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Theory of baryon number violation

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Baryon number is an accidental symmetry in the Standard Model and is thus fundamentally expected to be broken. In these proceedings, we summarize the current status of this topic. We review the latest experimental bounds on nucleon decay, the most sensitive probe of baryon number violation, and discuss future prospects. On the theoretical side, we outline the key conceptual steps in computing nucleon decay rates and explore how baryon number violation arises within Grand Unified Theories (GUTs). Finally, we highlight the challenges in making robust nucleon lifetime predictions within GUTs and propose possible strategies for their mitigation.

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1. Introduction

We present in these proceedings a short overview of the theory of baryon number violation. We introduce the main concept in Section 2, discuss the experimental constraints (present and future) in the form of nucleon decay in Section 3, provide a quick guide to calculating nucleon decay rates in Section 4, and show how baryon number violation automatically arises in the framework of Grand Unified Theories (GUTs) in Section sec:GUT.

Throughout this paper, we mainly emphasize the conceptual aspects, but we do not shy away from technical details when it serves as a useful bridge to referencing other work. A comprehensive review of proton decay is given by [1]. We refrain from compiling a list of individual models and their proton decay predictions; see [2] for a short reference.

2. Baryon and lepton number: a short introduction

2.1 B and L in the SM...

Where do the concepts of baryon number B and lepton number L come from?

To set the stage and conventions, we start by considering the Standard Model (SM). It is a non-Abelian Yang-Mills gauge theory based on the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, where the subscripts C, L and Y refer to color, "left" (weak interaction) and hypercharge, respectively. The scalar content of the SM consists of the Higgs doublet

$$H \sim (\mathbf{1}, \mathbf{2}, +\tfrac{1}{2}). \tag{1}$$

The fermion content consists of 3 families of the irreps

$$Q^a \sim (\mathbf{3}, \mathbf{2}, +\tfrac{1}{6}), \qquad L^a \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}), \tag{2}$$

$$(u^c)^a \sim (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}), \qquad (d^c)^a \sim (\bar{\mathbf{3}}, \mathbf{1}, +\tfrac{1}{3}), \qquad (e^c)^a \sim (\mathbf{1}, \mathbf{1}, +1), \tag{3}$$

where the family index a runs from 1 to 3. The fermion irreps are all written as Weyl fermions in left-handed form, i.e., they transform as $(\frac{1}{2}, 0)$ under the Lorentz group, and the Weyl indices are suppressed. For better visual clarity, we color quarks red and leptons blue, a convention that will carry over to coloring states with non-vanishing baryon or lepton number, respectively.

Given the above representations, the Yukawa sector can be schematically written at the renormalizable level as

$$\mathcal{L}_Y = (Y_U)_{ab}\, Q^a\, (u^c)^b\, H + (Y_D)_{ab}\, Q^a\, (d^c)^b\, H^* + (Y_E)_{ab}\, L^a\, (e^c)^b\, H^* + h.c.. \tag{4}$$

We suppressed gauge indices in the above notation, while a, b represent family indices. We denoted the Yukawa matrices by Y_U, Y_D and Y_E.

Consider now the group of global phase transformations $\mathrm{U}(1)_Q \times \mathrm{U}(1)_{u^c} \times \mathrm{U}(1)_{d^c} \times \mathrm{U}(1)_L \times \mathrm{U}(1)_{e^c} \times \mathrm{U}(1)_H \equiv \mathrm{U}(1)^6$, where generically $\mathrm{U}(1)_X$ is defined so that only the irrep X has a non-vanishing charge (assuming flavor universality for fermions). The SM kinetic terms and Higgs potential are invariant under this $\mathrm{U}(1)^6$, so the only terms that can violate such transformations are the Yukawa terms of Eq. (4). Each Yukawa term represents a violation of a different combination of charges in $\mathrm{U}(1)^6$, therefore only 3 combinations in the space $\mathrm{U}(1)^6$ of charges are preserved. One combination is the hypercharge Y, which is in any case gauged, while the other two represent "accidental" global symmetries of the SM. Conventionally we express them as baryon number B and lepton number L, which are defined by the charge assignments in Table 1.

	Q^a	$(u^c)^a$	$(d^c)^a$	L^a	$(e^c)^a$	H
B	$+\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0	0
L	0	0	0	$+1$	-1	0

Table 1: The assignment of B (baryon number) and L (lepton number) to SM fermions and scalars.

