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Color-magnetic correlations in SU(N) lattice QCD

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Motivated by color-magnetic instabilities in QCD, we investigate field-strength correlations in both SU(2) and SU(3) lattice QCD. In the Euclidean Landau gauge, we numerically calculate the perpendicular-type color-magnetic correlation, $C_\perp(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\perp})\rangle$ with $\perp \equiv x, y$, and the parallel-type one, $C_\parallel(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\parallel})\rangle$ with $\parallel \equiv z, t$. In the Landau gauge, all two-point field-strength correlations $g^2\langle G_{\mu\nu}^a(s)G_{\alpha\beta}^b(s')\rangle$ are described by these two quantities, due to the Lorentz and global SU(N_c) color symmetries. Curiously, the perpendicular-type color-magnetic correlation $C_\perp(r)$ is found to be always negative for arbitrary r, except for the same point of $r = 0$. The parallel-type color-magnetic correlation $C_\parallel(r)$ is always positive. In the infrared region, $C_\perp(r)$ and $C_\parallel(r)$ strongly cancel each other, which leads to an approximate cancellation for the sum of the field-strength correlations as $\sum_{\mu,\nu}\langle G_{\mu\nu}^a(s)G_{\mu\nu}^a(s')\rangle \propto C_\perp(|s-s'|)+C_\parallel(|s-s'|) \simeq 0$. Next, we decompose the perpendicular-type color-magnetic correlation $C_\perp(r)$ into quadratic, cubic and quartic terms of the gluon field A_μ. The quadratic term is always negative, which is explained by the Yukawa-type gluon propagator $\langle A_\mu^a(s)A_\mu^a(s')\rangle \propto e^{-mr}/r$ with $r \equiv |s-s'|$ in the Landau gauge. The quartic term gives a relatively small contribution. In the infrared region, the cubic term is positive and tends to cancel with the quadratic term, resulting in a small value of $C_\perp(r)$.

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1. Introduction

Since 1973, quantum chromodynamics (QCD) has been established as the fundamental theory of the strong interaction. Due to the asymptotic freedom in QCD, its coupling decreases with the renormalization scale, and perturbative QCD is applicable to the analysis of high-energy hadron reactions. At low energies, however, the coupling becomes strong, and QCD exhibits nonperturbative phenomena such as color confinement and dynamical chiral symmetry breaking [1].

In particular, due to the asymptotic freedom, QCD has a color-magnetic instability, which involves the spontaneous emergence of color-magnetic fields [2]. In other words, the system with zero color-magnetic field is energetically unstable. The non-zero color-magnetic QCD vacuum is called the Savvidy vacuum and/or the Copenhagen vacuum [3]. Actually, the gluon condensate $\frac{\alpha_s}{\pi}\langle G^a_{\mu\nu}G^{\mu\nu}_a\rangle$ is positive in the Minkowski space, implying significant excess of color-magnetic fields rather than color-electric fields. In the QCD vacuum, to recover the rotational symmetry, the color-magnetic systems are to form a fluctuating stochastic domain structure at a large scale [3].

Considering the fluctuating color fields in the QCD vacuum, Dosch and Simonov proposed the “stochastic vacuum model" for gauge-invariant field-strength correlators and showed that its infrared exponential damping leads to an asymptotic linear potential [4, 5]. Later, Di Giacomo et al. [6, 7] and Bali, Brambilla and Vairo [8] found that the gauge-invariant field-strength correlator exhibits infrared exponential damping in lattice QCD.

Motivated by these studies, we study the field-strength correlation and its overall behavior in SU(2) and SU(3) lattice QCD [9]. In this study, using lattice QCD, we mainly investigate the color-magnetic correlation in the Landau gauge, which has many advantages in terms of symmetries and minimal gauge-field fluctuations.

2. Color-magnetic correlations in the Landau gauge

In Euclidean QCD, the Landau gauge has a global definition to minimize

$$R[A^a_\mu] \equiv \int d^4x \, \{A^a_\mu(x)A^a_\mu(x)\} \tag{1}$$

by the gauge transformation. In the global definition, the Landau gauge has a clear physical interpretation that it strongly suppresses in total artificial gauge-field fluctuations associated with the gauge degrees of freedom [10].

