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Branching fractions and CP asymmetries in charm meson decays

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I present a consistent way to include η-η' mixing in global analyses of two-body decays of heavy hadrons employing the approximate flavour-SU(3) symmetry of QCD. The framework is applied to $D \to P\eta'$ decays, where P denotes a pseudoscalar meson. The result shows that flavour-SU(3) symmetry holds in the decay rates of these modes to better than 30%. With future data we expect the branching ratios of $D_s \to K^+\eta'$ and $D \to K^+\eta'$ to move upward and downward by $\sim 1\sigma$, respectively. Subsequently I discuss the implications of the LHCb measurements of the CP asymmetries in $D \to K^+K^-$ and $D \to \pi^+\pi^-$ for generic scenarios of new physics. New-physics contributions should have imprints on other CP asymmetries as well and can be tested through sum rules. Promising decays are $D_s^+ \to K^0\pi^+$, $D^+ \to \bar{K}^0K^+$, $D^0 \to K^0\bar{K}^{*0}$, $D^0 \to \bar{K}^0K^{*0}$, $D_s^+ \to K^{*0}\pi^+$, and $D^+ \to \bar{K}^{*0}K^+$.

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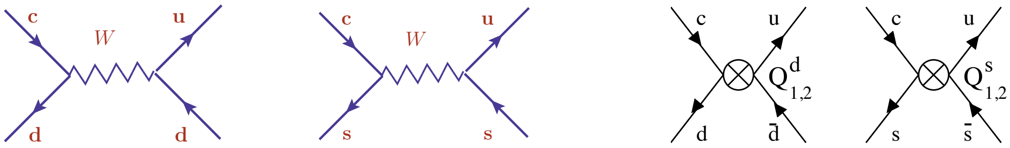

Figure 1: Tree-level contribution to singly Cabibbo-suppressed (SCS) charm decays in the Standard Model and the weak effective theory.

1. Overview

In the Standard Model (SM) flavour-changing transitions are encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{1}$$

with Wolfenstein parameters λ, A, ρ, η. Charm decays involve the red and blue CKM elements and have no stakes in Standard-Model (SM) CKM metrology. Yet they play a unique role in probing new physics in the flavour sector of up-type quarks.

I discuss decays of D^0, D^+, D_s^+ mesons into two pseudoscalar mesons, $D \to PP'$, or a pseudoscalar and a vector meson, $D \to PV$. All these decays are dominated by a W-mediated tree amplitude, categorised by the power of the Wolfenstein parameter $\lambda = 0.225$:

- Cabibbo-favoured (CF), $O(\lambda^0)$, $c \to s\bar{d}u$.
- Singly Cabibbo-suppressed (SCS), $O(\lambda^1)$, $c \to d\bar{d}u$ or $c \to s\bar{s}u$.
- Doubly Cabibbo-suppressed (DCS), $O(\lambda^2)$, $c \to d\bar{s}u$.

The tree diagrams for SCS decays are shown in Fig. 1. Since the energy scale of D meson decays is way below the mass of the W boson, we can describe these decays by a point-like four-fermion interaction in analogy to the Fermi interaction, resulting in the operators Q_2^d and Q_2^s with $Q_2^q \equiv \bar{q}_L^\alpha \gamma_\mu c_L^\alpha \, \bar{u}_L^\beta \gamma^\mu q_L^\beta$. Here α and β are colour indices. The resulting weak effective theory is set up to accomodate Quantum Chromodynamics (QCD) effects, which requires to include the colour-flipped operators $Q_1^q \equiv \bar{q}_L^\alpha \gamma_\mu c_L^\beta \, \bar{u}_L^\beta \gamma^\mu q_L^\alpha$ in our weak effective lagrangian.

Branching fractions in $D \to PP'$ or $D \to PV$ decays are insensitive to new physics and are "bread and butter" physics to test the calculational tools and check the data for consistency. On the contrary, CP asymmetries are tiny in the SM and thus very sensitive to new physics. SCS decays involve

$$\lambda_d = V_{cd}^* V_{ud}, \qquad \lambda_s = V_{cs}^* V_{us}, \qquad \lambda_b = V_{cb}^* V_{ub}. \tag{2}$$

Note that $|\lambda_d| \simeq |\lambda_s| \gg |\lambda_b|$. By using CKM unitarity, $\lambda_d = -\lambda_s - \lambda_b$, one verifies that *all* SM CP asymmetries are proportional to

$$\operatorname{Im} \frac{\lambda_b}{\lambda_s} = -6 \cdot 10^{-4}. \tag{3}$$

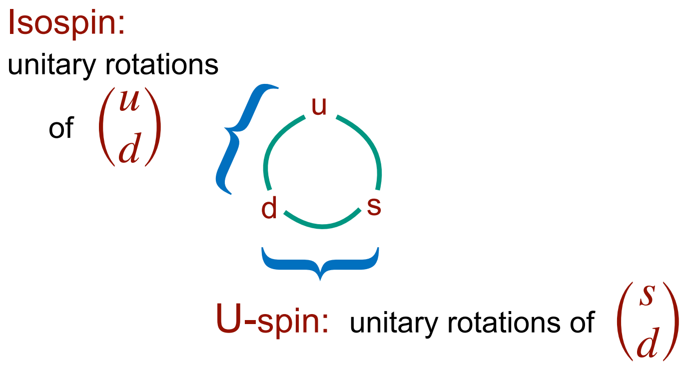

Figure 2: The $SU(3)_\mathrm{F}$ symmetry rotating the quark triplet $(u,d,s)^T$ would be an exact symmetry if the three quarks had the same mass. While isospin symmetry holds with an accuracy of about 2%, U-spin symmetry is broken by 20-30% due to $m_s \neq m_d$.

