PROCEEDINGS OF SCIENCE

Unifying Quark and Lepton Flavor Observables through Modular Symmetry

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In a predictive modular invariant flavour theory, correlations between quark and lepton observables are expected. However, with the exception of GUT models, almost all previous studies have analyzed these sectors separately, preventing a comprehensive investigation of their interplay. This work, based on analysis presented in [1], discusses the results of a truly unified analysis of quark and lepton observables within a modular flavour model based on the $2O$ symmetry, employing a minimal set of 14 real parameters.
The results demonstrate that the model aligns well with experimental data under a normal neutrino mass ordering, while also predicting other key leptonic parameters, including the Dirac and Majorana CP-violating phases, $(\delta_{\rm CP}, \eta_1, \eta_2)$, the lightest neutrino mass (m_1), and effective neutrino masses relevant to beta and neutrinoless double beta decay $(m_\beta, m_{\beta\beta})$.
A comparative analysis of separate (quark-only and lepton-only) versus combined (quark and lepton) fits reveals distinct shifts in best-fit values and parameter spaces, emphasizing a nontrivial correlation between quark and lepton sectors. Notably, our study uncovers strong correlations between several observables, such as the strange-to-bottom quark mass ratio (r_{sb}), which is negatively correlated with the three lepton mixing angles, $\delta_{\rm CP}, m_1, m_\beta, m_{\beta\beta}$, while showing positive correlations with η_1 and η_2. These significant interdependencies, which are overlooked in separate analyses, provide, in principle, testable predictions for ongoing and future experimental studies.

9th Symposium on Prospects in the Physics of Discrete Symmetries (DISCRETE2024)
2–6 Dec 2024
Ljubljana, Slovenia

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1. Introduction

The fermion flavor problem, which concerns the origin of quark and lepton mass patterns, mixings, and CP violation, remains a major challenge in particle physics. Modular invariance [3] has emerged as a promising framework, where Yukawa couplings and mass matrices are expressed through modular forms dependent on a single complex modulus τ. These modular forms transform under finite modular groups, which serve as effective flavor symmetries. The modular vacuum expectation value (τ_{VEV}) breaks these symmetries and can also be the only source of CP violation. Recent works (see [4] for a comprehensive reference list) have developed modular models with minimal free parameters, allowing for testable predictions. While modular invariance has been studied separately in quark and lepton sectors, their joint analysis was often not fully performed. Since both sectors are linked through τ and, in Grand Unified Theories (GUTs), through common gauge multiplets, correlations are expected. This study performs a joint analysis of quark and lepton observables within a minimal modular 2O model. The results highlight interdependencies between sectors, demonstrating that modular symmetry enforces nontrivial correlations, which could be tested by upcoming experiments.

2. The Model in the Lepton and Quark Sectors

The model is based on modular symmetry with the finite group $2O$, where the modulus τ determines Yukawa couplings and mass matrices. A detailed explanation of the weight and representation assignment can be found in [1, 2]. In the lepton sector, neutrino masses arise from a Type-I seesaw mechanism, where the Dirac mass matrix m_D and the Majorana mass matrix M_R are modular forms of specific weights. The effective light neutrino mass matrix is given by

$$m_\nu = -m_D M_R^{-1} m_D^T.$$

Charged lepton masses are generated through modular forms assigned to different generations, ensuring a hierarchical structure. In the quark sector, both up-type and down-type mass matrices originate from modular forms constrained by the 2O symmetry. The modular weights dictate the hierarchy of quark masses and mixings. The modulus τ serves as the only source of CP violation, influencing both the lepton Dirac and Majorana CP phases and the quark CP phase δ_{CP}.

The model is defined by 14 real parameters P_i:

- **Modulus:** $\text{Re}(\tau)$, $\text{Im}(\tau)$ (shared between sectors);
- **Lepton sector:** $g_1^E v_d$, g_2^E/g_1^E, g_3^E/g_1^E, $g v_u/\sqrt{\Lambda}$;
- **Quark sector:** $g_1^u v_u$, g_2^u/g_1^u, g_3^u/g_1^u, $g_1^d v_d$, g_2^d/g_1^d, g_3^d/g_1^d, g_4^d/g_1^d, g_5^d/g_1^d.

