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Strange sea quark-gluon effect on the charge radii and quadrupole moment of nucleons

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Characterizing the internal structure of nucleons in terms of sea quarks and gluons is a challenging task in hadronic physics. Both theoretical and experimental studies have validated the impact of valence and sea on different properties of nucleon. We employed the statistical model to investigate the contribution of strange sea to the charge radii and quadrupole moment of nucleons. Here, baryons are assumed to be expanded in terms of various quark-gluon Fock states as $|q\bar{q}g\rangle$, $|q\bar{q}gg\rangle$, $|q\bar{q}q\bar{q}\rangle$, $|ggg\rangle$. The statistical approach along with the detailed balance principle is used to find the relative probabilities of Fock states in flavor, spin and color space. The probabilities are computed by including subprocesses like g$\rightleftharpoons q\bar{q}$, g$\rightleftharpoons$ gg, q$\rightleftharpoons$ qg which rely on the chances of splitting and recombination of gluon into $q\bar{q}$ pairs and vice-versa. Due to heavy strange quark, a strangeness suppression factor $(1-C_l)^{n-1}$ is introduced to address the generation of $s\bar{s}$ pairs from gluon. It depends upon the no. of $s\bar{s}$ pairs present in the sea and the free energy of gluons. The present work also studied the contribution of sea in terms of scalar, vector and tensor sea. We compared our results with available experimental data and found to influenced almost 30% after considering the strange sea. Our results may contribute significantly to the forthcoming experimental advancements.

XVI^{th} Quark Confinement and the Hadron Spectrum Conference (QCHSC2024)
Cairns Convention Center, Cairns, Queensland, Australia
19 August- 24 August, 2024

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ISSN 1824-8039. Published by SISSA Medialab.

https://pos.sissa.it/

1. Introduction

Over the past decades, exploring the complete picture of the internal configuration of nucleons (proton & neutron) has been one of the most important domains in hadron physics, especially the strange content of the nucleon. In Quantum Chromodynamics (QCD), the quark sea plays a pivotal role in shaping hadron structures due to strong coupling effects. The very famous "Proton Spin Crisis?" has fascinated researchers for more than 40 years. Several studies on proton reveal that quark-antiquark pairs contribute approximately 30% to its total spin. In 2019, the STAR collaboration at the RHIC reported [1] the prime impact of sea antiquarks on the spin distribution of proton. Recently, in 2024, the CLAS Collaboration [2] conducted the first Deeply Virtual Compton Scattering (DVCS) measurement on the neutron to analyze the nucleonic structure in terms of quarks and gluons. As is well known, nucleons - fundamental components of atomic nuclei - are composed of three constituent quarks (qqq) and a dynamic 'sea' which consists of an infinite no. of quark-antiquark pairs ($u\bar{u}, d\bar{d}, s\bar{s}$) that are multiconnected through gluons. The importance of strange sea quarks becomes more evident when the non-zero quark contribution to the nucleon's spin is determined by NuTeV collaboration at Fermilab [3, 4] through a strange and non-strange quark content ratio. The ratio $\frac{2(s+\bar{s})}{u+\bar{u}+d+\bar{d}}$ = 0.477±0.063± 0.053 [5] affirmed the presence of strange quarks. It indicated the extent to which 'strangeness' contributes to overall nucleon momentum compared to other flavors of quarks. Recently, at the CERN SPS [39], the AMBER experiment detemined the charge radii of proton and mesons. Moreover, different experiments performed at MIT-Bates [6], JLab [7–9], MAMI [10] have determined the EM form factors and investigated the impact of strange quarks on the structure of nucleon.