Therefore even in decays in which the two large tree-level amplitudes $c \to d\bar{d}u$ and $c \to s\bar{s}u$ interfere, the resulting CP asymmetry involves the suppression factor in Eq. (3) owing to $\mathrm{Im}\,\frac{\lambda_d}{\lambda_s} = -\mathrm{Im}\,\frac{\lambda_b+\lambda_s}{\lambda_s} = -\mathrm{Im}\,\frac{\lambda_b}{\lambda_s}$. Prominent sample decays for this tree-tree interference are $D^0 \to K_S K_S$ and $D^0 \to K_S K^{*(0)}$ [1, 2].

There are no reliable methods to perform dynamical calculations for exclusive hadronic decays of charmed hadrons. But it is possible to use the approximate global symmetry $SU(3)_\mathrm{F}$ of QCD, which corresponds to unitary rotations of the quark triplet $(u,d,s)^T$, to relate the amplitudes of different decays to each other. The subscript "F", meaning "flavour", is added to distinguish $SU(3)_\mathrm{F}$ from the QCD colour gauge symmetry $SU(3)_c$. The two most prominent $SU(2)$ subgroups of $SU(3)_\mathrm{F}$ correspond to the *isospin* and *U-spin* subgroups explained in Fig. 2. There is a long history of global $SU(3)_\mathrm{F}$ analyses of hadronic two-body decays of heavy hadrons. In Ref. [3] an analysis of branching ratios of B decays including linear (*i.e.* first-order) $SU(3)_\mathrm{F}$ breaking has been presented. Corresponding analyses for D decays can be found in Refs. [4–7]. In the practical implementation of $SU(3)_\mathrm{F}$ breaking one treats the corresponding piece $H_{\cancel{SU(3)}_\mathrm{F}} = (m_s - m_d)\bar{s}s$ of the QCD hamiltonian as a perturbation. (We do not consider isospin breaking effects; in the $SU(3)_\mathrm{F}$ symmetry limit the three light quarks have the common mass m_d.) Including linear $SU(3)_\mathrm{F}$ breaking permits the reduction of the intrinsic $O(30\%)$ error of the predictions to an uncertainty of $O(10\%)$. Such global analyses involve theoretical building blocks, *i.e.* complex parameters entering the various decay amplitudes in different combinations. Higher orders in the $SU(3)_\mathrm{F}$ breaking parameter $m_s - m_d$ bring more such parameters into the game, so that successful predictions require good data on sufficiently many decay modes.

2. η-η' mixing angle and D $\to$ Pη' decays

In this section I discuss the results of Ref. [7].

The pseudoscalar meson $SU(3)_\mathrm{F}$ octet comprises the states of $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0$ and η_8. $SU(3)_\mathrm{F}$ breaking leads to mixing of the latter with the singlet η_1. This feature is commonly

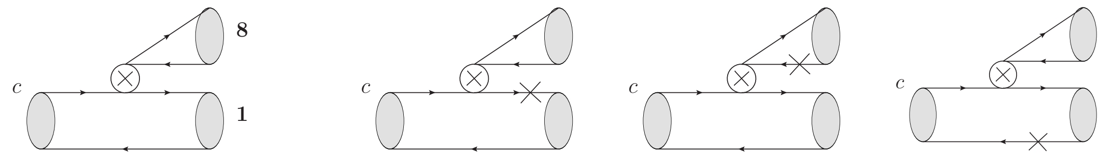

Figure 3: Sample topological amplitudes. The left diagram shows a $SU(3)_\mathrm{F}$ limit contribution for a D meson decay into a singlet-octet final state. The three other diagrams depict first-order $SU(3)_\mathrm{F}$-breaking contributions, with the cross on a strange-quark line indicating the Feynman rule for $H_{\not{\mathrm{SU(3)_F}}}$.

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parametrised in term of a mixing angle θ as

$$\begin{aligned} |\eta_8\rangle &= |\eta\rangle \cos\theta + |\eta'\rangle \sin\theta \\ |\eta_1\rangle &= -|\eta\rangle \sin\theta + |\eta'\rangle \cos\theta \end{aligned} \tag{4}$$

The mixing angle θ vanishes in the limit of exact $SU(3)_\mathrm{F}$ symmetry. $SU(3)_\mathrm{F}$ breaking leads to non-zero off-diagonal terms in the η, η' mass matrix and we define θ as the angle diagonalising this matrix.

