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Higher order corrections in dimensional reduction

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The usual approach to the computation of cosmological phase transitions (PT) in thermal field theory is through the construction of a dimensionally reduced effective field theory (3D EFT). The need for robust theoretical predictions of the gravitational wave (GW) spectra sourced by a first-order PT in the early Universe has recently pushed the construction of these 3D EFTs to unprecedented levels of precision in loops. However, as far as the authors know, the contributions from higher-dimensional effective operators that arise at the same order have generally been neglected in the literature. Here, we perform a quantitative analysis of the impact of effective interactions on the determination of PT parameters, and we develop a framework to consistently compute them. We find that they allow for strong PTs in a wider region of parameter space, and that both the peak energy density and frequency of the resulting GW power spectrum can change by more than one order of magnitude when they are included.

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1. Introduction

This text provides a summary of the work presented in [1]. There, we explore the impact of higher-order effective operators in the study of phase transitions (PTs) within the framework of dimensional reduction.

In a quantum field theory (QFT), the existence of two non-degenerate minima in the scalar potential can lead to a phase transition (PT) where the relevant scalar field changes its vacuum expectation value (VEV). In the imaginary-time or Matsubara formalism [2] one can describe such PTs when they are induced by thermal fluctuations. In this framework, time is compactified and fields are split in an infinite tower of static Fourier modes residing in 3-dimensional Euclidean space and with masses πT, where T is the temperature of the bath.

Within the high-temperature regime, where T is much larger than the zero-temperature mass scales of the fields, there appears a hierarchy of scales, and thus a perturbative treatment becomes possible only through the use of effective field theories (EFT). Indeed, one can build a *dimensionally-reduced*, Euclidean 3D EFT where the heavy thermal modes are integrated out and only the zeroth mode of bosonic fields are responsible for the PT (see [3] for further details and [4–6] for some recent applications).

In the construction of 3D EFTs, most works claim that higher-dimensional effective operators are irrelevant for PT computations based on naïve estimates. In our work we address these claims by studying a simple toy scalar model where we determine whether these effective operators can be safely neglected in strong PTs. Since we deal with higher-order derivative interactions, we furthermore develop a robust method to compute the *bounce* solution [7] perturbatively for effective operators with an arbitrary number of field and derivative insertions.

2. Theoretical setup

We consider a toy model consisting of a real scalar ϕ and a massless fermion ψ with Lagrangian

$$\mathcal{L}_4 = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \kappa\phi^3 - \lambda\phi^4 + \overline{\psi}i\partial\!\!\!/\psi - g\phi\overline{\psi}\psi\,. \tag{1}$$

In thermal equilibrium, this theory reduces to a Euclidean 3D theory with infinite thermal modes for ϕ and ψ. In the high temperature limit, every mode acquires a large mass and can be integrated out, except for the zeroth mode of ϕ, which we will refer to as φ.

The most general Lagrangian in the 3D EFT up to order $O(g^8)$ and adopting the standard power counting [3] is:

$$\begin{aligned}\mathcal{L}_3 = {} & \frac{1}{2}K_3(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4 \\ & + \alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi \\ & + \alpha_{81}\varphi^8 + \alpha_{82}\varphi^2\partial_\mu\partial_\nu\varphi\partial^\mu\partial^\nu\varphi + \beta_{81}\varphi\partial^6\varphi + \beta_{82}\varphi^3\partial^4\varphi + \beta_{83}\varphi^2\partial^2\varphi\partial^2\varphi + \beta_{84}\varphi^5\partial^2\varphi \\ & + \cdots\,.\end{aligned} \tag{2}$$

Up to this order in our power counting, only operators up to mass dimension 8 (in a 4D space-time) are generated. Going beyond dimension 6 does not only allow us to explore more accurately the parameter space, but, most importantly, it gives us control over the validity of the EFT expansion. Indeed, our 3D EFT will we valid as long as effects triggered by the dimension-8 interactions are significantly smaller than those coming from dimension 6.