Four of these parameters are dimensional. The following units are used for convenience:

$$g_1^E v_d \text{ (MeV)}, \quad \frac{g v_u}{\sqrt{\Lambda}} \text{ (}\sqrt{\text{eV}}\text{)}, \quad g_1^u v_u \text{ (GeV)}, \quad g_1^d v_d \text{ (GeV)}.$$

The observables O_i that are used as constraints include:

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- **Lepton sector (8 observables):**
  - Neutrino mixing angles: $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$;
  - Neutrino mass-squared differences: $\delta m^2 = m_2^2 - m_1^2$, $\Delta m^2 = m_3^2 - \frac{1}{2}(m_1^2 + m_2^2)$;
  - Charged lepton mass ratios: $r_{e\mu}$, $r_{\mu\tau}$, m_τ.
- **Quark sector (10 observables):**
  - Quark mixing parameters: θ_{12}^q, θ_{23}^q, θ_{13}^q, δ_{CP}^q;
  - Mass ratios: r_{uc}, r_{ct}, r_{ds}, r_{sb};
  - Masses: m_t, m_b.

Due to current experimental limitations, the leptonic CP phase δ_{CP} is not included in the fit, as well as the unknown absolute neutrino mass scale m_1. The analysis assumes a normal mass ordering (NO) for neutrinos, as the inverted ordering is not compatible with the model. The value $\sin^2\theta_{23} = 0.5$ is adopted, due to the current octant ambiguity. RG effects for neutrino observables are negligible at $\tan\beta = 10$ and are therefore ignored. RG running is included for charged fermion masses and quark mixing parameters, which are extrapolated to the GUT scale. The fit quality is assessed using a χ^2 test:

$$\chi^2_{\text{comb}}(\mathbf{P}) = \sum_{i=1}^{18} \left(\frac{O_i^{\text{theo}}(\mathbf{P}) - O_i^{\text{exp}}}{\sigma_i^{\text{exp}}} \right)^2 .$$

Separate fits are also performed for the lepton and quark sectors:

$$\chi^2_{\text{leptons}}(\mathbf{P}_{\text{leptons}}) = \sum_{i=1}^{8} \left(\frac{O_i^{\text{theo}} - O_i^{\text{exp}}}{\sigma_i^{\text{exp}}} \right)^2 , \quad \chi^2_{\text{quarks}}(\mathbf{P}_{\text{quarks}}) = \sum_{i=9}^{18} \left(\frac{O_i^{\text{theo}} - O_i^{\text{exp}}}{\sigma_i^{\text{exp}}} \right)^2 .$$

The total χ^2 is the sum of the two:

$$\chi^2_{\text{comb}} = \chi^2_{\text{leptons}} + \chi^2_{\text{quarks}}.$$

A Markov Chain Monte Carlo (MCMC) method is used to explore parameter space and identify the best-fit points. The modulus τ is varied over the reduced fundamental domain:

$$|\text{Re}(\tau)| \leq \frac{1}{2}, \quad \text{Im}(\tau) > 0, \quad |\tau| \geq 1,$$

with the further restriction $\text{Re}(\tau) \leq 0$ to avoid redundancy due to generalized CP symmetry.