Since the (renormalizable) SM Lagrangian preserves B and L, all perturbative processes derived from this Lagrangian will preserve them as well. At low energies, where QCD confines quarks into hadrons, with baryons, anti-baryons and mesons having the baryon number

$$[qqq]_B = 1, \qquad [\bar{q}\bar{q}\bar{q}]_B = -1, \qquad [q\bar{q}]_B = 0, \tag{5}$$

where q and $\bar{q}$ generically denote quarks and anti-quarks, respectively. A consequence of B-preservation is that the lightest baryon, namely the proton p, is (perturbatively) stable.

2.2 ... and their violation in SMEFT

At energies well above the electro-weak (EW) scale, the generic quantum field theory expectation is that B and L are eventually broken — a fate that befalls all global symmetries due to gravity if nothing else. More intriguingly, violation of B and L could arise from physics Beyond the SM (BSM) already at scales well below the Planck scale $M_{\mathrm{Pl}} \approx 1.2 \cdot 10^{19}$ GeV.

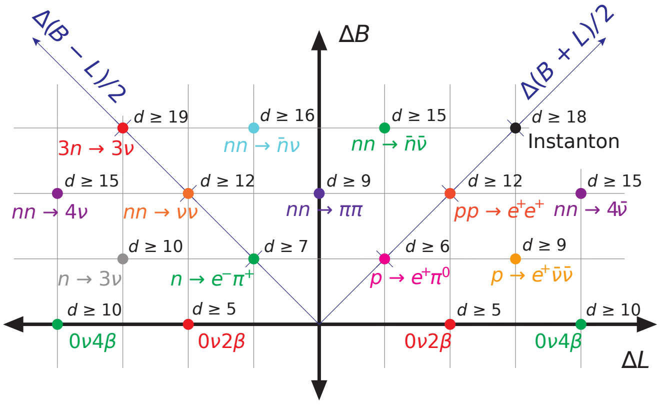


Figure 1: The $\not{B}$ and $\not{L}$ operators in SMEFT placed into the ΔL-ΔB grid, denotes as dots. The lowest dimension at which they arise and an example of an induced physical process are given. Source: [3].

An agnostic low-energy (EW-scale) approach to such a possibility is to consider the renormalizable SM as an effective theory, to which non-renormalizable operators with unknown coefficients are added — dubbed SM Effective Field Theory (SMEFT). At mass dimension $d > 4$, operators with $\not{B}$ and $\not{L}$ indeed do arise. The systematics of such operators are shown in Figure 1. An operator with total lepton number L and baryon number B will induce physical processes, which violate these numbers by that very amount, and thus labeled as ΔL and ΔB in the figure, respectively.

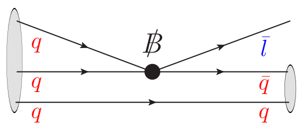
We see from Figure 1 that the process with $\not{L}$ arises already at $d = 5$; there is one such operator, namely the well known Weinberg operator $LLHH$ that may be the physical source of neutrino masses (if neutrinos are Majorana). Violation of baryon number, on the other hand, first arises at $d = 6$ from operators $qqql$, where q and l generically denote a quark and lepton, respectively; further elaboration is deferred to Section 4. Curiously, these operators separately violate B and L, but preserve the combination $B - L$, i.e. they are on the $\Delta(B + L)/2$ diagonal line in the figure.

3. $\not{B}$ in experiments: bounds and prospects

Having motivated why violation of B is expected to eventually arise from physics at high energy scales, we briefly summarize the experimental efforts dedicated to its possible observation.

Since the lowest-dimension $\not{B}$ operators arising in SMEFT are of the form $qqql$, see Section 2.2, the most promising process to search for is of the type

$$\text{baryon} \longrightarrow \text{meson} + \text{anti-lepton}. \tag{6}$$



The low decay rates of such processes are feasible to be observed only for baryons otherwise deemed stable. Experiments hence focus on observing the decay of either protons or of neutrons bound in a nucleus — they focus on *nucleon decay*.

Nucleon decay has thus far not been experimentally observed. The experiment setting all current bounds is Super-Kamiokande [4] (of neutrino-oscillation fame [5]), a water Cherenkov detector that has been accumulating data for nearly 30 years. It is essentially a huge cylindrical water tank, with its inner walls covered by photomultiplier tubes. These can detect Cherenkov light from charged decay products of a nucleon from anywhere in the enclosed water mass. The current bounds on various decay channels of the proton p and neutron n are summarized in Table 2. They are given in the form of lower bounds on $\tau/\mathcal{B}$ in years, where τ is the nucleon lifetime and $\mathcal{B}$ the branching ratio of the decay channel. For the sake of transparency, we list for each measurement the year of publication and exposure $\mathcal{E}$ in kiloton-years. The latter quantity is the time-integrated fiducial mass, i.e. the mass of the medium used for detection (effectively specifying the number of nucleons under observation).