Understanding the structure of nucleons is difficult due to the complex individual contributions from quarks, gluons, and sea-quark interactions. To enhance the clarity, the study of electromagnetic properties, particularly charge radii and quadrupole moment, is essential to understanding the internal composition of nucleons. These properties illustrate the spatial distribution of charge and magnetization within the nucleons and provide the structural (shape & size) information. The experimental measurements predicted the charge radii of the p, n and Σ^- with values r_p = 0.8409 ± 0.0004 fm, r_n^2 = -0.115 ± 0.0017 fm^2 and Σ^- = 0.78 ± 0.10 fm documented in PDG [11]. In the literature, different theoretical approaches have been employed to compute the charge radii and quadrupole moment of nucleons. These approaches include the χCQM model [12, 13] Lattice QCD [14, 15], GP method [16, 17], QCD sum rule [18, 19], the $1/N_c$ expansion method [20, 21] etc. In refs. [12, 13], the analysis of SU(3) flavor symmetry and its breaking effect is studied using the framework of χCQM. The GP method was applied in refs. [16, 17] to compute the quadrupole moment of nucleons. In spite of so much progress, the detailed structure of the strange sea in the nucleon is still not well understood. So, it is important to study diverse observables that are directly associated to the sea degrees of freedom.

In the present communication, we explore the contribution of sea quarks, particularly the strange sea, to the quadrupole moment & charge radii of nucleons using statistical techniques in conjunction with the principle of detailed balance. Here, the nucleon is treated as an ensemble of quark-gluon Fock states. The principle of detailed balance is applied to calculate the probabilities of all Fock states statistically and define them with statistical coefficients. We investigate the symmetry-breaking effect in the sea by introducing a suppression factor due to strange sea $q\bar{q}$ pairs. This factor amended the expectancy of Fock states, resulting in changes to the statistical parameters and thereby influencing the charge radii and quadrupole moment of nucleons. Statistical model have been successful in describing the parton structure functions of nucleons and various other properties of octet and decuplet baryons like masses [22], distribution of spin [23, 31], magnetic moment [24], semi-leptonic decays [29, 30], charge radii [27], quadrupole moment [28], by incorporating the effect of 'sea quarks'.

2. Theoretical Formalism

According to naive quark model, the composition of hadrons is defined by three valence quarks formed baryons (qqq) and a quark-antiquark pair formed mesons ($q\bar{q}$). The wavefunction of baryon considering only valence quarks with each component[32] is written as:

$$\Psi = \Phi(|\phi_{flavor}\rangle.|\chi_{spin}\rangle.|\psi_{color}\rangle.|\xi_{space}\rangle) \tag{1}$$

Over the years, advancement in experimental domain have revealed the existence of sea quarks surrounding the valence quark core, which contribute to the static properties and influence their internal structure. The sea comprises infinite quark-antiquark pairs of different flavors and specified by definite quantum numbers. The total spin-flavor-color wavefunction of baryons with sea component [32] is written as:

$$\begin{aligned}|\Phi^{(\uparrow)}_{1/2}> =& \frac{1}{N}[\Phi_1{}^{(\frac{1}{2}\uparrow)}H_0G_1 + a_8(\Phi_8{}^{(\frac{1}{2})} \otimes H_0)^{\uparrow}G_8 + a_{10}\Phi_{10}{}^{(\frac{1}{2}\uparrow)}H_0G_{\overline{10}} + \\ & b_1(\Phi_1{}^{(\frac{1}{2})} \otimes H_1)^{\uparrow}G_1 + b_8(\Phi_8{}^{(\frac{1}{2})} \otimes H_1)^{\uparrow}G_8 + b_{10}(\Phi_{10}{}^{(\frac{1}{2})} \otimes H_1)^{\uparrow}G_{\overline{10}} \\ & + c_8(\Phi_8{}^{(\frac{3}{2})} \otimes H_1)^{\uparrow}G_8 + d_8(\Phi_8{}^{(\frac{3}{2})} \otimes H_2)^{\uparrow}G_8\end{aligned} \tag{2}$$

where $N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2$
Here, N is the constant of normalization. Each term of the wavefunction is combined form of both valence and sea part. The spin and color wavefunction of sea is characterized by $H_{0,1,2}$ and $G_{1,8,10}$. For the lowest-lying baryons in S-wave, sea is presumed to be flavorless. If we consider the first term of the wavefunction, it contains the valence spin- $\frac{1}{2}$, flavor octet (8) and color singlet (1_c) state and sea having spin-0 (H_0) and singlet color (G_1) state:

$$\Phi_1^{(\frac{1}{2})}H_0G_1 = \Phi(8, \frac{1}{2}, 1_c)H_0G_1 \tag{3}$$

Other terms such as $a_{10}\Phi_{10}{}^{(\frac{1}{2}\uparrow)}H_0G_{\overline{10}}$, $b_1(\Phi_1^{(\frac{1}{2})} \otimes H_1)^{\uparrow}G_1$, $d_8(\Phi_8{}^{(\frac{3}{2})} \otimes H_2)^{\uparrow}$ etc. written by using appropriate C.G. coefficients. In addition, every combination (valence & sea quarks) in eq.(2) is structured in a way that ensures the antisymmetric nature of the complete wavefunction. The sea components with spin values 0, 1, and 2 correspond to the scalar, vector, and tensor sea, respectively. The statistical coefficients ($a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8$) mentioned in eq. (2), contain the contribution of scalar, vector and tensor sea. The parameters a_0, a_8, a_{10} correspond to scalar sea (spin- 0) contribution. Similarly, b_1, b_8, b_{10}, c_8 exhibit the contribution of vector sea (spin- 1) and d_8 signifies tensor sea (spin- 2) contribution.

3. Charge radii and quadrupole moment

The scrutiny of EM form factors is essential to probe the internal structure of baryons. They describe the geometric structure (shape & size), spatial charge distribution, magnetization within the baryons etc. The nucleonic vertex function is parameterized with EM Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ which is expressed as:

$$\Gamma^{\mu} = F_1(Q^2)\gamma^{\mu} + \kappa F_2(Q^2(i\frac{\sigma^{\mu\nu}q_{\nu}}{2m})) \tag{4}$$

where κ accounts for the anomalous aspect of the magnetic moment, γ^{μ} are Dirac matrices and $\sigma^{\mu\nu} = i(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})/2$ represent the spin tensor term. The Sach form factors $G_E(Q^2)$ (electric) and $G_M(Q^2)$ (magnetic) [33] can be associated as:

$$G_E = F_1 - \tau\kappa F_2 \tag{5}$$

$$G_M = F_1 + \kappa F_2 \tag{6}$$

where $\kappa = (\frac{Q}{2m})^2$. The charge of the baryon (Q), charge radii $\langle r_E^2 \rangle$, and the magnetic moment μ can be characterized by these form factors as [34]:

$$Q = G_E(0), \qquad \langle r_E^2 \rangle = 6\frac{d}{dq^2}G_E(q^2)|_{q^2=0}$$

In low-momentum expansion, the charge radii and quadrupole moment correspond to the fundamental moments of charge density (ρ) operator. The quadrupole moment of baryon is calculated by defining a general QCD unitary operator and baryon's quark-gluon states $|B\rangle$. The operator in spin-flavor space can be represented as [17, 35]:

$$\widehat{Q}_B = B\sum_{i\neq j}^{3} e_i(3\sigma_{iz}\sigma_{jz} - \sigma_i.\sigma_j) + C\sum_{i\neq j\neq k}^{3} e_i(3\sigma_{jz}\sigma_{kz} - \sigma_j.\sigma_k) \tag{7}$$

Here, σ_{iz} is the z-component of the Pauli spin matrix σ_i and e_i is the charge of the i^{th} quark where i = (u,d,s). The expansion of operator (eq. 7) for $J^P = \frac{1}{2}^+$ particles [12] expressed as:

$$\widehat{Q}_{1/2} = 3B\sum_{i\neq j} e_i\sigma_{iz}\sigma_{jz} + 3C\sum_{i\neq j\neq k} e_i\sigma_{jz}\sigma_{kz} + (-3B+3C)\sum_{i} e_i\sigma_{iz} + 3B\sum_{i} e_i \tag{8}$$