Global $SU(3)_\mathrm{F}$ analyses of heavy hadron decays relate matrix elements with π or K to those with η_8. To express the latter in terms of matrix elements with physical mesons, it is common practice to use Eq. (4) schematically as

$$\langle \eta \ldots | \ldots | \ldots \rangle = \langle \eta_8 \ldots | \ldots | \ldots \rangle \cos\theta - \langle \eta_1 \ldots | \ldots | \ldots \rangle \sin\theta, \tag{5}$$

with a similar expression for $\langle \eta' \ldots | \ldots | \ldots \rangle$ and to treat θ as a universal parameter. However, such an approach is inconsistent. Specifying to our case of interest, we write the D decay matrix elements as

$$\begin{aligned} \langle P\eta|H|D\rangle &= \cos\theta \langle P\eta_8 H|D\rangle - \sin\theta \langle P\eta_1|H|D\rangle, \\ \langle P\eta'|H|D\rangle &= \sin\theta \langle P\eta_8 H|D\rangle' + \cos\theta \langle P\eta_1|H|D\rangle'. \end{aligned}$$

Yet these matrix elements are three-point functions and depend on kinematic variables built from the momenta p_D, p_η, and $p_{\eta'}$ of the three mesons involved. Since η and η' have different masses, $p_\eta^2 \neq p_{\eta'}^2$, one concludes that

$$\begin{aligned} \langle P\eta_8|H|D\rangle' &\neq \langle P\eta_8 H|D\rangle \\ \langle P\eta_1|H|D\rangle' &\neq \langle P\eta_1 H|D\rangle. \end{aligned} \tag{6}$$

An immediate consequence of this observation is that there is no point in combining $D \to P\eta$ and $D \to P\eta'$ decays into a common analyses. However, one can still perform such a fit for $D \to P\eta'$ decays alone or instead do this for all $D \to P\eta$ decays in conjunction with other D decays into octet-octet final states.

The mixing-angle problem was first addressed in the context of η and η' decay constants [8–10] by introducing different mixing angles for different decay modes. This approach cannot be applied to global $SU(3)_\mathrm{F}$ analyses, in which the $|\eta_8\rangle$ component of $|\eta\rangle$ must be related to $|\pi\rangle$ and $|K\rangle$. In our approach of Ref. [7] we instead insisted on a universal mixing angle θ defined solely from the

η, η' mass matrix. Thus θ in Eq. (4) is solely defined in terms of the strong interaction, as opposed to definitions employing electromagnetic or weak decay matrix elements.* The drawback of our definition is that θ is not directly related to any physical observable. In our analysis θ counts as first order in the $SU(3)_\mathrm{F}$ breaking parameter and always appears together with hadronic quantities parametrising first-order corrections to decay matrix elements. Only the sum of the product of $\sin\theta$ with some $SU(3)_\mathrm{F}$-limit matrix elements and certain $SU(3)_\mathrm{F}$-breaking corrections to these matrix elements is physical, so that θ cannot be determined from the global fit.

We have performed a Frequentist statistical analysis using the branching fractions of

$$\begin{array}{llll} D^0 \to \pi^0\eta', & D^0 \to \eta\eta', & D^+ \to \pi^+\eta', & D_s^+ \to K^+\eta', \\ D^0 \to \bar{K}^0\eta', & D_s^+ \to \pi^+\eta', & D^0 \to K^0\eta', & D^+ \to K^+\eta'. \end{array}$$

The complex hadronic quantities serving as building blocks for the decay amplitudes are topological amplitudes [12, 13], see Fig. 3. The topological amplitudes describing $SU(3)_\mathrm{F}$-breaking corrections from $H_{\not{SU(3)}_\mathrm{F}} = (m_s - m_d)\bar{s}s$ involve a cross on a strange-quark line [3, 6]. Alternatively, one can use the reduced matrix elements of the Wigner-Eckart theorem as building blocks [4, 5]. In Ref. [6] the mapping between these reduced matrix elements and the topological amplitudes have been presented, showing the equivalence of both approaches.

The main results of our analyses are

- The global fit is consistent with $\leq$30% $SU(3)_\mathrm{F}$ breaking in the amplitudes.
- The $SU(3)_\mathrm{F}$ limit is ruled out by 5.6σ.
- The fit predicts the branching fractions of $D_s^+ \to K^+\eta'$ and $D^+ \to K^+\eta'$ by $\sim 1\sigma$ too low and too high, respectively, see Fig. 4.

3. CP asymmetries in hadronic two-body D decays

In this section I discuss the results of Ref. [14].