Considering the above-mentioned nontrivial color-magnetic structure in the QCD vacuum, we investigate the following type of color-magnetic correlation in the Landau gauge in lattice QCD:

1. Perpendicular-type color-magnetic correlation $C_\perp(r) \equiv g^2\langle H^a_z(s)H^a_z(s+r\hat{\perp})\rangle$ ($\hat{\perp}$: unit vector on the xy-plane),

2. Parallel-type color-magnetic correlation $C_\parallel(r) \equiv g^2\langle H^a_z(s)H^a_z(s+r\hat{\parallel})\rangle$ ($\hat{\parallel}$: unit vector on the zt-plane).

In the Euclidean metric, one finds $\langle H^a_z(s)H^a_z(s+r\hat{z})\rangle = \langle H^a_z(s)H^a_z(s+r\hat{t})\rangle$ using the four-dimensional rotational invariance. In the Landau gauge, all two-point field-strength correlations $g^2\langle G^a_{\mu\nu}(s)G^b_{\alpha\beta}(s')\rangle$ can be expressed with these two correlations $C_\perp(r)$ and $C_\parallel(r)$, due to the Lorentz and global SU(N_c) color symmetries.

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3. Lattice QCD setup

For the nonperturbative analysis of the color-magnetic correlation, we use SU(2) and SU(3) lattice QCD Monte Carlo calculations with the standard plaquette action at the quenched level. For the spatial correlation, we take both on-axis and off-axis lattice data. On the statistical error of the lattice data, the jackknife error estimate is adopted.

3.1 SU(3) lattice QCD setup

For the SU(3) lattice QCD calculations, we adopt the following four lattices:

i) $\beta = 5.7,\ L^4 = 16^4$ (i.e., $a \simeq 0.186$ fm, $La \simeq 3.0$ fm)
ii) $\beta = 5.8,\ L^4 = 16^4$ (i.e., $a \simeq 0.152$ fm, $La \simeq 2.4$ fm)
iii) $\beta = 6.0,\ L^4 = 24^4$ (i.e., $a \simeq 0.104$ fm, $La \simeq 2.5$ fm)
iv) $\beta = 6.2,\ L^4 = 48^4$ (i.e., $a \simeq 0.0726$ fm, $La \simeq 3.48$ fm)

The lattice spacing a is determined so as to reproduce the string tension $\sigma = 0.89$ GeV/fm [10, 11]. The gauge configurations are picked up with the interval of 1,000 sweeps, after the thermalization of 20,000 sweeps. In this study, 200 gauge configurations are used at β=5.7 and 5.8, and 800 gauge configurations at $\beta = 6.0$. For the gluon propagator, we also use 50 configurations at β=6.2 [12].

As for the Landau gauge fixing, we use the ordinary iterative maximization algorithm with an over-relaxation parameter of 1.6. In the Landau gauge, since the gluon field is globally minimized, we define SU(3) gluon fields with the link-variable as

$$\mathcal{A}_\mu(s) \equiv \frac{1}{2iag}[U_\mu(s) - U_\mu^\dagger(s)] - \frac{1}{2iagN_c}\mathrm{Tr}[U_\mu(s) - U_\mu^\dagger(s)] \in \mathrm{su}(N_c) \tag{2}$$

in the fundamental representation. This definition is often used in the Landau gauge.

3.2 SU(2) lattice QCD setup

For the SU(2) lattice QCD calculations, we adopt the following three lattices:

i) $\beta = 2.3,\ L^4 = 16^4$ (i.e., $a \simeq 0.18$ fm, $La \simeq 2.9$ fm)
ii) $\beta = 2.4,\ L^4 = 24^4$ (i.e., $a \simeq 0.127$ fm, $La \simeq 3.0$ fm)
iii) $\beta = 2.5,\ L^4 = 32^4$ (i.e., $a \simeq 0.09$ fm, $La \simeq 2.9$ fm)

The lattice spacing a is determined to reproduce the string tension $\sigma = 0.89$ GeV/fm [13]. The gauge configurations are picked up with the interval of 200 sweeps, after the thermalization of 2,000 sweeps. We use 400 gauge configurations at each β. The Landau gauge fixing is achieved by the ordinary iterative maximization algorithm with over-relaxation parameter of 1.7. In SU(2) lattice QCD, the gluon field $A_\mu(s)$ is directly obtained from the link-variable $U_\mu(s) = e^{iagA_\mu(s)}$ using the general relation of $e^{i\tau^a\theta^a} = \cos\theta + i\tau^a\hat{\theta}^a \sin\theta$ with $\theta \equiv (\theta^a\theta^a)^{1/2}$ and $\hat{\theta}^a \equiv \theta^a/\theta$.