Moreover, the charge radii operator for octet members[12] can be written as:

$$\widehat{r}^2_{1/2} = (A-3B)\sum_{i} e_i + 3(B-C)\sum_{i} e_i\sigma_{iz} \tag{9}$$

Here $\widehat{r}^2_{1/2}$ corresponds to operator of spin- $\frac{1}{2}$ particles. The parameters A, B, and C mentioned in the operators accounts the contribution of orbital and color space [16, 17]. We calculate the matrix elements of relevant operators on baryonic states as:

$$\langle\Phi_{1/2}^{(\uparrow)}|\tilde{O}|\Phi_{1/2}^{(\uparrow)}\rangle = \frac{1}{N^2}[{a_0}^2\langle{\Phi_1}^{(\frac{1}{2}\uparrow)}|\tilde{O}|{\Phi_1}^{(\frac{1}{2}\uparrow)}\rangle + {a_8}^2\langle{\Phi_8}^{(\frac{1}{2}\uparrow)}|\tilde{O}|{\Phi_8}^{(\frac{1}{2}\uparrow)}\rangle + {a_{10}}^2\langle{\Phi_{10}}^{(\frac{1}{2}\uparrow)}|\tilde{O}|{\Phi_{10}}^{(\frac{1}{2}\uparrow)}\rangle + {b_1}^2\langle{\Phi_1}^{(\frac{1}{2}\uparrow)}|\tilde{O}|{\Phi_1}^{(\frac{1}{2}\uparrow)}\rangle + b_8^2\langle{\Phi_8}^{(\frac{1}{2}\uparrow)}|\tilde{O}|{\Phi_8}^{(\frac{1}{2}\uparrow)}\rangle.... \tag{10}$$

Here, $\tilde{O}$ represent the operators mentioned in eq. (8) and eq. (10). We get the equations in terms of the statistical coefficients $(a_0, a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8)$ and the parameters A, B & C. A detailed explanation of these expressions can be found in Ref. [28]. In order to compute the statistical coefficients, we used the statistical approach, discussed in next section.

Particles	$\langle \Phi^{(\uparrow)}_{1/2} \vert r^2_B \vert \Phi^{(\uparrow)}_{1/2} \rangle N^2$
p	${a_0}^2$(0.333A +0.666B -1.666C)+ ${a_8}^2$(0.333A +2.110B -3.110C)+ ${a_{10}}^2$(0.333A- B) +${b_1}^2$(0.647A -2.498B +0.555C) +${b_8}^2$(1.352A -4.613B +0.555C) +${b_{10}}^2$(0.646A - 1.940B) + ${c_8}^2$(0.549A +0.018B -1.666C) + ${d_8}^2$(0.066A- 1.266B +1.066C)
n	${a_0}^2$(-1.33B + 1.33C) + ${a_8}^2$(-0.178B +0.178C) + ${b_1}^2$(0.444B - 0.444C) + ${b_8}^2$(0.059B -0.059C) + ${c_8}^2$(-0.222B +0.222C) + ${d_8}^2$ (0.134B - 0.134C)

Table 1: Expressions obtained for charge radii operator to nucleons [28]

Particles	$\langle \Phi^{(\uparrow)}_{1/2} \vert Q_B \vert \Phi^{(\uparrow)}_{1/2} \rangle N^2$
p	${a_0}^2$(-0.333B) + ${a_8}^2$(0.822B) + ${a_{10}}^2$(B) + ${b_1}^2$(0.9464B - 2.22C) + ${b_8}^2$(0.176B - 0.296C) + ${b_{10}}^2$(0.0574B) + ${c_8}^2$(1.889B + 2.445C) + ${d_8}^2$(4.4B -0.06C)
n	${a_0}^2$(0.666B) + ${a_8}^2$(0.3779B) + ${b_1}^2$(-1.111B + 1.77C) + ${b_8}^2$(-0.341B + 0.126C) + ${c_8}^2$(-0.111B + 0.444C) + ${d_8}^2$(-0.466B + 0.7998C)

Table 2: Expressions obtained for quadrupole moment operator to nucleons [28]