The experimental bounds for nucleon decay lifetime currently stand at around 10^{34} years, which is 24 orders-of-magnitude larger than the age of the universe. This remarkable sensitivity offers a possible window into physics at super-high energies up to $\sim 10^{16}$ GeV, cf. Sections 4 and 5.

Such long lifetime bounds arise from simultaneous observation of a vast number of nucleons. To build some intuition, observing 10^N nucleons for a year (assuming 100% detection efficiency) provides a lifetime bound comparable to that of observing a single nucleon for 10^N years. A kg of material, e.g. water, contains about $6 \cdot 10^{26}$ nucleons (as per Avogadro's number); therefore a cube $(10\,\mathrm{m})^3$ of water, which weighs 1 kt, corresponds to $6 \cdot 10^{32}$ nucleons. The current Super-K bounds are indeed consistent with observing a tank of water tens of meters in size for tens of years.

decay	bound $\tau/\mathcal{B}$ [y]	$\mathcal{E}$ [kt · y]	year	reference
$p \to \pi^0 e^+$	$2.4 \cdot 10^{34}$	450	2020	[6]
$p \to \pi^0 \mu^+$	$1.6 \cdot 10^{34}$	450	2020	[6]
$p \to \pi^+ \bar{\nu}$	$3.9 \cdot 10^{32}$	173	2013	[7]
$p \to K^+ \bar{\nu}$	$5.9 \cdot 10^{33}$	260	2014	[8]
$p \to K^0 e^+$	$1.0 \cdot 10^{33}$	92	2005	[9]
$p \to K^0 \mu^+$	$3.6 \cdot 10^{33}$	370	2022	[10]
$p \to \eta\, e^+$	$1.4 \cdot 10^{34}$	370	2024	[11]
$p \to \eta\, \mu^+$	$7.3 \cdot 10^{33}$	370	2024	[11]
$p \to e^- e^+ e^+$	$3.4 \cdot 10^{34}$	370	2020	[12]
$n \to \pi^0\, \bar{\nu}$	$1.1 \cdot 10^{33}$	173	2013	[7]
$n \to K^0\, \bar{\nu}$	$1.3 \cdot 10^{32}$	92	2005	[9]
$n \to \pi^- e^+$	$5.3 \cdot 10^{33}$	316	2017	[13]
$n \to \pi^- \mu^+$	$3.5 \cdot 10^{33}$	316	2017	[13]

Table 2: Current experimental bounds of various proton and neutron decay channels. The two body decays of Eq. (6) arise from $d = 6$ four-fermion operators, while the three-body decay $p \to e^- e^+ e^+$ is induced by a $d = 9$ six-fermion operator.

The crucial parameter for detection sensitivity is the aforementioned exposure $\mathcal{E}$. While increasing detection efficiency is valuable and technologically challenging, large parametric gains are predominantly achieved by a "brute force" approach — maximizing fiducial mass and observation time. Given these realities, the experimental efforts related to nucleon decay are (and will necessarily be) a marathon rather than a sprint.

To assess the experimental prospects, we compare in Table 2 the current Super-K experiment with future experiments currently under construction, cf. [3] for an overview. The main successor experiment will be Hyper-Kamiokande, currently scheduled for start in 2027, with a fiducial mass about 8× that of Super-K. It is thus expected to improve proton lifetime bounds by around 1 order-of-magnitude within 20 years of operation. Other notable experiments are DUNE and JUNO; operating on different detector technology, they boast a superior detection efficiency in some channels, but their comparative lack of bulk makes them less competitive overall.

experiment	fiducial mass [kt]	start year	technology	location
Super-Kamiokande	$22.5 \to 27.2$	1996	water Cherenkov	Japan
Hyper-Kamiokande	187	2027 ?	water Cherenkov	Japan
DUNE	40	2030s ?	liquid Ar TPC	Unites States
JUNO	20	2025 ?	liquid scintillator	China

Figure 2: An overview of current and planned experiments, as of March 2025. Hyper-K is the most competitive successor to the current Super-K experiment. The arrow for Super-K indicates an upgrade.