Our matching results are given in Eqs. (3)-(6) below, where we include only the dominant part of the one-loop contributions.

$$K_3 = 1 + \frac{g^2}{12\pi^2}\,, \quad m_3^2 = m^2 + \frac{g^2T^2}{6}\,, \quad \kappa_3 = \kappa\sqrt{T}\,, \quad \lambda_3 = \lambda T\,; \tag{3}$$

$$\alpha_{61} = -\frac{7\zeta(3)g^6}{192\pi^4}\,, \qquad \beta_{61} = -\frac{7\zeta(3)g^2}{384\pi^4T^2}\,, \qquad \beta_{62} = \frac{35\zeta(3)g^4}{576\pi^4T}\,; \tag{4}$$

$$\alpha_{81} = \frac{31\zeta(5)g^8}{2048\pi^6T}\,, \qquad \alpha_{82} = -\frac{31\zeta(5)g^4}{10240\pi^6T^3}\,, \qquad \beta_{81} = -\frac{31\zeta(5)g^2}{10240\pi^6T^4}\,, \tag{5}$$

$$\beta_{82} = \frac{217\zeta(5)g^4}{20480\pi^6T^3}\,, \qquad \beta_{83} = \frac{279\zeta(5)g^4}{20480\pi^6T^3}\,, \qquad \beta_{84} = -\frac{217\zeta(5)g^6}{5120\pi^6T^2}\,. \tag{6}$$

3. Phase-transition parameters

During a first-order phase transition (FOPT), the scalar field spontaneously changes its VEV in expanding $O(3)$-symmetric patches of space called *bubbles*. The PT dynamics can be described by four main physical parameters: the nucleation temperature (T_*), the latent heat or strength of the PT (α), the inverse duration time of the PT (β/H_*) and the terminal bubble wall velocity (v_w).

These parameters can be determined (up to quantum corrections of the scalar zeroth mode) by the effective action in the 3D EFT, $S_3[\varphi]$, evaluated at a classical solution of its equations of motion (EOM), φ_c, called *bounce* [7]. The bounce is an inhomogeneous static field configuration that approximately interpolates between the two minima of the potential. Following the definitions in [8–11] we have closed expressions for the relevant PT parameters:

- $$S_3[\varphi_c] \approx 100 - 4\log\frac{T_*}{100\,\text{GeV}}\,, \tag{7}$$
- $$\alpha = \frac{\Delta\left(V_3(\varphi) - \frac{T}{4}\frac{\mathrm{d}}{\mathrm{d}T}V_3(\varphi)\right)\Big|_{T_*}}{\rho_r(T_*)} \approx -0.03\frac{\Delta\left(V_3(\varphi) - \frac{T}{4}\frac{\mathrm{d}}{\mathrm{d}T}V_3(\varphi)\right)\Big|_{T_*}}{T_*^3}\,, \tag{8}$$
- $$\frac{\beta}{H_*} = T_*\frac{\mathrm{d}S_3[\varphi_c]}{\mathrm{d}T}\bigg|_{T_*}\,, \tag{9}$$
- $$v_w = \begin{cases} \sqrt{\frac{V_3(\varphi_T)}{\alpha\rho_r}} & \text{for} \quad \sqrt{\frac{V_3(\varphi_T)}{\alpha\rho_r}} < v_J(\alpha) \\ 1 & \text{for} \quad \sqrt{\frac{V_3(\varphi_T)}{\alpha\rho_r}} \geq v_J(\alpha) \end{cases}\,, \tag{10}$$

where in the second line $\rho_r = g(T)\pi^2T^4/30$ is the energy density of the radiation plasma, and in the last line v_J is the Jouguet velocity, defined as

$$v_J = \frac{1}{\sqrt{3}}\frac{1+\sqrt{3\alpha^2+2\alpha}}{1+\alpha}\,. \tag{11}$$