3.3 Landau-gauge gluon propagator

Before proceeding the color-magnetic correlation, we examine the Landau-gauge gluon propagator in SU(2) and SU(3) lattice QCD. In the Landau gauge, all the gluon two-point functions of $D_{\mu\nu}^{ab}(s - s') \equiv g^2\langle A_\mu^a(s)A_\nu^b(s')\rangle$ are expressed with the scalar combination $g^2\langle A_\mu^a(s)A_\mu^a(s')\rangle$ [10], which is a single-valued function of the four-dimensional Euclidean space-time distance $r \equiv |s - s'|$. Figure 1 shows the gluon propagator in the Landau gauge in lattice QCD.

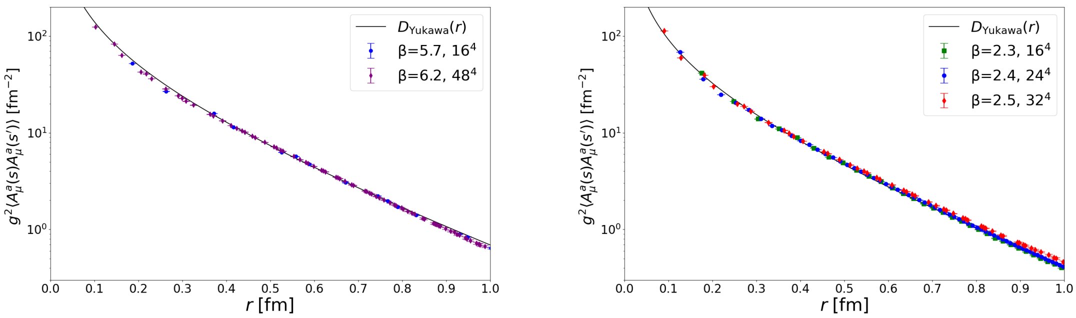

Figure 1: Landau-gauge gluon propagator $D(r) \equiv g^2\langle A_\mu^a(s)A_\mu^a(s')\rangle$ plotted against $r \equiv |s-s'|$ in SU(3) (left) and SU(2) (right) lattice QCD. The curve is the best-fit Yukawa function.

In both SU(2) and SU(3) QCD, the Landau-gauge gluon propagator is well described with a Yukawa-type function,

$$D(r) \equiv g^2\langle A_\mu^a(s)A_\mu^a(s')\rangle \simeq D_{\rm Yukawa}(r), \quad D_{\rm Yukawa}(r) \equiv A\frac{m}{r}e^{-mr}, \quad r \equiv |s-s'|, \tag{3}$$

in the wide region of $r = 0.1 - 1.0$ fm. The gluonic mass parameter is estimated as $m \simeq 0.660$ GeV for SU(3) QCD and $m \simeq 0.676$ GeV for SU(2) QCD. Since the Yukawa-type gluon propagation is natural in the three-dimensional space-time instead of the four-dimensional one, this might relate to some dimensional reduction hidden in nonperturbative QCD [14].

4. Lattice QCD result for color-magnetic correlations

In this section, we study the color-magnetic correlations, $C_\perp(r)$ and $C_\parallel(r)$, in the Landau gauge using SU(2) and SU(3) lattice QCD at the quenched level. Note again that all the two-point field-strength correlations of $g^2\langle G_{\mu\nu}^a(s)G_{\alpha\beta}^b(s')\rangle$ are expressed with these two correlations. In the adopted β region, $C_\perp(r)$ and $C_\parallel(r)$ at different β values are found to be approximately single-valued functions of r in lattice QCD.

4.1 Perpendicular-type color-magnetic correlation

To begin with, we investigate the perpendicular-type color-magnetic correlation

$$C_\perp(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\perp})\rangle \quad (\hat{\perp} : \text{unit vector on the } xy\text{-plane}) \tag{4}$$

in the Landau gauge. Figure 2 shows the numerical result in SU(3) and SU(2) lattice QCD.

Curiously, the perpendicular-type color-magnetic correlation is *always negative*, $C_\perp(r) < 0$, for all values of r, except for the same point of $r = 0$. In fact, an “always negative" correlation would be rare in physics, whereas “always positive correlation" and “alternating correlation" have been observed in various areas of physics.