For $SU(3)_\mathrm{F}$ analyses it is useful to decompose the decay amplitude $\mathcal{A}^\mathrm{SCS}$ of a $D \to PP'$ or $D \to PV$ decays in terms of U-spin representations as [15]

$$\mathcal{A}^\mathrm{SCS} \equiv \lambda_{sd} A_{sd} - \frac{\lambda_b}{2} A_b \tag{7}$$

with

$$\lambda_{sd} = \frac{\lambda_s - \lambda_d}{2} \simeq \lambda_s, \qquad -\frac{\lambda_b}{2} = \frac{\lambda_s + \lambda_d}{2}. \tag{8}$$

For A_{sd} and A_b are $|\Delta U| = 1$ (triplet) and $\Delta U = 0$ (singlet) transitions, respectively. Since D^0 carries no U-spin, in D^0 decays these quantum numbers directly translate into those of the final state. Eq. (8) translates to the more commonly used "tree" and "penguin" amplitudes as

$$\text{"tree"} \simeq A_{sd}, \qquad \text{"penguin"} \simeq -\frac{A_b}{2}. \tag{9}$$

*Also in lattice gauge theory the η-η' mixing angle is defined via physical matrix elements and therefore process-dependent, see *e.g.* [11].

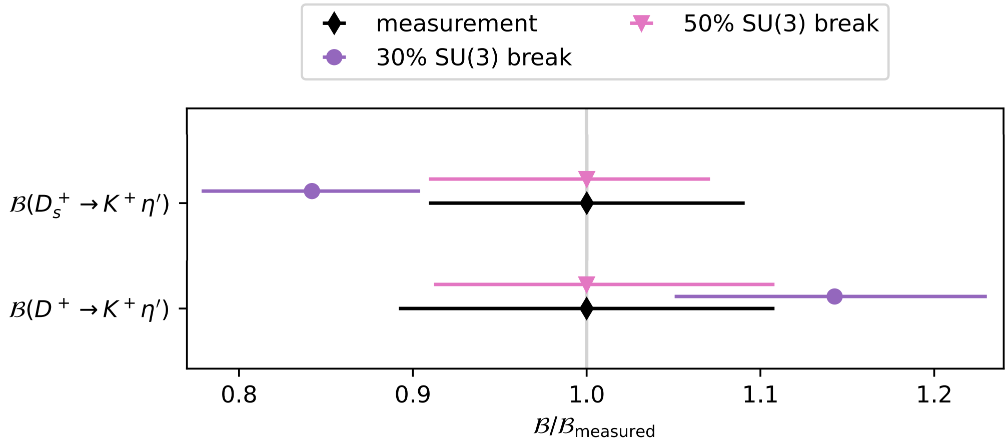

Figure 4: If the size of the $SU(3)_F$-breaking contributions is limited to 30%, the predictions for the two shown branching ratios deviate slightly from their measurements. If one permits 50% $SU(3)_F$ breaking, the global fit essentially reproduces the experimental input. See Ref. [7] for details.

In the SM direct CP violation stems from the interference of A_{sd} and A_b. The corresponding CP asymmetry for the decay $D \to f$ reads

$$a_{CP}^{\text{dir}}(D \to f) \equiv \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})} \simeq \text{Im}\frac{\lambda_b}{\lambda_{sd}} \, \text{Im}\frac{A_b(D \to f)}{A_{sd}(D \to f)}. \tag{10}$$

Here $\Gamma(D \to f)$ denotes the decay rate and $\bar{f}$ is the CP-conjugate final state to f. In Eqs. (8) to (10) "$\simeq$" means that sub-leading terms in λ_b/λ_{sd} have been neglected.

3.1 $\mathbf{D^0 \to \pi^+\pi^-}$ and $\mathbf{D^0 \to K^+K^-}$

On March 21, 2019, the LHCb collaboration announced the discovery of charm CP violation through the measurement [16]

$$\Delta a_{CP} \equiv a_{CP}^{\text{dir}}(D^0 \to K^+K^-) - a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = (-15.4 \pm 2.9) \cdot 10^{-4}. \tag{11}$$

LHCb has measured the time-integrated decays, so that Δa_{CP} may contain a contamination from mixing-induced CP violation. This potential contribution is much smaller than the error in Eq. (11) and we omit the superscript "dir" occasionally. In the U-spin limit $m_s = m_d$ one finds

$$A_b(D^0 \to K^+K^-) = A_b(D^0 \to \pi^+\pi^-), \qquad A_{sd}(D^0 \to K^+K^-) = -A_{sd}(D^0 \to \pi^+\pi^-). \tag{12}$$

so that

$$\Delta a_{\text{CP}} = 2a_{\text{CP}}(D^0 \to K^+K^-) = -2a_{\text{CP}}(D^0 \to \pi^+\pi^-) \tag{13}$$

