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The standard model in Wilson lattice gauge theory

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The $SU(3) \otimes SU(2) \otimes U(1)$ standard model maps smoothly onto a conventional Wilson lattice gauge formalism, including the parity violation of the weak interactions. The formulation makes use of the pseudo-reality of the weak group and requires the inclusion a full generation of both leptons and quarks. As in continuum discussions, chiral eigenstates of the Dirac operator generate known anomalies, although with rough gauge configurations these are no longer exact zero modes.

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1. Introduction

This talk concerns how the usual $SU(3) \otimes SU(2) \otimes U(1)$ standard model fits well with a lattice formulation using Wilson fermions and including the parity violation of the weak interaction [1]. The weak interactions are known violate parity with neutrinos being left handed, and it is desirable to include this in a mathematically well defined lattice formulation. In addition, while very small, in principle the weak interactions include non-perturbative proton decay [2]. The lattice provides a framework to study this mechanism, rather non-intuitive in perturbation theory.

A goal of many lattice approaches is to give a mathematical definition of a field theory as a continuum limit. Unfortunately we do not realize this here because the approach relies heavily on the scalar field associated with the Higgs mechanism. This is used as usual to give masses to the fermions, but also to give large masses to the familiar fermion doublers. Being a scalar field, the Higgs does not satisfy asymptotic freedom, and therefore it is unclear how to take continuum limit. A possible direction is a composite Higgs model, but this goes beyond the current discussion. The $U(1)$ field of the standard model is also not asymptotically free, but this presumably might be sidestepped via a unified model.

The approach here is similar in spirit to Refs. [3–6], with the main difference being a non trivial mingling of the weak and strong groups. For this, the appearance of an even number of fundamental weak doublets in a single generation is essential. Witten [7] has discussed how a closed path in $SU(2)$ field space can change the sign of the fermion determinant. This phase ambiguity is extensively reviewed in [8] and is closely related to the 't Hooft vertex [2] with its connection to anomalies.

A useful ingredient is the pseudo reality of the weak group. For every left handed $SU(2)$ doublet of particles, their antiparticles also form an $SU(2)$ doublet, although right handed. Thus we have an equal number of left and right handed fields. Combining them into gauge invariant singlet operators is what enables the small baryon decay process.

On the lattice all field configurations are simply connected. This means that the usual continuum discussions relating anomalies to gauge field topology must be modified. The index theorems relating zero modes of the Dirac operator to topology are incomplete. We will argue that corresponding to these modes are real eigenvalues of the Dirac operator separated by chirality.

2. Fields

Consider a full generation of fermions as a set of 8 4-component fields on lattice sites

$$u^r, u^g, u^b, d^r, d^g, d^b, \nu, e^-. \tag{1}$$

Here we include the three colors for up and down quarks, denoted $\{r, g, b\}$. We allow all fermions to have right-handed parts, including the neutrino, but the weak gauge fields will not couple to these components. In addition we include a complex Higgs doublet of scalar fields $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$ also occupying the lattice sites.

In addition to these "matter" fields, we consider gauge fields placed on the lattice bonds. Remaining as close as possible to traditional lattice methods [9–11], the bond variables include

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$SU(3)$ matrices, denoted $U_{su(3}$ for the strong interactions, $SU(2)$ matrices, denoted $U_{su(2}$ for the weak, and finally a phase factor U_Y for the hyper charge $U(1)$. These fields reflect the fields of the usual continuum standard model [12]. The gauge fields self interact separately via the standard plaquette action. This gives three independent gauge coupling constants.

Our single generation of fermions breaks down into two vector-like strong $SU(3)$ triplets

$$u = \begin{pmatrix} u^r \\ u^g \\ u^b \end{pmatrix} \qquad d = \begin{pmatrix} d^r \\ d^g \\ d^b \end{pmatrix} \tag{2}$$

as well as four left handed weak $SU(2)$ doublets

$$r = \begin{pmatrix} u^r \\ d^r \end{pmatrix}_L \ g = \begin{pmatrix} u^g \\ d^g \end{pmatrix}_L \ b = \begin{pmatrix} u^b \\ d^b \end{pmatrix}_L \ l = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L . \tag{3}$$

3. Local gauge symmetries

The formulation preserves exact local gauge symmetries on each site. These symmetries are independent for each group. In particular, a strong gauge transformation multiplies each triplet by an arbitrary $SU(3)$ element $g_{su(3)}$ on the corresponding site

$$\psi_{ud} \longrightarrow g_{su3}\psi_{ud}. \tag{4}$$

Similarly, the weak group acts on the four left handed doublets

$$\psi_{rgbl} \longrightarrow \left(g_{su2}\frac{1-\gamma_5}{2} + \frac{1+\gamma_5}{2} \right)\psi_{rgbl}. \tag{5}$$