4. A guide to computing nucleon decay rates

This section offers a brief conceptual overview of computing nucleon decay. We strive for sufficient detail, so that coupled with external software and/or references, the reader has the tools to undertake a computation of nucleon decay in their model of choice.

We limit ourselves to cases where the effective description is via four-fermion operators. There are 4 independent $\not{B}$ operators at $d = 6$ in SMEFT, all of type $qqql$ [14]:

$$(O_{RL})^{abcd} := (d^{\alpha a} u^{\beta b})(Q^{\gamma i c} L^{jd})\, \epsilon_{\alpha\beta\gamma}\, \epsilon_{ij}, \tag{7}$$

$$(O_{LR})^{abcd} := (Q^{\alpha i a} Q^{\beta j b})(u^{\gamma c} e^{d})\, \epsilon_{\alpha\beta\gamma}\, \epsilon_{ij}, \tag{8}$$

$$(O_{LL})^{abcd} := (Q^{\alpha i a} Q^{\beta j b})(Q^{\gamma k c} L^{ld})\, \epsilon_{\alpha\beta\gamma}\, \epsilon_{il}\, \epsilon_{jk}, \tag{9}$$

$$(O_{RR})^{abcd} := (d^{\alpha a} u^{\beta b})(u^{\gamma c} e^{d})\, \epsilon_{\alpha\beta\gamma}. \tag{10}$$

The fields u, d and e are those of Eq. (3) written in right-handed form (with a dotted Weyl index, see e.g. [15]; $(x^c)^c = x$). Parentheses denote pairs of Weyl fermions is Lorentz-contracted, and subscripts under the label O reflect whether the fermions in parentheses are right- or left-handed. Conventions: upper indices are fundamental, Weyl indices (dotted or undotted) are suppressed, $a, b, c, d \in \{1, 2, 3\}$ are family indices, $\alpha, \beta, \gamma \in \{1, 2, 3\}$ are $SU(3)_C$ indices, $i, j, k, l \in \{1, 2\}$ are $SU(2)_L$ indices, ϵ with n indices is the n-dimensional Levi-Civita tensor, and $\epsilon_{123} = \epsilon_{12} = 1$.

When computing the nucleon decay rate in a given model, the goal is to ultimately reduce the description to the SMEFT B-violating Lagrangian

$$\mathcal{L}_{\not{B},\mathrm{SMEFT}} = \sum_I C^I_{abcd}\, O_I^{abcd}, \qquad I \in \{RL, LR, LL, RR\}, \tag{11}$$

so that a universal computational procedure based on a top-down tower of EFTs can take over from that point onward. One should follow the following steps [1, 16, 17]:

(1) Perform matching of $\not{B}$ operators to determine the SMEFT coefficients C^I in Eq. (11). This must be done at a scale M_{match} where SMEFT can be consistently applied, i.e. the description uses the SM gauge group and any additional states (e.g. mediating $\not{B}$) have been integrated out.

(2) Run the operators O to the EW-scale $M_{\mathrm{EW}} \approx 91.2\,\mathrm{GeV}$ (Z-boson mass) using renormalization group equations (RGE) found in [14]. Some generalizations for RGE are also given in e.g. [18].

(3) Switch the description in SMEFT to one in LEFT ("low energy EFT") by computing the $\tilde{C}$-coefficients, see below. LEFT is applicable in the EW-broken phase, is based on the gauge group $SU(3)_C \times U(1)_{EM}$, and the Higgs boson h, the top quark t, and the massive gauge bosons $W^\pm_\mu, Z^0_\mu$ have been integrated out. The $\not{B}$ terms in LEFT are given by

$$\mathcal{L}_{\not{B},\mathrm{LEFT}} = \frac{1}{16\pi^2} \sum_J \tilde{C}^J_{abcd} \tilde{O}_J^{abcd}. \tag{12}$$

There are 7 operators of the type $\tilde{O}_J$. Their definitions, as well as the matching conditions between the $\tilde{C}$- and C-coefficients, are explicitly given in Table 3 for convenience.

(4) Run the coefficients $\tilde{C}^J_{abcd}$ via RGE from M_{EW} down to the proton-mass scale $m_p = 938\,\mathrm{MeV}$, integrating out the bottom quark b, tau lepton τ, and charm quark c when crossing their mass thresholds.

(5) Rotate flavor indices a, b, c, d in coefficients $\tilde{C}^J_{abcd}$ to the mass eigenbasis. Due to kinematic constraints, only index values referring to states with masses smaller than m_p are relevant.