4. Statistical model and principle of detailed balance

Zhang et. al formulated the principle of detailed balance [36], which assumed the expansion of hadrons in terms of various quark- gluon Fock states as:

$$|h\rangle = \sum_{i,j,k,l} C_{i,j,k,l} |\{q^3\}, \{i, j, k, l\}\rangle \tag{11}$$

Here, q^3 represent the three core quark and i, j, l and k denote the no. of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs and gluons respectively. Each Fock state is characterized by its quantum numbers, i.e., spin, flavor & color. The detailed balance principle depends on the premise that neighboring quark-gluon Fock states uphold equilibrium condition [37], ensuring that the probability of baryon occupying any particular Fock state remains constant over time and expressed as:

$$\rho_{i,j,k,l} |\{q^3\}, \{i, j, k, l\}\rangle \xleftrightarrow{balance} \rho_{i',j',k',l'} |\{q^3\}, \{i', j', k', l'\}\rangle$$

Transition like g $\rightleftharpoons q\bar{q}$, g $\rightleftharpoons$ gg, and q $\rightleftharpoons$ qg are considered to calculate the probabilities of Fock state in flavor space. These transitions involve two processes: splitting, where the probability is proportional to the no. of partons splitting and recombination, where the probability depends on no. of two kind of partons that combine to form a final state. Further, incorporating the strange sea quark into picture changes the dynamics due to its considerable mass. The occurrence of process g $\rightleftharpoons s\bar{s}$ requires the free energy of gluons that should be greater than the mass of strange quark, i.e., $\varepsilon_g > 2M_s$. Due to the suppression factor $k(1 - C_l)^{n-1}$, the production of $s\bar{s}$ pairs via gluons is restricted [36, 37]. The value of $C_{l-1} = \frac{2M_s}{M_B - 2(l-1)M_s}$, where M_B is the mass of baryon. It is essential to highlight that an increase in the no. of sea $\bar{s}s$ pairs modifies the value of $(1 - C_l)^{n-1}$, thereby affects the overall probability of Fock states. The accommodation of strange sea condensates is so engrossing that it enables the SU(3) flavor symmetry breaking within the sea. Now, the relative probabilities of several Fock states ($|u\bar{u}g\rangle$, $|d\bar{d}g\rangle$, $|s\bar{s}g\rangle$, $|u\bar{u}d\bar{d}\rangle$..) in spin and color space is calculated statistically by defining suitable multiplicities. The sum of probabilities of each Fock state represented by statistical coefficients $a_{8,10}b_{1,8,10}, c_8, d_8$. Thus, these coefficients are of immense significance since they correlate with the probabilities of each possible Fock state. A detailed analysis of the statistical approach can be found in Ref. [26, 29, 30, 38]. The statistical model is designed to be applicable at energy scale of 1 GeV^2.

5. Result and discussion

Charge radii and quadrupole moment are essential observables that reveal critical details about the inner structure of baryons. These quantities are shaped by various factors such as quark spin orientation, orbital angular momentum, sea quark-gluon interaction, etc. In the present study, we have estimated the charge radii and quadrupole moment of nucleons using the statistical framework together with the detailed balance principle. The probabilities of each Fock state is computed in terms of statistical coefficients ($a_{8,10}, b_{1,8,10}, c_8, d_8$). The independent participation of sea components (scalar, vector & tensor) is calculated. To isolate the contribution of the scalar sea (spin-0), the value of coefficients $b_{1,8,10}, c_8, d_8$ is assumed to be zero. Similarly, the individual contribution of vector (spin-1) and tensor sea (spin-2) is computed by taking the coefficients $a_{1,8,10}, d_8$ and $a_{1,8,10}, b_{1,8,10}, c_8$ to zero respectively. Further, a suppression factor is introduced $k(1 - C_l)^{n-1}$ to examine the symmtery breaking effect due to the presence of strange $q\bar{q}$ pairs in sea. Statistical model have different forms i.e. Model C, D and P which are used to understand the sea dynamics to low-energy properties. In this work, Model C is used, based on the assumption of equal probability for each Fock state with specific spin & color quantum numbers. In this work, Model C is used to investigate the above-mentioned properties. We used the same set of parameters A, B, and C as input, as discussed in our earlier publications [27, 28].