One might suspect that the gauge fixing has some unphysical effect. Then, we also examine gauge-invariant field-strength correlation extracted from the plaquette correlators, as shown in Fig. 3, and obtain a similar result. Indeed, the correlation corresponding to the perpendicular-type color-magnetic correlation is always negative.

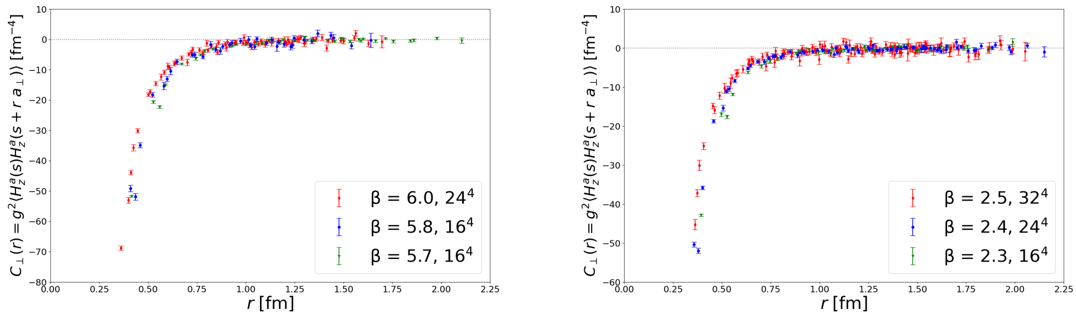

Figure 2: The perpendicular-type color-magnetic correlation $C_\perp(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\perp})\rangle$ ($\perp \equiv x, y$) in the Landau gauge in SU(3) (left) and SU(2) (right) lattice QCD.

Figure 3: An example of the plaquette correlator to extract the gauge-invariant field-strength correlation in lattice QCD.

4.2 Parallel-type color-magnetic correlation

Next, we show in Fig. 4 the parallel-type color-magnetic correlation in the Landau gauge,

$$C_\parallel(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\parallel})\rangle \quad (\hat{\parallel} : \text{unit vector on the } zt\text{-plane}), \tag{5}$$

in SU(3) and SU(2) lattice QCD. The parallel-type of $C_\parallel(r)$ is found to be always positive, $C_\parallel(r) > 0$.

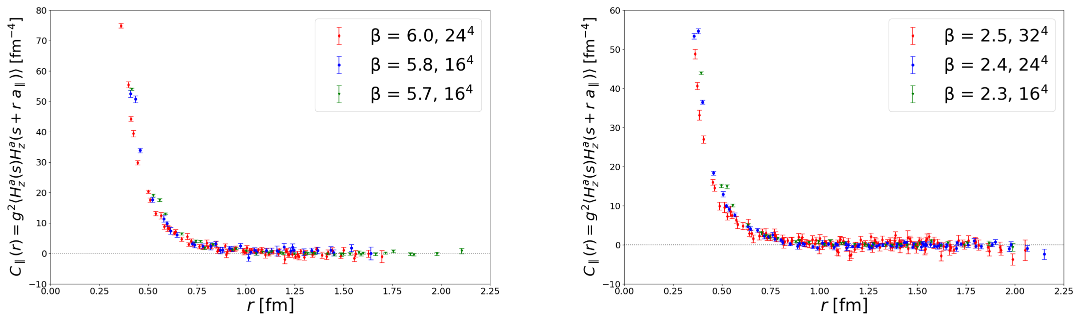

Figure 4: The parallel-type color-magnetic correlation $C_\parallel(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\parallel})\rangle$ ($\parallel \equiv z, t$) in the Landau gauge in SU(3) (left) and SU(2) (right) lattice QCD.

In the infrared region of $r \gtrsim 0.4$ fm, the parallel-type color-magnetic correlation $C_\parallel(r)$ strongly cancels with the perpendicular-type one as

$$C_\parallel(r) \simeq -C_\perp(r), \qquad \text{e.g.,} \quad \langle H_z^a(s)H_z^a(s+r\hat{z})\rangle \simeq -\langle H_z^a(s)H_z^a(s+r\hat{x})\rangle, \tag{6}$$

which leads to an approximate cancellation for the sum of Landau-gauge field-strength correlations,

$$\sum_{\mu,\nu} g^2\langle G_{\mu\nu}^a(s)G_{\mu\nu}^a(s')\rangle = 6\,[C_\perp(r) + C_\parallel(r)] \simeq 0. \tag{7}$$

5. Analysis of color-magnetic correlations in QCD

In this section, we try to analyze the lattice QCD result of the color-magnetic correlation in the Landau gauge, particularly considering the origin of the negative correlation of $C_\perp(r) < 0$.