(6) Insert coefficients $\tilde{C}^J_{abcd}$ into decay width expressions for different channels derived from chiral perturbation theory. Values for hadronic matrix elements are provided by lattice QCD.

definition of $\tilde{O}_J^{abcd}$	matching of $\tilde{C}^J_{abcd}$ and C^I_{abcd}
$\tilde{O}_{RL}(udue)^{abcd} := (u_R{}^{\alpha a} d_R{}^{\beta b})(u_L{}^{\gamma c} e_L{}^d)\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RL(udue)}_{abcd} = -16\pi^2\, C^{RL}_{bacd}$
$\tilde{O}_{LR}(udue)^{abcd} := (u_L{}^{\alpha a} d_L{}^{\beta b})(u_R{}^{\gamma c} e_R{}^d)\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{LR(udue)}_{abcd} = 16\pi^2 (C^{LR}_{abcd} + C^{LR}_{bacd})$
$\tilde{O}_{LL}(udue)^{abcd} := (u_L{}^{\alpha a} d_L{}^{\beta b})(u_L{}^{\gamma c} e_L{}^d)\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{LL(udue)}_{abcd} = 16\pi^2 (- C^{LL}_{abcd} + C^{LL}_{acbd} - C^{LL}_{cabd})$
$\tilde{O}_{RR}(udue)^{abcd} := (u_R{}^{\alpha a} d_R{}^{\beta b})(u_R{}^{\gamma c} e_R{}^d)\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RR(udue)}_{abcd} = -16\pi^2\, C^{RR}_{bacd}$
$\tilde{O}_{RL}(uddv)^{abcd} := (u_R{}^{\alpha a} d_R{}^{\beta b})(d_L{}^{\gamma d} \nu_L{}^d)\, \epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RL(uddv)}_{abcd} = 16\pi^2\, C^{RL}_{bacd}$
$\tilde{O}_{LL}(uddv)^{abcd} := (u_L{}^{\alpha a} d_L{}^{\beta b})(d_L{}^{\gamma d} \nu_L{}^d)\, \epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{LL(uddv)}_{abcd} = 16\pi^2 (C^{LL}_{bacd} + C^{LL}_{cbad} - C^{LL}_{bcad})$
$\tilde{O}_{RL}(dduv)^{abcd} := \frac{1}{2}(d_R{}^{\alpha a} d_R{}^{\beta b})(u_L{}^{\gamma c} \nu_L{}^d)\, \epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RL(dduv)}_{abcd} = 0$

Table 3: Definition of $\not{B}$ operators in LEFT and the matching of $\tilde{C}$- and C-coefficients, see Eq. (12). The fermion subscripts L and R refer not just to their chirality, but also their origin in SM irreps.

Note that step (1) is model dependent, and so one must take special care of both the matching and the choice of scale M_{match}. In the softly broken minimal supersymmetric SM (MSSM), for example, the presence of scalar superpartners allows for effective $d = 5$ $\not{B}$ operators (see $d = 5$ proton decay in e.g. [17, 19]), requiring these to be part of the description, hence $M_{\text{match}} = M_{\text{SUSY}}$. In the case of extended gauge symmetry, the matching scale might be the symmetry breaking scale to the SM, assuming no exotic states.

Steps (4)–(6) are elaborated on further in [1, 16, 17], but the entire procedure is also automatized as part of the *Mathematica* package `ProtonDecay` [19], wherein conventions are consistent with Table 3. A sister package `SusyTCProton` also exists to automate the computation of $d = 5$ proton decay in SUSY, along with solving RGE and computing the spectrum in the softly broken MSSM.

5. Embedding $\not{B}$ into a GUT framework

5.1 Opening up the four-fermion operators

The four-fermion SMEFT operators of Eqs. (7)–(10) are an effective description of $\not{B}$. One is ultimately interested, however, in the underlying physics leading to such operators.

