (2014), 125023 [arXiv:1410.4193 [hep-ph]].
- [28] Y. Liao and X. D. Ma, JHEP **11** (2016), 043 [arXiv:1607.07309 [hep-ph]].
- [29] Y. Miyazaki *et al.* [Belle], Phys. Lett. B **632** (2006), 51-57 [arXiv:hep-ex/0508044].