3. Correlated flavor structure: quark and lepton analysis

We perform three distinct fits: one for the lepton sector, one for the quark sector, and a combined fit including both. In each case, the number of observables equals or exceeds the number of free model parameters: specifically, 10 observables vs. 10 parameters in the quark-only analysis, 8 vs. 6 in the lepton-only, and 18 vs. 14 in the combined analysis. This means the quark-only fit is exactly constrained, while the lepton-only and combined fits are overconstrained, making a good

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fit non-trivial. Nonetheless, successful fits can lead not only to consistent descriptions of known observables but also to predictions for currently unmeasured quantities, and potential correlations between quark and lepton parameters. For clarity and compactness, the following plots overlay results from all three analyses. Figure 1 displays the allowed regions for the complex modulus τ across the three analyses. The combined fit of lepton and quark observables yields a minimum

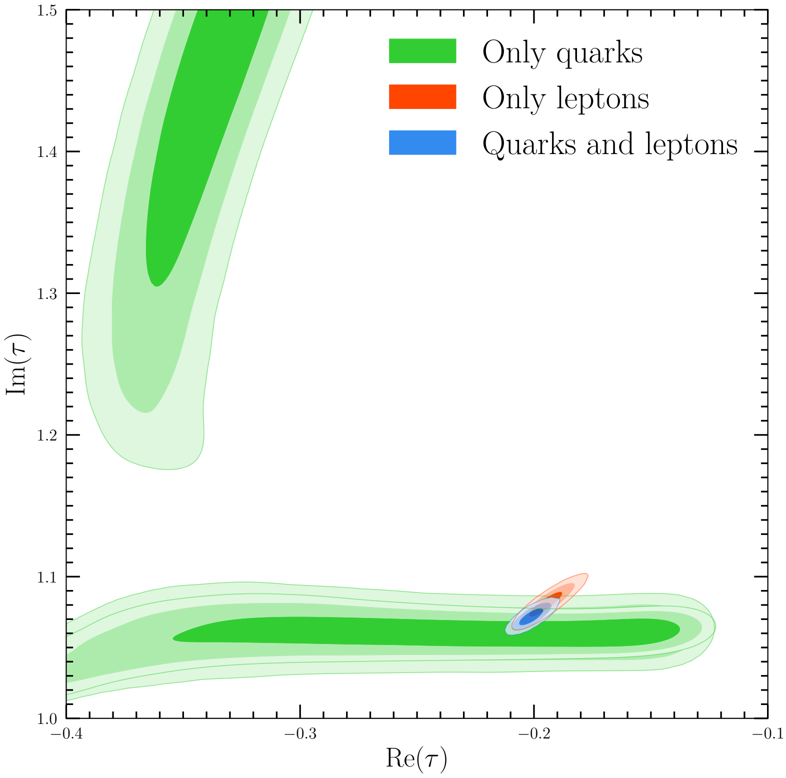

Figure 1: Allowed regions in the τ plane from the lepton-only (red), quark-only (green), and combined (blue) analyses. Different color shadings correspond to the 1σ, 2σ, and 3σ confidence levels.

value of $\chi^2_{\rm comb} = 20.3$ at $\tau = -0.202 + 1.071i$, indicating a good global agreement between the model and experimental data. This value is significantly higher than the sum of the individual minima, reflecting a mild tension between the optimal regions of the two sectors. Nonetheless, the model successfully accommodates all 18 observables with only 14 parameters, reproducing most of them within 1–2σ of their measured values. Only two observables, $\sin^2\theta_{12}$ and r_{sb}, show larger deviations, at the level of about 3σ. These results demonstrate that the model is capable of providing a coherent and predictive framework, even under the more stringent constraints of a global fit. The combined analysis reveals a non-trivial interplay between the quark and lepton sectors, going beyond a simple aggregation of the individual fits. This interaction emerges clearly from the behavior of the global χ^2 minimum, which does not coincide with the sum of the lepton-only and quark-only minima, but instead results from a shifted, joint best-fit point in the τ plane. This shift affects certain observables, worsening the fit for a few while preserving the overall consistency with experimental data. Crucially, the combined fit allows us to uncover correlations between model parameters and physical observables that are not visible in the separate analyses. As shown in