The simplest possibility to "open up" the four-fermion operator is to introduce a gauge boson X or a scalar T to mediate the interaction at tree level. The corresponding diagrams are shown on the left and center in Figure 3, while the right diagram corresponds to $d = 5$ proton decay in SUSY, cf. Section 4, which is a loop-mediated example. For each diagram in Figure 3, proton (or nucleon) decay rates Γ_p can be estimated on dimensional grounds for each process as

$$\Gamma_p^{(X)} \sim \frac{g^4 m_p^5}{m_X^4}, \qquad \Gamma_p^{(T)} \sim \frac{|y_1 y_2|^2 m_p^5}{m_T^4}, \qquad \Gamma_p^{(5)} \propto \left(\frac{1}{16\pi^2}\right)^2 \frac{m_p^5}{m_T^2 m_{\text{SUSY}}^2}. \tag{13}$$

Crucially, the decay rate for tree-mediation is proportional to the inverse 4th power of mediator mass. For nucleon decay, the couplings y_1 and y_2 connect the scalar T to the 1st or 2nd generation fermions; if they are of similar size to the Yukawa couplings of the Higgs H to these families, the gauge-boson mediated process is expected to dominate over the scalar-mediated one. Conversely, the $d = 5$ process has a different parametric suppression, and thus dominates in SUSY models (assuming also a small enough R-parity violation, see e.g. 4.1 in [1]).

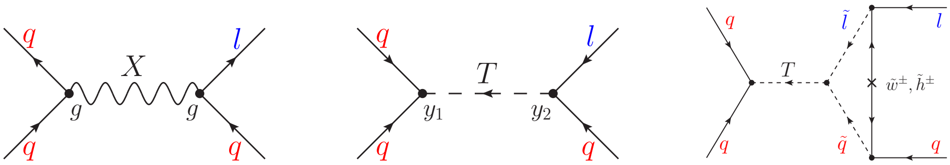


Figure 3: Examples of $\not{B}$ diagrams emerging from underlying BSM physics. Tree-level mediation by a gauge boson X (left) and scalar T (center) are the simplest possibility. A loop diagram involving a scalar T, as well as a squark $\tilde{q}$ and slepton $\tilde{l}$ is a more complicated example (i.e. $d = 5$ proton decay in SUSY).

Since the interactions of fermions q or l with X or T in Figure 3 must be gauge invariant, the mediators X and T can only be in certain SM irreps. Systematically going through the possibilities to open up the operators O of Eqs. (7)–(10) yields the list of possibilities in Table 4. Gauge boson mediation can induce only operators O_{LR} and O_{RL}, i.e. operators with one left- and one right-handed pair of fermion fields.

X	induced O
$(\mathbf{3}, \mathbf{2}, -\frac{5}{6})$	$O_{RL,LR}$
$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	O_{RL}

T	induced O
$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$O_{RL,LR,LL,RR}$
$(\mathbf{3}, \mathbf{1}, -\frac{4}{3})$	O_{RR}
$(\mathbf{3}, \mathbf{3}, -\frac{1}{3})$	O_{LL}

Table 4: The SM irrep possibilities for the gauge mediator X (left) and scalar mediator T (right). In each case we specify which operators from Eqs. (7)–(10) are induced.

5.2 The GUT framework

The framework of Grand Unified Theories (GUTs) [20, 21] is the natural environment in which some of the states of Table 4 are always present, and thus automatically comes with an embedded theory of baryon number violation.

The idea of a GUT is to unify all SM gauge interactions inside a simple gauge group G. Clearly, the unified group must contain the SM as its subgroup, i.e. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \subset G$, and G must allow for complex representations in order for the chiral SM to emerge at low energy (in 4D spacetime). Simple groups have been classified, and the aforementioned constraints are satisfied by $\mathrm{SU}(n)$ for $n \geq 5$, $\mathrm{SO}(4m+2)$ for $m \geq 2$, and E_6. Taking the minimal candidate in each class, the main GUT protagonists emerge: $\mathrm{SU}(5)$, $\mathrm{SO}(10)$ and E_6. They can be ordered into a chain of subgroups:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \quad \subset \quad \mathrm{SU}(5) \quad \subset \quad \mathrm{SO}(10) \quad \subset \quad \mathrm{E}_6. \tag{14}$$

Remarkably, the SM fermion irreps in Eqs. (2)–(3) of one generation can be incorporated into $\mathbf{10} \oplus \bar{\mathbf{5}}$ of SU(5), as shown in the left panel of Figure 4. In SO(10), all matter of one generation unifies together with SM-singlet ("right-handed") neutrinos ν^c into a $\mathbf{16}$, while E_6's $\mathbf{27}$ adds an additional singlet neutrino n and vector like pairs of exotic fermions. The emergence of the correct chiral SM fermions from GUT irreps in an anomaly-free way is a non-trivial phenomenon [22].