The interfering diagrams contributing to $a_{\text{CP}}(D^0 \to K^+K^-)$ are shown in Fig. 5. The measurement exceeds the QCD light-cone sum rule prediction [17]

$$|\Delta a_{CP}| \leq (2.0 \pm 0.3) \cdot 10^{-4} \tag{14}$$

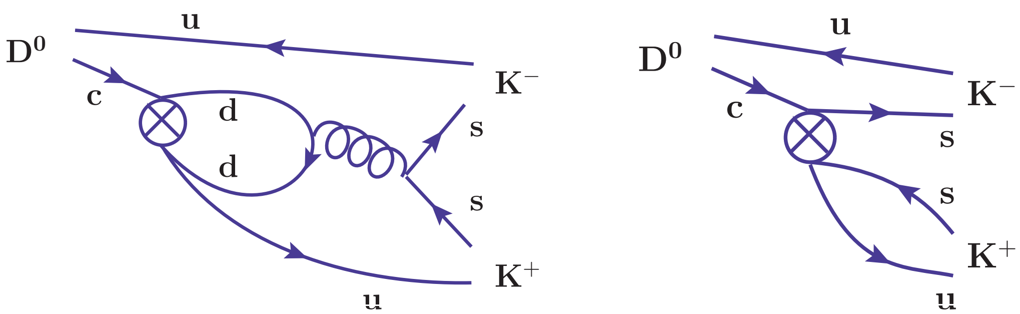

Figure 5: Dominant SM contributions to A_b (left) and A_{sd} (right). $a_{\rm CP}(D^0 \to K^+K^-)$ is proportional to $\text{Im}\,\frac{\lambda_d}{\lambda_s} = -\text{Im}\,\frac{\lambda_b}{\lambda_s}$ and $\text{Im}\,\frac{A_b}{A_{sd}}$, see Eq. (3). A non-vanishing $\text{Im}\,\frac{A_b}{A_{sd}}$ requires rescattering, meaning that the intermediate $\bar{u}d\bar{d}u$ state is an on-shell $\pi\pi$ or multi-hadron state which scatters into K^+K^-.

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by a factor of 7.† QCD sum rules constitute a sound method of dynamical calculations of hadronic quantities, which has proven to yield correct predictions for various observables in B physics. In the field of charm physics QCD sum rules are not well tested yet; a key feature of this method is that the sum over certain hadronic contributions is calculated in perturbation theory. Since D mesons are lighter than their beautiful counterparts, an individual resonance could dominate an amplitude and this approach might fail in charm physics. In Ref. [19] it has been suggested that $U = 0$ resonances like $f_0(1710)$ and $f_0(1790)$ could enhance $A_b(D^0 \to K^+K^-)$ and $A_b(D^0 \to \pi^+\pi^-)$. Thus the first puzzle of charm CP violation is

Is Δa_{CP} in Eq. (11) due to new physics or poorly understood QCD dynamics enhancing penguin amplitudes?

In 2022 LHCb has measured [20]

$$a_{CP}(D^0 \to K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4} \tag{15}$$

entailing

$$a_{\rm CP}(D^0 \to \pi^+\pi^-) = (23.1 \pm 6.1) \cdot 10^{-4} \tag{16}$$

from Δa_{CP} in Eq. (11). From Eqs. (13) and (11) we conclude that one expects $a_{CP}(D^0 \to K^+K^-) = -a_{\rm CP}(D^0 \to \pi^+\pi^-) = -(7.7 \pm 1.5) \cdot 10^{-4}$ in the limit of exact $SU(3)_{\rm F}$ symmetry. The central values in Eqs. (15) and (16) are far away from these $SU(3)_{\rm F}$ limit values, so that the situation is very different from branching fractions for which $SU(3)_{\rm F}$ breaking is at the nominal value of 30% or smaller [4, 6, 7]. The important observations are

- Eq. (15) complies with the QCD sum rule calculation in [17] at 1.1σ.
- With future data $a_{CP}(D^0 \to K^+K^-)$ in Eq. (15) must flip sign to comply with the approximate U-spin symmetry prediction $a_{\rm CP}(D^0 \to K^+K^-) \approx -a_{\rm CP}(D^0 \to \pi^+\pi^-)$.

†After the LHCb measurement this calculation was critically reviewed and essentially confirmed, with a slightly weaker bound, $|\Delta a_{CP}| \leq 3.6 \cdot 10^{-4}$ [18].

Thus the second puzzle of charm CP violation is

What causes the large violation of U-spin symmetry in
$a_{\rm CP}(D^0 \to K^+K^-)$ *vs.* $a_{\rm CP}(D^0 \to \pi^+\pi^-)$*?*

In view of the large error in Eq. (15) the second puzzle is not a severe problem yet, a future shift of $a_{\rm CP}(D^0 \to K^+K^-)$ by 2σ (to a negative value) will eliminate the second puzzle [14, 21]. If, however, we have to give up U-spin symmetry, also explanations of $\Delta a_{\rm CP}$ in terms of resonant enhancements [19] cannot be upheld, because large U-spin breaking associated with these resonances would also unduly enhance $|A_{sd}|$ and thereby branching fractions, spoiling the good agreement of the latter with approximate $SU(3)_{\rm F}$ symmetry.

