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Isospin violation effects in the pion–nucleon sigma-term

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A precision determination of the pion–nucleon sigma term requires the consideration of isospin-breaking corrections, given that such effects are enhanced due to the chiral suppression of the isoscalar pion–nucleon amplitude. In particular, when comparing phenomenological and lattice-QCD determinations, it is critical that consistent definitions be employed. In this contribution, we present an update on the determination of the sigma term from pion–nucleon scattering, focusing on the role of isospin violation.

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1. Introduction

While it is not possible to directly probe scalar currents in the Standard Model (SM), such currents routinely arise in extensions beyond, the most prominent example being the interaction of a Majorana weakly interacting massive particle with SM fields via a scalar quark operator [1–6]. Similarly, scalar currents can play a role in the search for lepton flavor violation in $\mu \to e$ conversion in nuclei [7–12], in neutrino scattering [13, 14], and become useful for the evaluation of matrix elements in the context of electric dipole moments [15–18]. In all these cases, the scalar couplings f_q^N of the nucleon are required, defined by the matrix element

$$\langle N|m_q\bar{q}q|N\rangle = f_q^N m_N. \tag{1}$$

For the light quarks, the isosymmetric combination $\sigma_{\pi N} = m_N(f_u^N + f_d^N)$ is called the pion–nucleon (πN) sigma term, and offers a unique opportunity to validate lattice-QCD calculations of beyond-SM matrix elements against phenomenology, since $\sigma_{\pi N}$ can be still be extracted from experiment, albeit indirectly, via the Cheng–Dashen low-energy theorem [19, 20] and the analytic continuation of πN scattering into the subthreshold region. This program has been pursued in the past decade in the framework of Roy–Steiner (RS) equations [21–32], culminating in the most recent determination of $\sigma_{\pi N} = 59.0(3.5)$ MeV [33].

2. Definition of the isospin limit

Since $\sigma_{\pi N}$ is defined as an isosymmetric quantity, one needs to specify how the isospin limit is taken. For phenomenological applications, this definition conventionally follows the charged-particle masses, see Ref. [34] for $\sigma_{\pi N}$, simply because most data are available for charged-particle processes, thus minimizing the need for isospin-breaking (IB) corrections. This choice is particularly relevant for the pion mass, e.g., for the $\pi\pi$ scattering lengths a_0^I the most precise Roy-equation determination [35] pertains to the charged-pion-mass limit, while corrections need to be applied when comparing to $K \to 3\pi$ and $K_{\ell 4}$ decays [36, 37]. Enhancements of IB corrections can then occur in the presence of neutral-particle thresholds or if the quantity in question vanishes in the chiral limit. The latter applies to $\sigma_{\pi N}$, in such a way that IB corrections of size $\Delta_\pi/M_\pi^2 \simeq 6\%$, $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$, are expected if the isospin limit is instead defined by the mass of the neutral pion, as preferred in lattice QCD (we denote the corresponding value by $\bar{\sigma}_{\pi N}$). In Ref. [33] we showed that the difference can be estimated in chiral perturbation theory (ChPT) according to

$$\Delta\sigma_{\pi N} \equiv \sigma_{\pi N} - \bar{\sigma}_{\pi N} = 3.1(5)\,\text{MeV}, \tag{2}$$

and is thus not negligible when comparing lattice-QCD results with phenomenology [38]. In the updated FLAG review [39], the correction (2) is now taken into account, reducing the mismatch between phenomenological and lattice-QCD determinations.