The proof of existence of the bounce solution for an action with a kinetic term and an arbitrary potential was provided by Coleman in [7]. This solution in spherical coordinates is given by the differential equation

$$\ddot{\varphi}(r) + \frac{2}{r}\dot{\varphi}(r) = V_3'(\varphi(r)) \,, \tag{12}$$

with boundary conditions $\dot{\varphi}(0) = 0$ and $\lim_{r\to\infty} \varphi(r) = \varphi_F$ (hereafter we will choose $\varphi_F = 0$). However, no such proof has been given for a general action with higher-order derivative terms, and the dedicated tools for bounce computations (e.g. [12]) do not implement this kind of interactions.

To overcome this issue, we developed a consistent way to compute the bounce solution perturbatively for a general effective action S_3. Being ϵ the perturbative parameter and expanding

$$\varphi_c = \varphi_c^{(0)} + \epsilon\varphi_c^{(1)} + \epsilon^2\varphi_c^{(2)} + \cdots \,, \qquad S_3 = S_3^{(0)} + \epsilon S_3^{(1)} + \epsilon^2 S_3^{(2)} + \cdots \,, \tag{13}$$

we find that $\varphi_c^{(0)}$ corresponds to the usual bounce solution of $S_3^{(0)}$ (Eq. (12)), and that the next order in the expansion of φ_c is given by the differential equation

$$\ddot{\varphi}_c^{(1)} + \frac{2}{r}\dot{\varphi}_c^{(1)} - V_3^{(0)''}(\varphi_c^{(0)})\,\varphi_c^{(1)} - \frac{1}{4\pi r^2}\frac{\delta S_3^{(1)}}{\delta\varphi}\bigg|_{\varphi_c^{(0)}} = 0 \,, \tag{14}$$

with boundary conditions

$$\dot{\varphi}_c^{(1)}(0) = \lim_{r\to\infty} \varphi_c^{(1)}(r) = 0. \tag{15}$$

Similar relations can be found at each order in ϵ, and they were included, along with their derivation, in the appendix of the original text [1].

Our approach has several advantages over dealing directly with the Euler-Lagrange EOM obtained from the general action S_3. First, proceeding this way we can keep track of the exact contribution at each order in ϵ in our EFT expansion. This is a key point to test the validity of the EFT expansion in the results, by assuring that dimension-8 contributions are suppressed with respect to dimension-6 ones. Second, solving the differential equation arising from the Euler-Lagrange relations by brute force can easily turn very cumbersome and unmanageable, depending on the nature and the number of higher-order derivative terms. Finally, thanks to the perturbative expansion, the PT parameters are computed in an invariant way under field-redefinitions, which serves as a cross-check of our calculations. For the latter to be true, it is necessary to evaluate $S_3[\varphi_c]$ and $V_3(\varphi_T)$ in a perturbative way as well:

$$S_3[\varphi_c] = S_3^{(0)}[\varphi_c^{(0)}] + \epsilon S_3^{(1)}[\varphi_c^{0}] + \epsilon^2 \left\{ S_3^{(2)}[\varphi_c^{(0)}] + 2\pi \int_0^\infty dr\, r^2 \varphi_c^{(1)} \frac{\delta S_3^{(1)}}{\delta\varphi}\bigg|_{\varphi_c^{(0)}} \right\} + O(\epsilon^3) \,,$$

$$V_3(\varphi_T) = V_3^{(0)}(\varphi_T^{(0)}) + \epsilon V_3^{(1)}(\varphi_T^{(0)}) + \epsilon^2 \left\{ V_3^{(2)}(\varphi_T^{(0)}) - \frac{1}{2}\frac{\left(V_3^{(1)'}(\varphi_T^{(0)})\right)^2}{V_3^{(0)''}(\varphi_T^{(0)})} \right\} + O(\epsilon^3) \,. \tag{16}$$