The gauge matrices transform as usual under the transformations associated with the ends of the corresponding bonds

$$\begin{array}{c} U^{ij}_{su3} \longrightarrow g^i_{su3}\, U^{ij}_{su3}\, g^j_{su3}{}^\dagger \\ U^{ij}_{su2} \longrightarrow g^i_{su2}\, U^{ij}_{su2}\, g^j r_{su2}{}^\dagger . \end{array} \tag{6}$$

Finally, the weak $SU(2)$ gauge group also acts on Higgs fields

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \longrightarrow g_{su2}H. \tag{7}$$

The pseudo-reality of $SU(2)$ says that each element is similar to its complex conjugate

$$g^* = \tau_2 g \tau_2. \tag{8}$$

This immediately implies that another combination of the Higgs field

$$H' \equiv \tau_2 H^* \tau_2 = \begin{pmatrix} -H_2^* \\ H_1^* \end{pmatrix} \tag{9}$$

transforms under gauge transformations equivalently to H

$$H' \longrightarrow g_{su2}H'. \tag{10}$$

Finally the hyper-charge introduces phases g_Y on the fields proportional the corresponding hyper-charge $\psi \longrightarrow g_Y \psi$. These values take the same values they take in conventional continuum discussions. For the fundamental fields

$$u^r, u^g, u^b, d^r, d^g, d^b, \nu, e^- \tag{11}$$

the corresponding hyper-charge values are

$$Y_L = (1/3, 1/3, 1/3, 1/3, 1/3, 1/3, -1, -1) = 2Q \pm 1 \tag{12}$$

for the left hand parts and

$$Y_R = (4/3, 4/3, 4/3, -2/3, -2/3, -2/3, 0, -2) = 2Q \tag{13}$$

for the right hand parts. The Higgs fields also have hyper-charge, with $Y_H = 1$, $Y_{H'} = -1$. Finally the gauge fields are all neutral under hyper-charge.

An important observation is that the gauge symmetries under $SU(3)$, $SU(2)$ and $U(1)$ groups all commute. In particular the weak group $SU(2)$ doesn't change $SU(3)$ colors, the strong group doesn't break weak chirality, and hyper-charge is constant on each multiplet, whether strong or weak.

4. Composite fields

From each fermion doublet we can form an $SU(2)$ gauge singlet with the Higgs field in two different ways (H, ψ_L) and (H', ψ_L). More precisely, the singlets are defined

$$(H, \psi_L) \equiv H_1^* \psi_1 + H_2^* \psi_2 \tag{14}$$

and

$$(H', \psi_L) \equiv -H_2 \psi_1 + H_1 \psi_2. \tag{15}$$

There are thus two SU(2) invariant combinations per doublet. These combinations represent the physical left handed particles, which might be interpreted as "composite." To make closer contact with continuum discussions, one can divide out the Higgs "vacuum expectation" $v = |H|$ and obtain physical combinations

$$\begin{aligned}
e_L &= (H, l)/v & Q &= (Y_l - Y_H)/2 = -1 \\
\nu_L &= (H', l)/v & Q &= (Y_l + Y_H)/2 = 0 \\
u_{rgb_L} &= (H, rgb)/v & Q &= (Y_{rgb} - Y_H)/2 = 2/3 \\
d_{rgb_L} &= (H', rgb)/v & Q &= (Y_{rgb} + Y_H)/2 = -1/3.
\end{aligned} \tag{16}$$

This extraction is equivalent to the perturbative "unitary" gauge. As the Higgs field spins around in gauge space, the fermion fields follow along with them.

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5. Masses and doublers

As in the usual continuum discussions, the fermions are given masses via the Higgs mechanism [13–17]. For this lattice construction we use the above $SU(2)$ invariant combinations

$$\chi_L = \frac{1}{v} \begin{pmatrix} (H, \psi_L) \\ (H', \psi_L) \end{pmatrix} \tag{17}$$

to mix with the left handed fermions and give gauge singlet mass terms

$$\overline{\psi}_R M \chi_L + h.c. \tag{18}$$

.

Once we have given the fermions their masses, we can use a similar mechanism to drive the doubler masses to large values, in the “cutoff” region. This uses the conventional Wilson term [10] modified to use these “physical” fields

$$\overline{\psi}_{R i+e_\mu}(1+\gamma_\mu)\chi_{Li}/2 + \overline{\psi}_{Ri}(1-\gamma_\mu)\chi_{L i+e_\mu}/2 + h.c. \tag{19}$$

This term mimics the “formally irrelevant” operator that is only important in the large momentum doubler region. As in usual discussions of Wilson fermions, this terms breaks chiral symmetry and requires an additive mass tuning. In particular, the approach offers no explanation of why neutrinos are so light.