5.1 Decomposition of the field-strength correlation in terms of the gluon field

We decompose the field-strength correlation $\langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle$ into three parts, i.e., quadratic, cubic and quartic terms of the gluon field A_μ:

$$\langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle = \langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle_{\rm quad} + \langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle_{\rm cubic} + \langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle_{\rm quartic}. \quad (8)$$

Here, the quadratic, cubic and quartic terms are defined by

$$\begin{aligned}
\langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle_{\rm quad} &\equiv \langle(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(s)(\partial_\alpha A^a_\beta - \partial_\beta A^a_\alpha)(s')\rangle, \\
\langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle_{\rm cubic} &\equiv 2ig\langle {\rm Tr}\,\{(\partial_\mu A_\nu - \partial_\nu A_\mu)(s)[A_\alpha, A_\beta](s')\}\rangle \\
&+ 2ig\langle {\rm Tr}\,\{[A_\mu, A_\nu](s)(\partial_\alpha A_\beta - \partial_\beta A_\alpha)(s')\}\rangle, \\
\langle G^a_{\mu\nu}(s)G^a_{\alpha\beta}(s')\rangle_{\rm quartic} &\equiv -2g^2\langle {\rm Tr}\,\{[A_\mu, A_\nu](s)[A_\alpha, A_\beta](s')\}\rangle, \quad (9)
\end{aligned}$$

where the factor 2 comes from ${\rm Tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$. Among the three terms, the quadratic term can be directly expressed with the gluon propagator $D^{ab}_{\mu\nu}(s-s') \equiv g^2\langle A^a_\mu(s)A^b_\nu(s')\rangle$ as

$$\begin{aligned}
& g^2\langle G^a_{\mu\nu}(s)G^b_{\alpha\beta}(s')\rangle_{\rm quad} \\
= \;& \partial^s_\mu\partial^{s'}_\alpha D^{ab}_{\nu\beta}(s-s') - \partial^s_\mu\partial^{s'}_\beta D^{ab}_{\nu\alpha}(s-s') - \partial^s_\nu\partial^{s'}_\alpha D^{ab}_{\mu\beta}(s-s') + \partial^s_\nu\partial^{s'}_\beta D^{ab}_{\mu\alpha}(s-s'). \quad (10)
\end{aligned}$$

In the Laudau gauge, due to the Lorentz symmetry, this quantity can be expressed using the scalar combination of the gluon propagator $D(r) \equiv g^2\langle A^a_\mu(s)A^a_\mu(s')\rangle$, which is a single-valued function of the four-dimensional Euclidean distance $r = |s - s'|$.

5.2 Decomposition of perpendicular-type color-magnetic correlation in the Landau gauge

For the color-magnetic correlation,

$$\begin{aligned}
\langle H^a_z(s)H^a_z(s')\rangle &= \langle(\partial_x A^a_y - \partial_y A^a_x)(s)(\partial_x A^a_y - \partial_y A^a_x)(s')\rangle \\
&+ 4ig\langle {\rm Tr}\,\{(\partial_x A_y - \partial_y A_x)(s)[A_x, A_y](s')\}\rangle \\
&- 2g^2\langle {\rm Tr}\,\{[A_x, A_y](s)[A_x, A_y](s')\}\rangle, \quad (11)
\end{aligned}$$

the quadratic term can be described with the gluon propagator. Using the Yukawa-type gluon propagator $D_{\rm Yukawa}(r)$ in Eq. (3) in the Landau gauge, we find that the quadratic term in the perpendicular-type color-magnetic correlation $C_\perp(r)$ becomes always negative:

$$\begin{aligned}
g^2\langle H^a_z(s)H^a_z(s+r\hat{\perp})\rangle_{\rm quad} &= g^2\langle(\partial_x A^a_y - \partial_y A^a_x)(s)(\partial_x A^a_y - \partial_y A^a_x)(s+r\hat{\perp})\rangle \\
&= -\frac{Am^4}{3}\frac{e^{-mr}}{mr}\left(1 + \frac{1}{mr} + \frac{1}{m^2r^2}\right) < 0 \quad (\perp \equiv x, y). \quad (12)
\end{aligned}$$

Then, if the quadratic term is dominant, the negative behavior of the perpendicular-type color-magnetic correlation, $C_\perp(r) < 0$, could be explained. However, the real situation is not so simple.