4. Results

Using the formalism presented in the previous section, we study the effects of higher-dimensional operators in the Lagrangian in Eq. (2). In particular, we focus on their impact within the region

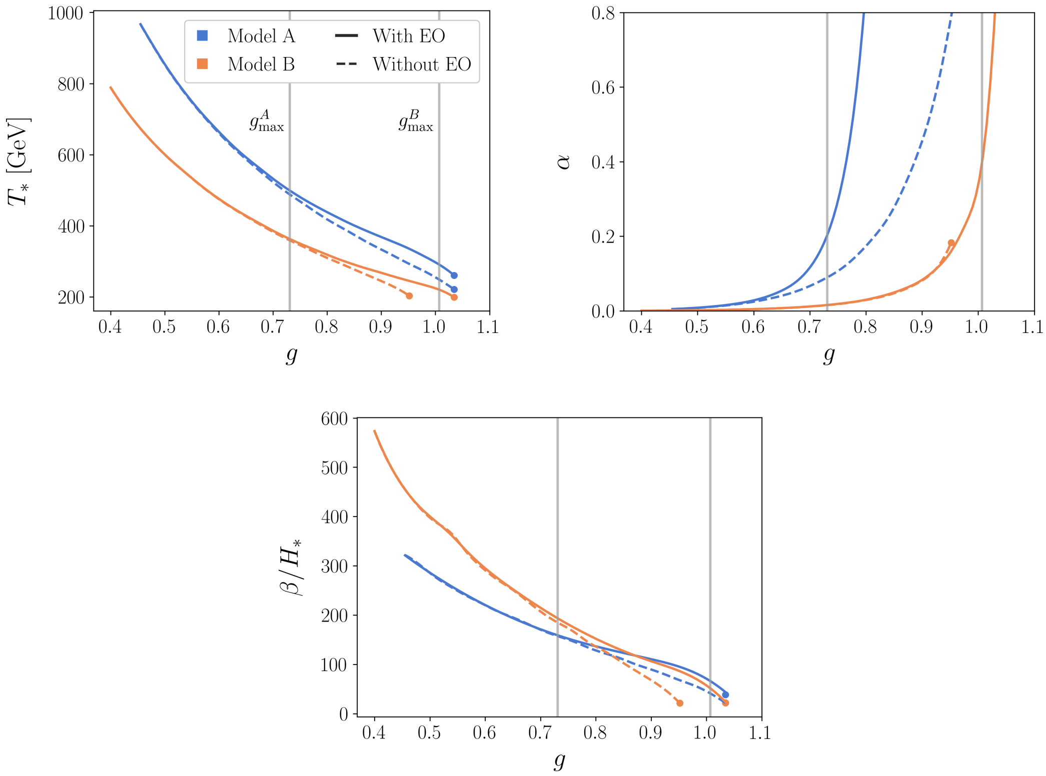

Figure 1: *Plots comparing PT parameters with and without effective operators (EO). We represent two different regions, with parameters* $(m^2, \kappa, \lambda)_A = (20\,000\ \text{GeV}^2, -40\ \text{GeV}, 0.01)$ *and* $(m^2, \kappa, \lambda)_B = (31\,643.5\ \text{GeV}^2, -71.1\ \text{GeV}, 0.045)$*, and varying* g*. The vertical lines represent* $g_{\rm max}$*, where lies the limit of validity of EFT expansion, defined as the value of* g *such that* $V_3^{(2)}(\varphi_T^{(0)})/V_3^{(1)}(\varphi_T^{(0)}) \sim 0.5$.

of the parameter space where a strong PT takes place (we will say a PT is strong if $\alpha \gtrsim 0.1$). To restrict such a space we make use of the scan over $(\alpha, m_3^2, \kappa_3, \lambda_3)$ carried out in Ref. [13]. We take those points where the PT is strong and where, in addition, $m/(\pi T_*) < 1, \kappa/(\pi T_*) < 1$, ensuring the validity of the EFT expansion. The value of m^2, κ and λ can obtained straightforwardly from Eq. (3) for a given g, which we explore in the range $0 < g < 2$.