5.1 Charge radii

The numerical outcomes of charge radii of nucleon is presented in Table 3, along with individual contribution of sea. The impact of both SU(3) flavor symmetry and its breaking is also studied. For proton, the vector sea (spin-1) is dominant as compared to scalar (spin-0) and tensor sea (spin-2). The role of vector sea is more than 50%. It may be possible because the dominance in the sea is due to the generation of virtual gluons, and the major contribution comes from the spin-1 coefficients, i.e., $b_{1,8,10}, c_8$. On the other side, the effect of tensor sea is less due to the spin-flip of quarks. In the case of neutron (neutral particle), the major contribution is observed from spin-0, i.e., scalar sea, indicating the large interaction of gluons. The pure scalar sea accounts over 90%, while the influence of vector and tensor sea is almost negligible. The most prominent aspect is the participation of strange sea $q\bar{q}$ pairs. The probabilities of various Fock states are governed by the interaction of gluons and $s\bar{s}$ pairs through the process $g \rightleftharpoons s\bar{s}$. The suppression factor $(1 - C_l)^{n-1}$, where $C_{l-1} = \frac{2M_s}{M_B - 2(l-1)M_s}$, limits the formation of strange condensates which in turns affect the charge radii. The computed values of charge radii deviated upto 24% from SU(3) symmetry predicitions.

However, when sea is excluded completely, more than 50% deviation is observed in the charge radii [28]. We have further compared our results with different theoretical models and observed consistency in sign and magnitude [12, 13, 21].

Baryons	Charge radii	Statistical Model					Without sea Ref. [28]
		SU(3) symmetry breaking $(g \to u\bar{u}, d\bar{d}, s\bar{s})$	Scalar Sea	Vector Sea	Tensor Sea	SU(3) symmetry $(g \to u\bar{u}, d\bar{d})$	
p	0.7509A - 1.5850B - 0.6692C	0.6934	0.3266	0.3929	-0.0261	0.5230	0.766
n	-0.2112B + 0.2112C	-0.0836	-0.0866	-0.0005	0.0035	-0.0846	-0.116

Table 3: Charge radii of nucleons in the units of [fm^2] with parameters A= 0.754, B= 0.062 and C= -0.337

5.2 Quadrupole moment

Quadrupole moment reflects the deformities in the shape of baryons. Table 4 illustrates the predicted values of quadrupole moment of nucleons. The table clearly identifies the vector sea as main contributor to the total sea. This suggests that the quadrupole moment is primarily affected by the vectorial parameters, i.e., $b_{1,8,10}, c_8$ having spin-1, which shows the dominance of virtual gluon emission. The contribution of vector sea is more than 70% while the effect of scalar and tensor sea can be neglected. It is interesting to note that when the strange mass corrections are

Baryons	Quadrupole moment	Statistical Model					Without sea Ref. [28]
		SU(3) symmetry breaking $(g \to u\bar{u}, d\bar{d}, s\bar{s})$	Scalar Sea	Vector Sea	Tensor Sea	SU(3) symmetry $(g \to u\bar{u}, d\bar{d})$	
p	0.8147B + 0.2349C	-0.0269	-0.0005	-0.0250	-0.0013	-0.0109	-0.032
n	-0.0650B + 0.2402C	-0.0221	-0.0008	-0.0164	-0.0048	-0.0176	-0.019

Table 4: Quadrupole moment of nucleons in the units of [fm^2] with parameters B= -0.006 and C= -0.094

applied in the sea, a notable increase in the quadrupole moment values is observed, as shown in table 4. This shows the importance of strange sea to static properties. The statistical approach also predicted an oblate shape for the nucleons. The values obtained are consistent in sign and magnitude with several phenomenological approaches [16, 17]. No experimental data are available for the quadrupole moment, our predicted results may be useful to future experimentation.

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