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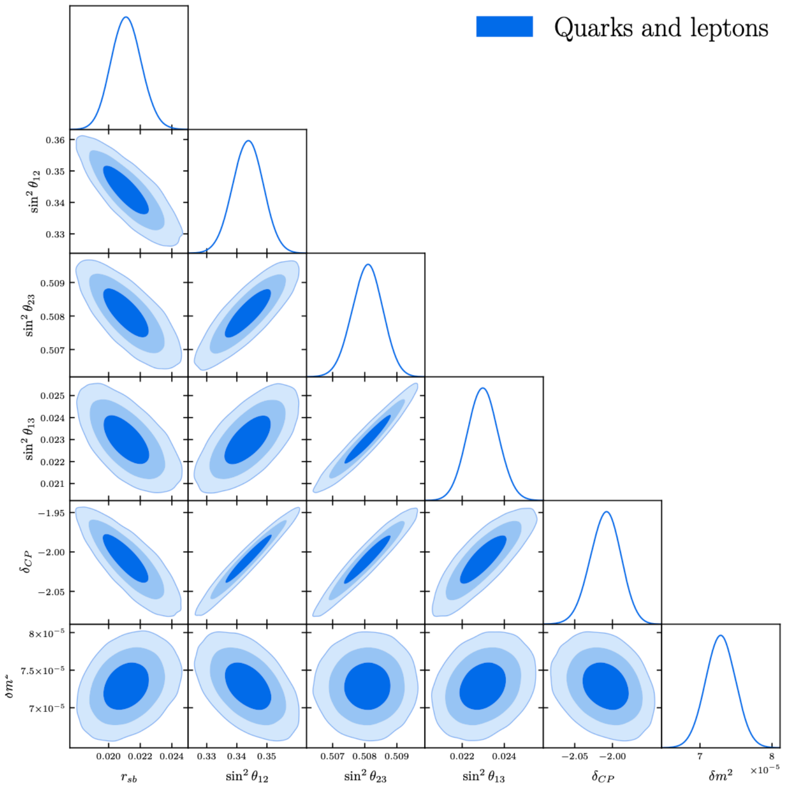

Figure 2: Allowed regions for most correlated quark-lepton observables. Different blue shadings correspond to the 1σ, 2σ, and 3σ confidence levels.

Figure 6 of [1], two observables in particular, $\sin^2\theta_{12}$ and r_{sb}, exhibit significant correlations with the real and imaginary parts of τ:

- $\sin^2\theta_{12}$ is **negatively correlated** with both $\mathrm{Re}(\tau)$ and $\mathrm{Im}(\tau)$;
- r_{sb} is **positively correlated** with the same.

As τ shifts in the combined analysis, both $\mathrm{Re}(\tau)$ and $\mathrm{Im}(\tau)$ decrease slightly. Due to their opposite correlation signs, this leads to an increase in both $\sin^2\theta_{12}$ and r_{sb}, which in turn worsens their fit compared to the separate analyses. These effects propagate through the model via correlations between τ and the Lagrangian parameters. For instance (see Figure 3 and the correlation matrices in 2):

- r_{sb} is positively correlated with all quark sector parameters, such as g_3^u/g_1^u, $g_1^d v_d$, g_2^d/g_1^d, g_3^d/g_1^d, and g_4^d/g_1^d. In the combined fit, these parameters increase, thereby raising r_{sb} and worsening its fit.
- $\sin^2\theta_{12}$ is negatively correlated with $g_1^E v_d$, g_2^E/g_1^E, and g_3^E/g_1^E, but positively correlated with $g v_u/\sqrt{\Lambda}$. The combined fit decreases the former three and increases the latter, causing $\sin^2\theta_{12}$ to rise.