Figure 5 shows individual contributions of the quadratic, cubic and quartic terms in the perpendicular-type color-magnetic correlation $C_\perp(r)$ in lattice QCD. The quadratic term is always negative, as was demonstrated with the Yukawa-type gluon propagator in the Landau gauge. The cubic term is comparable to the quadratic term, whereas the quartic term gives a relatively small contribution. In the infrared region, the cubic term is positive and tends to cancel with the quadratic term, resulting in a small value of $C_\perp(r)$. Since the cubic term of the gauge field is unique to non-abelian gauge theories, its significant contribution in the QCD vacuum indicates the distinction between QCD and abelian gauge theories.

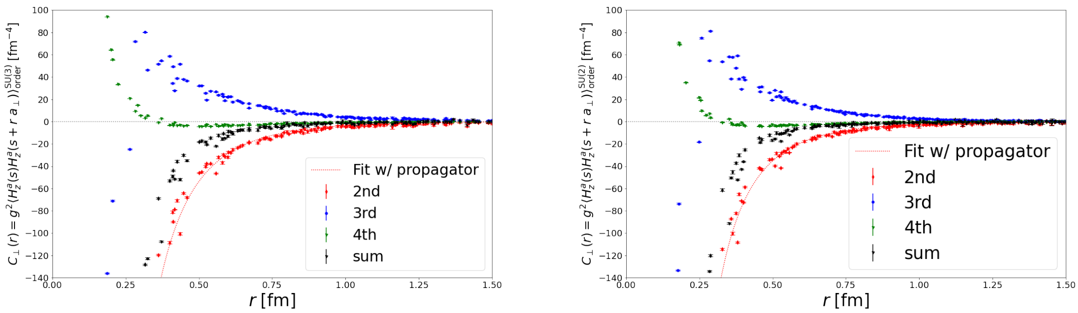

Figure 5: Each contribution of the quadratic (red), cubic (blue) and quartic (green) terms of the perpendicular-type color-magnetic correlation $C_\perp(r)$ (black) in the Landau gauge in SU(3) (left) and SU(2) (right) lattice QCD. The red dotted line denotes the curve of Eq. (12) derived from the Yukawa-type propagator $D_{\rm Yukawa}(r)$.

6. Summary and Conclusion

To examine color-magnetic instabilities in QCD, we have studied field-strength correlations in both SU(2) and SU(3) lattice QCD. In the Euclidean Landau gauge, we have numerically calculated the perpendicular-type color-magnetic correlation, $C_\perp(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\perp})\rangle$ ($\perp \equiv x, y$), and the parallel-type one, $C_\parallel(r) \equiv g^2\langle H_z^a(s)H_z^a(s+r\hat{\parallel})\rangle$ ($\parallel \equiv z, t$).

Curiously, we have found that the perpendicular-type color-magnetic correlation $C_\perp(r)$ is always negative for all values of r, except for $r = 0$. In contrast, we have found that the parallel-type color-magnetic correlation $C_\parallel(r)$ is always positive. In the infrared region, $C_\perp(r)$ and $C_\parallel(r)$ strongly cancel each other, which leads to an approximate cancellation for the sum of the field-strength correlations as $\sum_{\mu,\nu}\langle G_{\mu\nu}^a(s)G_{\mu\nu}^a(s')\rangle \propto C_\perp(|s-s'|) + C_\parallel(|s-s'|) \simeq 0$.

Next, we have decomposed the perpendicular-type color-magnetic correlation $C_\perp(r)$ into the quadratic, cubic and quartic terms of the gluon field A_μ. The quadratic term is always negative, which can be explained with the Yukawa-type gluon propagator in the Landau gauge. The quartic term gives a relatively small contribution. In the infrared region, the cubic term is positive and tends to cancel with the quadratic term, resulting in small $C_\perp(r)$.

Finally, we consider the negativity of the perpendicular-type color-magnetic correlation, $C_\perp(r) < 0$. If it were an abelian gauge theory, this could be be explained by the magnetic-flux conservation, but such an argument cannot be applied to QCD. The negative correlation seems to contradict the simple constant-magnetic or multi-vortex picture. Instead, the negative correlation $C_\perp(r) < 0$ indicates that color-magnetic fields are highly stochastic in the QCD vacuum [3–8].

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