6. Physical gauge fields

The bare gauge fields are located on the lattice bonds, which we label by their endpoints ij. As with the fermion fields, we can combine the bond variables with the Higgs field to create gauge invariant operators for the physical weak bosons. For example, the combination

$$W^+ = (H'_i, U_{su2ij} H_j) \tag{20}$$

has electric charge one and represents a field creating the W^+ meson. Correspondingly,

$$W^- = (H_i, U_{su2ij} H'_j) \tag{21}$$

creates the negative W boson.

Following this general scheme, there are two neutral weak operators available on the bonds

$$\begin{array}{c} (H'_i, U_{su2ij} H'_j) \\ (H_i, U_{su2ij} H_j). \end{array} \tag{22}$$

These in general will mix, with one combination representing the Z and the other the hopping parameter for the physical Higgs boson.

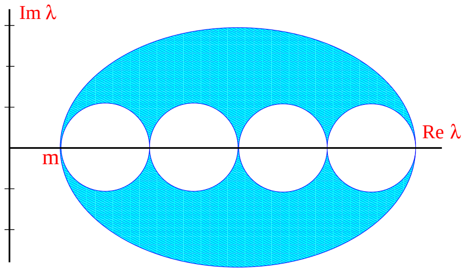

Figure 1: The spectrum of free Wilson fermions. The Wilson term moves the doubler eigenvalues to large values in the cutoff region.

7. Anomalies and Dirac eigenvalues

This basically completes the model, but it is instructive to consider how the usual quantum anomalies come into play. With dimensional regularization these effects appear via the fermionic measure not being chirally symmetric [18]. With Wilson fermions, the anomalies are moved into the behavior of heavy doubler states. This is similar in spirit to discussions of anomalies with Pauli-Villars regulation [19–21], with additional heavy states added near the cutoff.

In continuum discussions anomalies are frequently tied to topology in the gauge fields and the index theorem [22, 23] relating to zero modes in the Dirac operator. On the lattice the space of allowed configurations is simply connected and does not support separate topological sectors absent some sort of smoothing condition [24]. However such restrictions destroy reflection positivity [25] and will interfere with any Hamiltonian formulation.

It is interesting to contrast this picture with the overlap approach of Neuberger [26–28] where one projects the relevant eigenvalues onto exact zero modes. This has been successful to all orders in perturbation theory [29]. It does, however, eliminate ultra-locality of the Dirac operator [30, 31]. In addition the projection process encounters singularities as one transits between topological sectors [32]. In the current approach, the Dirac operator remains local while robust zero modes are lost.

Our Dirac operator has been constructed to satisfy gamma 5 hermeticity

$$\gamma_5 D \gamma_5 = D^\dagger. \tag{23}$$

Indeed most lattice fermion prescriptions, with the exception of twisted mass [33], satisfy this. This implies that all eigenvalues of D are either in complex conjugate pairs or real. As a reminder, in Fig. 1 we sketch the eigenvalue spectrum for free Wilson fermions. When non-trivial gauge fields are turned on, these eigenvalues move around. If the gauge fields are sufficiently smooth, the index theorem does apply and modes of non-trivial chirality are well known. However, since the space of lattice fields is simply connected, there must exist a continuous path connecting a configuration without such chiral states to one with them. As discussed in Ref. [32] in terms of the eigenvalues of D, a complex conjugate pair of eigenvalues can join on the real axis and split apart as two real eigenvalues. One of these can move to have a small real part while the other moves off to the doubler region, as sketched in Fig. 2. Ref. [34] demonstrated such a path does not need to pass through a barrier of of large action, although it does require local fields to violate the smoothness condition of [24].

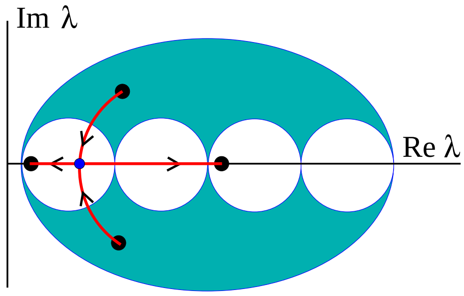

Figure 2: As we travel through configuration space, two complex conjugate Dirac modes can join on the real axis and become a pair of real eigenvalues of opposite chirality.

Restricted to the space spanned by real eigenmodes, γ_5 commutes with the Dirac operator. On this space γ_5 and D can be simultaneously diagonalized. Therefore all the real eigenvalues can be sorted by chirality. More precisely, if we have some isolated mode satisfying

$$D\psi = \lambda\psi \tag{24}$$

with lambda real, this mode also satisfies

$$\gamma_5\psi = \pm 1. \tag{25}$$

The concept of "topology" from the continuum field theory is replaced by an excess of small eigenvalues of one winding. On the lattice, these chiral real modes remain robust under small deformations of the fields. For smooth fields with a differentiable continuum limit, this reduces to the index theorem.