3. Isospin violation in the πN scattering lengths and the Cheng–Dashen theorem

Enhanced IB corrections can further occur at loop level, which needs to be taken into account for the πN scattering lengths [40–42] entering as input in the solution of the RS equations. Accordingly,

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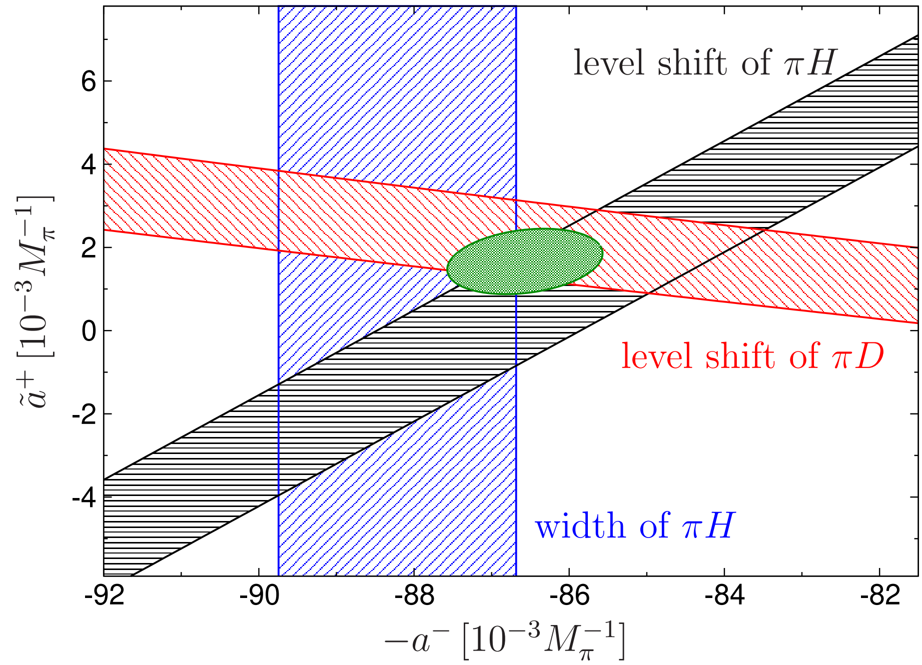

Figure 1: Combined constraints on the isoscalar ($\tilde{a}^+$) and isovector (a^-) πN scattering lengths from data on the width and level shift of πH, as well as the πD level shift. Figure taken from Ref. [33].

IB corrections need to be considered in the analysis of data from pionic atoms to determine these scattering lengths, via level shift [43] and width [44] of pionic hydrogen (πH) as well as the level shift of pionic deuterium (πD) [45], see Fig. 1, together with many-body corrections for πD [46–52]. Updating the experimental inputs in Refs. [53, 54], one obtains for the isoscalar and isovector scattering lengths [33]

$$\tilde{a}^+ = 1.7(8) \times 10^{-3} M_\pi^{-1}, \qquad a^- = 86.6(1.0) \times 10^{-3} M_\pi^{-1}, \tag{3}$$

where $\tilde{a}^+$ subsumes a set of universal IB corrections. As input for the master formula for $\sigma_{\pi N}$ derived from the RS analysis [23, 25, 26], in addition, virtual photons need to be subtracted to match the same isospin conventions, leading to isospin-1/2 and -3/2 scattering lengths

$$a^{1/2} = 169.9(2.0) \times 10^{-3} M_\pi^{-1}, \qquad a^{3/2} = -86.5(1.8) \times 10^{-3} M_\pi^{-1}. \tag{4}$$

Finally, to ensure that all corrections are defined with respect to the charged-particle masses, also the Cheng–Dashen theorem requires modifications, reflected by the shifts [23]

$$\begin{aligned}
\Delta_\sigma^p &= \sigma_p(2M_\pi^2) - \sigma_p = \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{128\pi F_\pi^2}\Big(-7 + 2\sqrt{2}\log\big(1+\sqrt{2}\big)\Big),\\
\Delta_D^p &= F_\pi^2\Big\{D_p(0, 2M_\pi^2) - d_{00}^p - 2M_\pi^2 d_{01}^p\Big\} = \frac{23 g_A^2 M_\pi^3}{384\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}\Big(3 + 4\sqrt{2}\log\big(1+\sqrt{2}\big)\Big),
\end{aligned} \tag{5}$$

in the scalar form factor $\sigma(t)$ and the πN amplitude $D(\nu, t)$, respectively. While all logarithms drop out in the difference, a net effect of $\frac{81 g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2} \simeq 3.4\,\text{MeV}$ in $\sigma_{\pi N}$ remains, again a surprisingly large loop correction enhanced by a factor of π that originates from the non-analytic loop functions.