Postulating new physics in $|\Delta U| = 1$ interactions solves the second puzzle. A generic new $|\Delta U| = 1$ interaction is proportional to

$$\overline{u}\Gamma c \left(\overline{d}\Gamma' d - \overline{s}\Gamma' s\right), \tag{17}$$

where the Dirac structures Γ and Γ' need not be specified for the presented symmetry-based analysis. The generic $\Delta U = 0$ interaction involves a "+" sign between the two terms and may have a second term proportional to $\overline{u}\Gamma c\, \overline{u}\Gamma u$ with an independent coupling strength. Subsuming these cases into

$$\mathcal{A}^{\rm SCS} \equiv \lambda_{sd} A_{sd} + a A_{\rm NP} \tag{18}$$

one finds, neglecting the SM penguin contribution,

$$a_{CP}^{\rm dir} = -2\,{\rm Im}\frac{a}{\lambda_{sd}}\,{\rm Im}\frac{A_{\rm NP}}{A_{sd}} \tag{19}$$

in analogy to Eq. (10). The two different cases come with different relative signs in the $A_{\rm NP}$ amplitudes of U-spin related decays. For example $A_{\rm NP}(D^0 \to K^+K^-) = -A_{\rm NP}(D^0 \to \pi^+\pi^-)$ in the U-spin limit for $|\Delta U| = 1$ new physics, while these amplitude are the same for the $\Delta U = 0$ case. The essential features of these scenarios is that $\Delta U = 0$ new physics is indistinguishable from an enhanced SM penguin amplitude A_b, while $|\Delta U| = 1$ new physics leads to different relations between CP asymmetries, in particular

$$a_{\rm CP}(D^0 \to K^+K^-) = a_{\rm CP}(D^0 \to \pi^+\pi^-). \tag{20}$$

in the U-spin limit if $|\Delta U| = 1$ new physics dominates over the SM A_b amplitude. Fig. 6 confronts the measurements in Eqs. (11) and (15) with the SM and new-physics scenarios. Clearly, Eq. (20) forbids an explanation in terms of $|\Delta U| = 1$ new physics alone and the corresponding orange line in Fig. 6 does not intersect the experimentally allowed range. We may speculate about new physics in $c \to u\bar{d}d$ decays, which contributes to $D^0 \to \pi^+\pi^-$ at tree level and to $D^0 \to K^+K^-$ through a penguin loop. The latter is suppressed by a colour factor of $1/N_c = 1/3$ w.r.t. the tree amplitude. If furthermore the strong phases of tree and penguin amplitude are similar, one will find $a_{\rm CP}(D^0 \to \pi^+\pi^-) \approx 3a_{\rm CP}(D^0 \to K^+K^-)$ which fits the data well, since the purple line in Fig. 6 spikes the center of the error ellipses. The $c \to u\bar{d}d$ case is an example of a scenario with both $|\Delta U| = 1$ and $\Delta U = 0$ new physics. Several authors have considered specific new-physics models to address Eqs. (11) and (15) [18, 22–26].

To summarise:

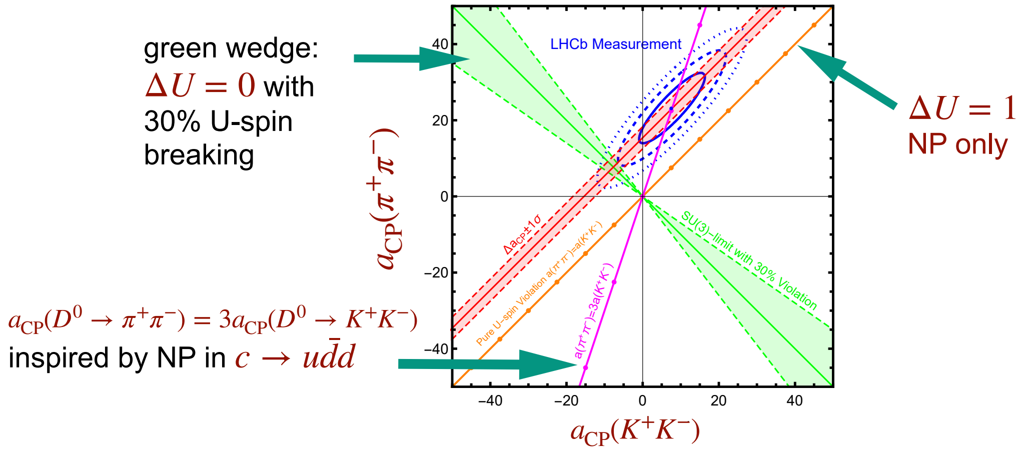

Figure 6: The blue ellipses show the 1σ, 2σ, and 3σ ranges of Eqs. (11) and (15). The green wedge covers the SM (allowing an arbitrarily large penguin amplitude A_b) and the case of $\Delta U = 0$ new physics, permitting 30% violation of U-spin symmetry. The orange line corresponds to $|\Delta U| = 1$ new physics with no $\Delta U = 0$ contribution. The scenario $a_{CP}(D^0 \to \pi^+\pi^-) = 3a_{CP}(D^0 \to K^+K^-)$ explains the data. Plot from Ref. [14].

- If $a_{CP}(D^0 \to \pi^+\pi^-)$ is governed by the SM...

  ... the QCD sum rule calculation does not work *and*
  ... either U-spin symmetry fails for A_b or in future measurements $a_{CP}(D^0 \to K^+K^-)$ will move down by 2σ from the value in Eq. (15) and flip sign.