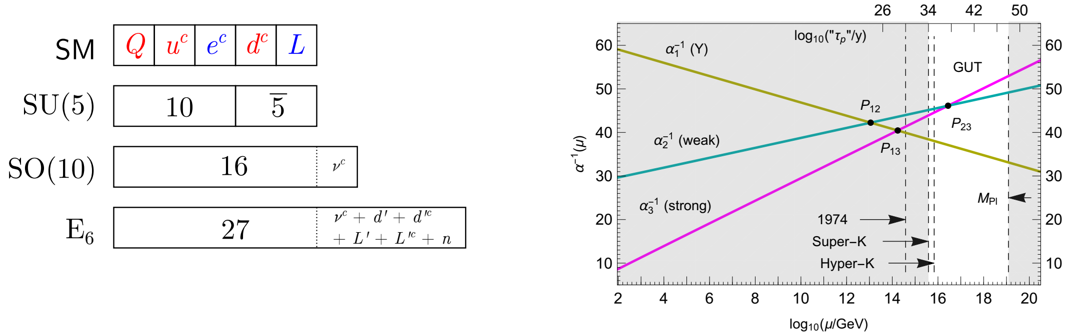


Figure 4: *Left:* the incorporation of SM fermion irreps into GUT irreps (as indicated by vertical alignment of boxes). *Right:* 2-loop RG-evolution of SM gauge couplings α_i^{-1} with scale μ. The vertical white band shows the viable unification region compatible with both proton decay bounds and below M_{Pl}.

The idea of GUT itself originates from another non-trivial observation: evolving the SM gauge couplings α_i^{-1} to higher energies via RGE, the gauge couplings come closer in value, as shown in the right panel of Figure 4. The triangle of intersections $\{P_{12}, P_{23}, P_{13}\}$ is small enough that introduction of new states can easily modify the running to result in exact unification of all three in one point, necessary for a consistent origin from GUT.

Having established strong motivation for GUTs, we now demonstrate that $\not{B}$ is an inherent part of this framework:

- The gauge boson representations X from Table 4 that mediate nucleon decay turn out to be present in all GUTs, as shown in Table 5. Indeed, the gauge boson transforming as $(\mathbf{3}, \mathbf{2}, -\frac{5}{6})$ under the SM group is present already in SU(5), and by inclusions of Eq. (14) in all GUTs. For completeness, we added to the table also two cases of flipped symmetry (denoted by primes): $\mathrm{SU}(5)' \times \mathrm{U}(1)'$ and $\mathrm{SO}(10)' \times \mathrm{U}(1)'$. The two cases are maximal subgroups of SO(10) and E_6, respectively, and are characterized by $\mathrm{U}(1)_Y$ being embedded across both factors. Although not true GUTs, the flipped cases are nevertheless further examples of SM gauge extensions with $\not{B}$.
- The presence of scalars T from Table 4 in GUT is model dependent, but the triplet $(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$ should be present in any realistic GUT. The reason is that any SU(5) irrep (at least those of dimension 280 or lower [23]) containing the Higgs doublet $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$ also comes with the aforementioned triplet. The statement no longer holds true for flipped cases.

We now return to gauge coupling unification in the right panel of Figure 4. The numbers on the upper axis represent the estimate for proton lifetime based on the gauge-mediated scenario in SU(5). The excluded grey region from the left is due to proton decay bounds, where the estimated future sensitivity from Hyper-K is also shown. The viable region for unification, consistent with current proton decay bounds and below M_{Pl}, is shown as a white band, and it is a non-trivial feature that the triangle of α^{-1}-intersections falls into this correct ballpark.

X	SU(5)	SO(10)	E_6	$SU(5)' \times U(1)'$	$SO(10)' \times U(1)'$
$(\mathbf{3}, \mathbf{2}, -\frac{5}{6})$	✓	✓	✓		
$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$		✓	✓✓	✓	✓✓

Table 5: The presence of two types of gauge bosons X that mediate nucleon decay in various GUTs. We included the flipped cases as well, see main text. Multiple checkmarks indicate multiple irrep copies.

5.3 Decay rate predictions and robustness

We conclude our survey of GUTs by elaborating on nucleon decay rate predictions. The intention is not to compile a list of predictions from different models, but to instead point out the general features of these predictions, see e.g. [3]:

- The dominant decay mode of the proton in non-SUSY GUTs (usually) comes out to be $p \to \pi^0 e^+$ (π-modes), while for non-SUSY theories it is $p \to K^+ \bar{\nu}$ (K-modes).
- Unification of gauge couplings in any specific model provides an important constraint. Namely, it sets the range of possible mediator masses m_X in Eq. (13), and hence the predicted range for the proton's lifetime. Such predictions for a model, however, can still span several orders of magnitude, see Figure 5 as an illustration. We caution that despite the simplified labeling of models in the figure, the lifetime range depends not only on the gauge group, but on the choice of fermion and scalar representations as well, i.e. it is fully model dependent.