Now, we plug each of these values into the Lagrangian in Eq. (2), compute the bounce φ_c from (14) and (15), and obtain values for T_*, α and β/H_*. In Fig. 1 we plot these magnitudes as functions of g, both taking and not taking into account effective operators, for two representative points of the (m^2, κ, λ) parameter space. We see how the range of values of g for which the FOPT occurs is larger when effective interactions are included, also yielding larger values values of α. This is because in these points, effective interactions tend to decrease the value of the action noticeably, thus allowing for the nucleation criterion $S_3[\varphi_c] \sim 100$ to be satisfied for a wider range of values of g. With this, it becomes apparent that the inclusion of effective interactions can turn crucial in the predictions of PT parameters, while within the regime of validity of the EFT expansion.

Another important aspect of thermally-induced FOPTs is the production of gravitational waves (GW). The nucleation of bubbles of vacuum energy, their expansion and eventual collision sources a stochastic GW background [10]. In Fig. 2 we show the predicted GW power spectra for the same

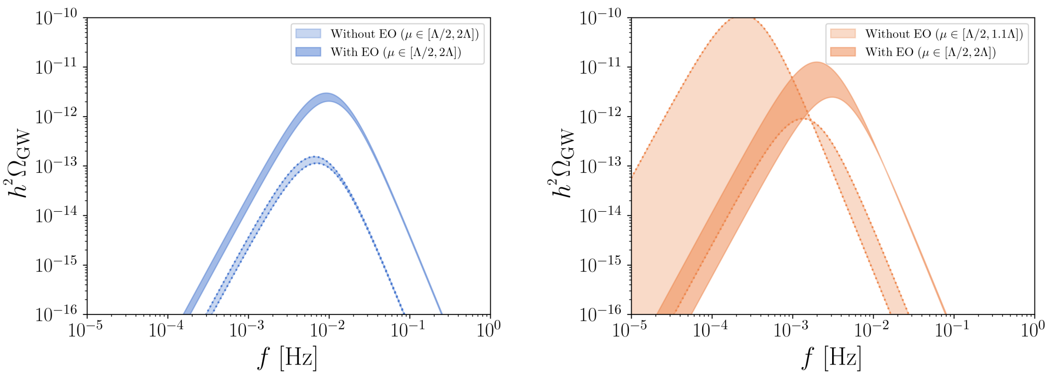

Figure 2: *Sound wave GW power spectra for regions A and B computed with and without effective operators (EO). The bands represent the uncertainty ensuing from running the matching scale μ in the range* $[0.5\Lambda, 2\Lambda]$.

two regions as before (with $g_A = 0.73$ and $g_B = 0.95$), computed from the dominant sound wave contribution for a given frequency f [14] along with their scale dependence. From these plots we learn that effective interactions can also change by orders of magnitude the power spectra of GW, whether in terms of the peak energy density or in the value of the peak frequency.

5. Conclusions

We have computed the matching to a full basis of higher-order effective operators in the dimensional reduction of a toy model with a real scalar and a fermion, and quantified its impact on the dynamics of strong FOPTs. We have found that the presence of such operators does not only allow for FOPTs at higher values of the Yukawa coupling, but also that it can substantially change the prediction of the relevant PT parameters $T_*, \alpha, \beta/H_*$ and v_w. These corrections are seen to consequently modify the peak energy density and frequency of the GW power spectra by orders of magnitude.

Our work highlights the importance of EFT effects in strong FOPT studies. To isolate these effects, we omitted higher-loop corrections in matching and in the effective potential. Future directions could seek the incorporation of these effects, the inclusion of quantum corrections in the effective action or the application of the techniques herein developed to other physically relevant models, as the SMEFT.

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