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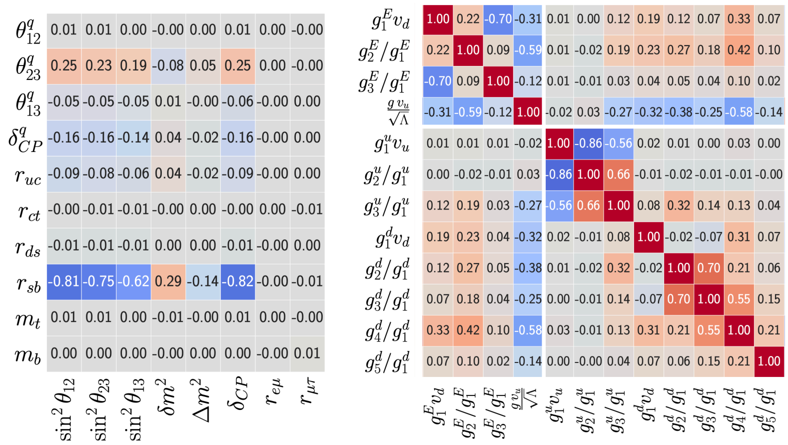

Figure 3: Allowed regions in the τ plane from the lepton-only (red), quark-only (green), and combined (blue) analyses. Different color shadings correspond to the 1σ, 2σ, and 3σ confidence levels.

Beyond known observables, the combined analysis also yields predictions for unknown neutrino parameters, including Majorana phases and mass observables. We find that:

- The J_{CP} invariant and the Majorana phases η_1, η_2 slightly decrease;
- m_1, Σ, m_β, and $m_{\beta\beta}$ slightly increase.

These shifts are consistent with the observed positive correlations between each of these observables and $\sin^2\theta_{12}$, the lepton observable most affected by the combined fit. We also identify the following correlation patterns:

i) The three lepton mixing angles are strongly positively correlated with each other;

ii) Each lepton mixing angle is positively correlated with δ_{CP} and negatively correlated with η_1, η_2;

iii) The mixing angles are also positively correlated with m_1, Σ, m_β, and $m_{\beta\beta}$.

Regarding quark-lepton interplay, r_{sb} serves as a bridge:

i) Strong negative correlation with all three lepton mixing angles;

ii) Negative correlation with δ_{CP}, positive with η_1 and η_2;

iii) Negative correlation with m_1, Σ, m_β, $m_{\beta\beta}$;

iv) Positive correlation with J_{CP}.

These correlations offer a window into the deeper structure of the model and strongly suggest that a successful combined fit is not just a numerical achievement, but an indicator of an underlying organizing principle—possibly modular flavor symmetry. A future confirmation of these correlation patterns, especially those linking quark and lepton sectors, would provide compelling support for the model. Notably, the predicted range for $m_{\beta\beta}$ lies between 9 and 11 meV, below the sensitivity of current experiments but within reach of future ton-scale neutrinoless double-beta decay searches.

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4. Conclusions

We presented a combined analysis of quark and lepton observables in a minimal bottom-up flavour model based on modular invariance and $2O$ symmetry. Unlike previous studies treating the two sectors separately, our joint fit reveals a nontrivial interplay between them, driven by their shared dependence on the complex modulus τ and by global experimental constraints. The combined fit yields an excellent overall agreement with data, reproducing most observables within 1–2σ, though some tensions remain for $\sin^2\theta_{12}$ and r_{sb} at the level of 3.2σ and 2.7σ. The model also makes testable predictions for unknown neutrino parameters. These results highlight the predictive power of modular symmetries and the necessity of global analyses to capture quark-lepton correlations in constrained flavour models.

Acknowledgements

The work of S.T.P. was supported in part by the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 860881-HIDDeN, by the Italian INFN program on Theoretical Astroparticle Physics and by the World Premier International Research Center Initiative (WPI Initiative, MEXT), Japan. The work of E.L. and A.M. was partially supported by the research grant number 2022E2J4RK "PANTHEON: Perspectives in Astroparticle and Neutrino THEory with Old and New messengers" under the program PRIN 2022 funded by the Italian Ministero dell'Università e della Ricerca (MUR) and by the European Union – Next Generation EU, as well as by the Theoretical Astroparticle Physics (TAsP) initiative of the Istituto Nazionale di Fisica Nucleare (INFN). G.J.D. is supported by the National Natural Science Foundation of China under Grant No. 2375104. A.M. is grateful to theWorkshop organizersfor theirkind hospitality

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