- If $a_{CP}(D^0 \to \pi^+\pi^-)$ is dominated by new physics...

  ... the new-physics contribution necessarily has a $|\Delta U| = 1$ piece *and*
  ... there is an additional $\Delta U| = 0$ new-physics contribution or some enhancement of A_b over the sum rule prediction.

3.2 Sum rules for CP asymmetries

In order to shed light on the two puzzles mentioned above one must measure CP asymmetries in as many decays of charmed hadrons as possible. Employing U-spin symmetry we have derived sum rules between direct CP asymmetries of SCS D meson decays in Ref. [14] for the generic $|\Delta U| = 1$ and $\Delta U = 0$ new-physics scenarios. Such sum rules have been derived for the SM (and thereby also for $\Delta U = 0$ new physics) in Refs. [4, 27, 28]. Our findings for the $\Delta U = 0$ sum rules comply with those of Ref. [27], but we have found one extra sum rule each for $D \to PP'$ and $D \to PV$ decays. The sum rules for the $|\Delta U| = 1$ new-physics scenario are derived for the case of Eq. (17). The sum rules involve between two and ten CP asymmetries. Here I present only those which are easiest to test at LHCb. To this end it is useful to define

$$A_{CP}(D \to f) \equiv 2\bar{\Gamma}(D \to f)a_{CP}(D \to f) \;=\; \Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f}) \qquad (21)$$

with $\bar{\Gamma}(D \to f)$ being the average of $\Gamma(D \to f)$ and $\Gamma(\bar{D} \to \bar{f})$.

We find

$\Delta U = 0$	$\lvert\Delta U\rvert = 1$
$A_{\rm CP}(D^0 \to K^+K^-) + A_{\rm CP}(D^0 \to \pi^+\pi^-) = 0$	$A_{\rm CP}(D^0 \to K^+K^-) - A_{\rm CP}(D^0 \to \pi^+\pi^-) = 0$
$A_{\rm CP}(D_s^+ \to K^0\pi^+) + A_{\rm CP}(D^+ \to \bar{K}^0K^+) = 0$	$A_{\rm CP}(D_s^+ \to K^0\pi^+) - A_{\rm CP}(D^+ \to \bar{K}^0K^+) = 0$
$A_{\rm CP}(D^0 \to K^0\bar{K}^{*0}) + A_{\rm CP}(D^0 \to \bar{K}^0K^{*0}) = 0$	$A_{\rm CP}(D^0 \to K^0\bar{K}^{*0}) - A_{\rm CP}(D^0 \to \bar{K}^0K^{*0}) = 0$
$A_{\rm CP}(D_s^+ \to K^{*0}\pi^+) + A_{\rm CP}(D^+ \to \bar{K}^{*0}K^+) = 0$	$A_{\rm CP}(D_s^+ \to K^{*0}\pi^+) - A_{\rm CP}(D^+ \to \bar{K}^{*0}K^+) = 0$

For the remaining sum rules see Ref. [14]. To explain Eqs. (11) and (15) we need both $\Delta U = 0$ and $|\Delta U| = 1$ contributions. Since A_{CP} is linear in $A_{\rm NP} = A_{\rm NP}^{\Delta U=0} + A_{\rm NP}^{|\Delta U|=1}$ (and A_b), we can write

$$A_{CP}(D \to f) = A_{CP}(D \to f)^{\Delta U=0} + A_{CP}(D \to f)^{|\Delta U|=1} \tag{22}$$

with the first and second term obeying the sum rule of the first and second column of the table, respectively. Solving these equations for the decays in the first row, one finds

$$\begin{aligned} A_{CP}(D \to \pi^+\pi^-)^{\Delta U=0} &= -A_{CP}(D \to K^+K^-)^{\Delta U=0} \\ &= \frac{A_{CP}(D \to \pi^+\pi^-) - A_{CP}(D \to K^+K^-)}{2} \\ A_{CP}(D \to \pi^+\pi^-)^{|\Delta U|=1} &= A_{CP}(D \to K^+K^-)^{|\Delta U|=1} \\ &= \frac{A_{CP}(D \to \pi^+\pi^-) + A_{CP}(D \to K^+K^-)}{2} \end{aligned} \tag{23}$$

Taking $A_{CP}(D \to \pi^+\pi^-) = 2A_{CP}(D \to K^+K^-)$ as a numerical example one finds $A_{CP}(D \to \pi^+\pi^-)^{|\Delta U|=1} = 3A_{CP}(D \to K^+K^-)^{\Delta U=0}$, which thus fixes the relative size of the contributions from $A_{CP}^{|\Delta U|=1}$ and $A_b + A_{\rm CP}^{\Delta U=0}$ in the two studied decay modes. Since $SU(3)_{\rm F}$ symmetry is approximate one should include an uncertainty of order 30%.