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4. Update on $\sigma_{\pi N}$ and scalar couplings of the nucleon

Using the updated πN scattering lengths from Eq. (4), the RS analysis gives

$$\sigma_{\pi N} = 59.0(3.5)\,\text{MeV}, \tag{6}$$

where a contribution of about 2.2 MeV to the quadratic sum of uncertainties is a direct consequence of IB. Similarly, IB corrections are critical in the analysis of low-energy cross section data [29]. The resulting value $\sigma_{\pi N} = 58(5)\,\text{MeV}$ is consistent with Eq. (6), and apart from systematic uncertainties in the cross section normalizations, IB corrections indeed contribute substantially to the larger overall uncertainty.

While $\sigma_{\pi N}$ depends on the isospin conventions, the scalar couplings of the nucleon (1) are physical. They can be expressed in the form [55]

$$\begin{aligned} m_N f_u^N &= \frac{\tilde{\sigma}_{\pi N}}{2}(1-\xi) \pm Bc_5(m_d-m_u)\Big(1-\frac{1}{\xi}\Big),\\ m_N f_d^N &= \frac{\tilde{\sigma}_{\pi N}}{2}(1+\xi) \pm Bc_5(m_d-m_u)\Big(1+\frac{1}{\xi}\Big), \end{aligned} \tag{7}$$

where

$$\tilde{\sigma}_{\pi N} = \sigma_{\pi N} + \bigg(4c_1 + \frac{21 g_A^2 M_\pi}{128\pi F_\pi^2}\bigg)\Delta_\pi = \sigma_{\pi N} - 3.6(2)\,\text{MeV}, \tag{8}$$

and [38, 56, 57]

$$\xi = \frac{m_d - m_u}{m_d + m_u} = 0.365(22). \tag{9}$$

In combination with $Bc_5(m_d - m_u) = -0.51(8)\,\text{MeV}$ as determined from the electromagnetic part of the proton–neutron mass difference $(m_p - m_n)^{\text{em}} = 0.76(30)\,\text{MeV}$ [58],[1] one obtains

$$\begin{aligned} f_u^p &= 19.7(1.4)\times 10^{-3}, & f_d^p &= 38.3(2.7)\times 10^{-3},\\ f_u^n &= 17.8(1.3)\times 10^{-3}, & f_d^n &= 42.3(2.6)\times 10^{-3}. \end{aligned} \tag{10}$$

5. Excited-state contamination

Despite the small shift in the right direction due to Eq. (2), a significant tension between most lattice-QCD calculations [65–75] (with few exceptions [76]) and phenomenological determinations of $\sigma_{\pi N}$ remains. As shown in Refs. [77, 78], excited-state contamination could play an important role in resolving this puzzle, based on the observations that (i) current lattice calculations cannot reliably resolve the spectrum of excited states, especially near the physical point, and (ii) ChPT predicts sizable corrections in the right direction, enhanced by non-analytic loop effects and the $\Delta(1232)$. While it is not yet possible to distinguish between different strategies for excited-state contamination in a statistically significant manner, it is striking that comparing results in Ref. [75] between "window" and "two-state" fit (the latter corresponding better to the ChPT expectation) does entail a systematic increase of (5–10) MeV when restricting results to low pion masses, where no assumptions on SU(3) extrapolations are required and the sensitivity to excited-state effects is indeed expected to be most severe.

[1] Within uncertainties, this value is consistent with lattice QCD [59–61] as well as more recent evaluations employing the Cottingham formula [62–64].

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6. Conclusions

The careful consideration of isospin violation is clearly important for an accurate determination of $\sigma_{\pi N}$, both within the phenomenological approach via the Cheng–Dashen theorem itself and in the comparison to lattice QCD. While such IB corrections did help reduce the tension between lattice QCD and phenomenology [33], the validation can certainly not be considered satisfactory yet and more work is required, including the role of excited-state contamination in direct lattice calculations of $\sigma_{\pi N}$.

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