8. The ’t Hooft process

Small eigenvalues of D are suppressed in the partition function

$$Z = \int (dA)(d\overline{\psi}d\psi)\, e^{-S_g+\overline{\psi}D\psi} = \int (dA)\, e^{-S_g(A)} \prod \lambda_i. \tag{26}$$

This would suggest that any approximate zero modes would become irrelevant and small eigenvalues are of little importance. This naive view was shown to be incorrect by ’t Hooft [2]. The point being that certain observables can enhance the small modes.

To see this, introduce sources η and $\overline{\eta}$, differentiation with respect to which generates Green’s functions

$$Z(\eta,\overline{\eta}) = \int (dA)\ (d\overline{\psi}d\psi)\ e^{-S_g+\overline{\psi}D\psi+\overline{\psi}\eta+\overline{\eta}\psi}. \tag{27}$$

Completing the square gives

$$Z = \int (dA)\ e^{-S_g+\overline{\eta}D^{-1}\eta/4} \prod \lambda_i. \tag{28}$$

The factor of D^{-1} can bring in the inverse of small eigenvalues and cancel any suppression.

How this works to give the chiral anomaly in the strong interactions is well understood. The needed observables couple left and right fermions $\langle \psi_R D^{-1} \psi_L \rangle \neq 0$. The effective vertex applies to all strong triplets and we get a coupling that give a mass to the flavor singlet eta prime meson, as sketched in Fig. 3

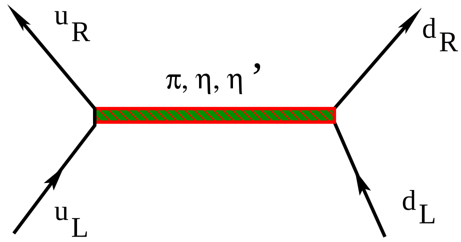

Figure 3: Small chiral eigenvalues involving the strong group generate an effective interaction that contributes to the mass of the flavor singlet eta prime meson. Figure taken from Ref. [35]

9. Weak interactions

Because the electroweak coupling is so small, the effects of the anomaly for the weak interactions are highly suppressed and essentially unobservable. However the consequences include the non-conservation of baryon and lepton number and are rather non-intuitive.

We start with our four left handed doublets. For each of them, their antiparticles are also a doublet, although right handed

$$\psi_R^c = \tau_2 \gamma_2 \psi_L^*. \tag{29}$$

For our observable, we want to combine these fields into a Lorentz invariant and gauge invariant combination. We start by pairing each doublet with a second conjugate doublet

$$\overline{\psi}_{iR}^c D^{-1} \psi_{jL} \tag{30}$$

where the indices run over the four doublets $i, j \in \{r, g, b, l\}$. The D^{-1} removes zero mode suppression. Antisymmetrization over the doublets restores strong gauge invariance

$$\epsilon_{ijkl} \, \langle \overline{\psi}_i^c D^{-1} \psi_j \, \overline{\psi}_k^c D^{-1} \psi_l \rangle \, \neq 0. \tag{31}$$

This effective vertex changes baryon number and lepton number each by 1 but preserves $B - L$. In a Hamiltonian approach this involves modes crossing into and out of the Dirac sea [36, 37]. In the process Fermion number changes by 2. This is consistent with $SU(2)$ since the group is pseudo-real. It is also consistent with $SU(3)$ since $\overline{3} \in 3 \otimes 3$ and we are taking two flavors to one anti-flavor. More physically, the process involves an "effective" neutron anti-neutrino and proton positron mixing

$$\begin{pmatrix} n \\ p \end{pmatrix} \Longleftrightarrow \begin{pmatrix} \overline{\nu} \\ e^+ \end{pmatrix}. \tag{32}$$

Through this mixing the process

$$p \rightarrow e^+ + \pi \tag{33}$$

is allowed. Of course it is st a very small rate of order $O(e^{-c/\alpha})$ with c involving the action of the chiral eigenvalue. A version of this vertex appears in a proposed domain wall approach to the weak interactions [38–40].

10. Summary

We have discussed how a single generation of the standard model fits nicely onto a Wilson lattice. In the construction the $SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetries commute and remain exact. It requires including a full full generation of quarks and leptons. We find a mechanism for baryon and lepton number violation, while the combination $B - L$ is preserved. The model has essentially the same parameters as in continuum discussion: fermion masses, gauge couplings, and the Higgs potential.

The main remaining issue concerns asymptotic freedom, absent both in electromagnetism and the Higgs quartic self coupling. This leaves an obstacle towards defining the theory as a continuum limit. For electromagnetism this suggests a possible unification with further gauge fields at high energies. For the Higgs it hints at composite models or possibly involving gravity at the highest energies [41, 42].

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