An experimental advantage of Δa_{CP} compared to the individual CP asymmetries is the cancellation of the D^0 vs. $\bar{D}^0$ production asymmetry from the measured quantity. In Ref. [14] we have proposed similar combinations for the CP asymmetries entering our sum rules. For example, instead of measuring $a_{\rm CP}(D^+ \to \bar{K}^{*0}K^+)$ one could measure

$$\Delta a_{\rm CP,9}(D^+) \equiv a_{\rm CP}(D^+ \to \bar{K}^{*0}K^+) - a_{\rm CP}(D^+ \to \bar{K}^{*0}\pi^+) \tag{24}$$

The decay $D^+ \to \bar{K}^{*0}\pi^+$ has no SM CP violation and also new-physics contributions can barely compete with the CF tree amplitude governing this decay. So it is safe to assume that the subtracted CP asymmetry in Eq. (24) is much smaller than $a_{\rm CP}(D^+ \to \bar{K}^{*0}K^+)$. Playing the same game for $a_{\rm CP}(D_s^+ \to K^{*0}\pi^+)$ requires the subtraction of $a_{\rm CP}(D_s^+ \to K^0K^{*+})$ which, however, is DCS. DCS decays have no penguin amplitude but there could be a tree contribution from new physics [29]. A future measurement of a non-zero $\Delta a_{\rm CP,5}(D_s^+) \equiv a_{\rm CP}(D_s^+ \to K^{*0}\pi^+) - a_{\rm CP}(D_s^+ \to K^0K^{*+})$ will trigger an interesting discussion on which of the two decays is responsible for the finding. As a final remark, the production asymmetry is not an issue in the $\Delta U = 0$ scenario for $a_{\rm CP}(D^0 \to K^0\bar{K}^{*0})$ and $a_{\rm CP}(D^0 \to \bar{K}^0K^{*0})$, since these CP asymmetries can be measured without flavour tagging [2]. But searching for $|\Delta U| = 1$ new physics does require to distinguish D^0 from $\bar{D}^0$ decays, because $A_{\rm NP}^{|\Delta U|=1}$ drops out from $a_{\rm CP}\left(\overset{(-)}{D}{}^0 \to K^0\bar{K}^{*0}\right)$, where $\overset{(-)}{D}{}^0$ denotes an untagged D or $\bar{D}$ meson.

4. Summary

1. A universal η-η' mixing angle defined through unitary rotations of matrix elements with η_8 and η_1 is known since 27 years to be ill-defined. It is nevertheless commonly used in global $SU(3)_{\rm F}$ analyses of D or B decay data.
2. We have devised a consistent treatment of $\eta-\eta'$ mixing, which permits a global analysis of $D \to P\eta'$ or $D \to P\eta$ data, while it is not possible to relate the former decays to the latter.
3. A global fit to the branching ratios of $D^0 \to \pi^0\eta', D^0 \to \eta\eta', D^+ \to \pi^+\eta', D_s^+ \to K^+\eta', D^0 \to \bar{K}^0\eta', D_s^+ \to \pi^+\eta', D^0 \to K^0\eta'$, and $D^+ \to K^+\eta'$ complies with $\leq$30% $SU(3)_{\rm F}$ breaking, with slight tensions in $D_s^+ \to K^+\eta'$ and $D^+ \to K^+\eta'$.
4. The LHCb measurements $\Delta a_{CP} = (-15.4 \pm 2.9) \cdot 10^{-4}$ and $a_{CP}(D^0 \to K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4}$ are not consistent with the SM if U-spin symmetry holds approximately.
5. New physics explanations involve a $|\Delta U| = 1$ amplitude (with a different phase than $\arg(\pm V_{cs}^* V_{us})$) and a $\Delta U = 0$ amplitude (SM or NP) as well.
6. One can check this in the future in other decay modes in which CP asymmetries are not yet measured to be non-zero. To this end we have proposed sum rules between CP asymmetries.
7. Especially interesting for LHCb are sum rules relating $a_{\rm CP}(D_s^+ \to K^0\pi^+)$ to $a_{\rm CP}(D^+ \to \bar{K}^0K^+)$ as well as sum rules involving CP asymmetries in $D^0 \to K^0\bar{K}^{*0}$, $D^0 \to \bar{K}^0K^{*0}$, $D_s^+ \to K^{*0}\pi^+$, and $D^+ \to \bar{K}^{*0}K^+$.
8. In an experimental analysis one may choose to study differences like $\Delta a_{\rm CP,9}(D^+) \equiv a_{\rm CP}(D^+ \to \bar{K}^{*0}K^+) - a_{\rm CP}(D^+ \to \bar{K}^{*0}\pi^+)$ to eliminate production asymmetries.

Finally I mention the parallel talks on charm physics at this conference:
Eleftheria Solomonidi, *Implications of cascade topologies for rare charm decays and CP violation*, which is a theory talk,
Luca Balzani, *Particle-antiparticle asymmetries in hadronic charm decays at LHCb* covering $D-\bar{D}$ mixing and CP violation, and
Marco Colonna, *Rare charm decays at LHCb*, discussing $D \to hh'e^+e^-$ and more.

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