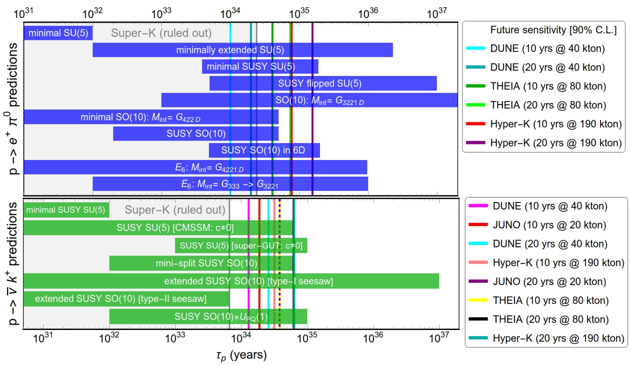


Figure 5: Proton lifetime predictions in different models based on the dominant mode. Source: [3].

The uncertainties in predicting the proton lifetime can thus be disappointingly large for a given model, especially given the implications for experimental searches. In light of this, it is important to understand the sources of the uncertainty [24], in the hopes of finding mitigation strategies:

(a) The lifetime $\tau_p = \Gamma_p^{-1}$ (when gauge mediation dominates) is proportional to $\propto m_X^4 \sim M_{\mathrm{GUT}}^4$. The unification scale, however, depends on the spectrum of additional, usually scalar, states introduced by the GUT irreps; these in turn depend on the (unknown values of) parameters of the scalar potential. Even under assumptions such as minimal fine-tuning, the uncertainty in M_{GUT} can easily be a factor 10, which translates to a factor 10^4 in τ_p. One intriguing possibility, in the spirit of the recent works in [25, 26], is that the parameter space (and thus spectrum) is constrained by perturbativity at the quantum level to a greater extent than is typically appreciated.

(b) There are uncertainties from flavor [27], introduced when rotating from the flavor to the mass eigenbasis in step (3) of Section 4. They may involve the right mixing angles that are unobservable in the SM. In flipped-SU(5), proton decay can even be rotated away [28]. Some of these ambiguities can be reduced by (i) performing a flavor fit, (ii) considering ratios of decay rates for different channels [19], and (iii) considering neutrino channels (the flavor drops out in an incoherent sum over neutrino flavors).

(c) Non-renormalizable operators can present a nuisance. The source of these can be an EFT description with a cut-off before M_{Pl}, or gravity itself, hence the effect is usually referred to as *gravitational smearing*. The most dangerous operator of this type is $F_{\mu\nu}F^{\mu\nu}\Phi/M_{\mathrm{Pl}}$, where Φ is a scalar irrep. The induced shift in gauge couplings α_i^{-1} is of relative size $\langle\Phi\rangle/M_{\mathrm{Pl}}$, perhaps as much as $\sim 1\,\%$, and hence the shift in $\log_{10}\delta M_{\mathrm{GUT}}/M_{\mathrm{GUT}}$ is similar (depending on the angle the gauge couplings meet at). The appearance of the effective operator can be prevented in model building by limiting the irrep choices Φ in the scalar sector, so that $F_{\mu\nu}F^{\mu\nu}\Phi$ is not an invariant.

(d) All partial widths of nucleon decay channels are proportional to $\Gamma \cdot \mathcal{B} \propto |\alpha\mathcal{A}_1 + \beta\mathcal{A}_2|$ (see e.g. [17]), where $\mathcal{A}_i$ are channel-dependent amplitude expressions, and α, β are hadronic matrix elements, which are currently computed up to 10-20 % accuracy [29]. This uncertainty is model universal; it will reduce over time with improved lattice QCD calculations.

6. Conclusion

Baryon number is an accidental symmetry in the SM, and is fundamentally expected to be broken. Proton decay (and more broadly nucleon decay) offers an extremely sensitive probe of this violation. The current Super-K bound on the proton's lifetime, around 10^{34} years, provides a possible window into physics of ultra-high energies up to $10^{15\div16}$ GeV. Detection efforts will be upgraded in a few years with Hyper-K, improving sensitivity by order of magnitude over 20 years of data collection. On the computational side, however, challenges remain in achieving robust predictions for decay rates (or associated observables).

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