PROCEEDINGS OF SCIENCE

Gravitational form factors of the nucleon and their mechanical structure: Twist-2 case

Hyun-Chul Kim,[a,*] **June-Young Kim**[b] **and Ho-Yeon Won**[c]

[a] *Department of Physics, Inha University, Incheon 22212, Republic of Korea*

[b] *Theory Center, Jefferson Lab, Newport News, VA 23606, USA*

[c] *CPHT, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France*

E-mail: hchkim@inha.ac.kr, jykim@jlab.org, hoyeon.won@polytechnique.edu

We present a series of recent works on the gravitational form factors (GFFs) of the nucleon within a pion mean-field approach, which is also called the chiral quark-soliton model. We investigate the flavor structure of the mass, angular momentum, and D-term form factors of the nucleon. The main findings of the present work are given as follows: the contribution of the strange quark is rather small for the mass and angular momentum form factors, it plays an essential role in the D-term form factors. It indicates that the D-term form factor is sensitive to the outer part of the nucleon. The flavor blindness, i.e, $D^{u-d} \simeq 0$, is valid only if the strange quark is considered. We also discuss the effects of twist-4 operators. Though the gluonic contributions are suppressed by the packing fraction of the instanton vacuum in the twist-2 case, contributions from twist-4 operators are significant.

The XVIth Quark Confinement and the Hadron Spectrum Conference (QCHSC24)
19-24 August, 2024 Cairns Convention Centre, Cairns, Queensland, Australia

*Speaker

https://pos.sissa.it/

PoS(QCHSC24)074

1. Introduction

The gravitational form factors (GFFs) [1, 2] of the nucleon, also known as the energy-momentum tensor (EMT) form factors, provide crucial information on the mass, spin, mechanical properties of the nucleon [3, 4]. To investigate them, one has to consider the EMT operators in QCD, which are defined by

$$T_q^{\mu\nu} = \frac{i}{4}\bar{\psi}_q \left(\gamma^{\{\mu}\overleftrightarrow{\mathcal{D}}^{\nu\}}\right)\psi_q, \quad T_g^{\mu\nu} = -F^{\mu\rho,b}F^{\nu,b}_{\rho} + \frac{1}{4}g^{\mu\nu}F^{\lambda\rho,b}F^{b}_{\lambda\rho}, \tag{1}$$

where $\overleftrightarrow{\mathcal{D}}^\mu = \overleftrightarrow{\partial}^\mu - 2igA^\mu$ denotes the covariant derivative with $\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$, and $A^{\{\mu}B^{\nu\}} = A^\mu B^\nu + A^\nu B^\mu$. $F^{b,\mu\nu}$ represents the gluon field strength, where the superscript b is the color index. The GFFs of the nucleon can be derived by computing the matrix elements of the EMT operators:

$$\begin{aligned}\langle N(p',J_3')|T_a^{\mu\nu}(0)|N(p,J_3)\rangle = \bar{u}(p',J_3')\bigg[&A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M_N} \\ &+ D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + \bar{c}^a(t)M_N g^{\mu\nu}\bigg]u(p,J_3),\end{aligned} \tag{2}$$

where the subscript a denotes either the quark or gluon part ($a = q, g$). A^a, J^a, D^a, and $\bar{c}^a$ are called the mass, angular momentum, D-term, and $\bar{c}^a$ form factors of the nucleon, respectively. The EMT operator is conserved only if we consider both the quark and gluon EMT operators

$$T^{\mu\nu} = \sum_q T_q^{\mu\nu} + T_g^{\mu\nu}, \quad \partial_\mu T^{\mu\nu} = 0. \tag{3}$$

If one takes into account each term separately, it is not conserved. Thus, one needs to renormalize each of them, which introduces the scale dependence of the renormalized EMT operator [5, 6]. The conservation of the EMT current (3) implies that the sum of $\bar{c}^a$ vanishes: $\sum_{a=q,g}\bar{c}^a = 0$.

While the GFFs can be regarded as the second moments of the vector generalized parton distributions (GPDs) [7], the EMT operator consists of the leading-twist (spin-2) and twist-4 (spin-0) components:

$$T_a^{\mu\nu} = \bar{T}_a^{\mu\nu} + \hat{T}_a^{\mu\nu}, \tag{4}$$

where the twist-2 ($\bar{T}_a^{\mu\nu}$) and twist-4 ($\hat{T}_a^{\mu\nu}$) parts are defined by

$$\bar{T}_a^{\mu\nu} = T_a^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^{\alpha}_{a,\alpha}, \quad \hat{T}_a^{\mu\nu} = \frac{1}{4}g^{\mu\nu}T^{\alpha}_{a,\alpha}. \tag{5}$$

Thus, the leading-twist vector GPDs do not provide all GFFs.

The GFFs can be decomposed in terms of the flavors [8–10]:

$$F^{\chi=0} = F^u + F^d + F^s, \quad F^{\chi=3} = F^u - F^d, \quad F^{\chi=8} = \frac{1}{\sqrt{3}}\left(F^u + F^d - 2F^s\right), \tag{6}$$

where F^χ denotes a generic GFF. As pointed out in Ref. [10], it is nontrivial to derive the effective nonsinglet EMT currents corresponding to QCD ones, in particular, the twist-4 parts of them. In the present talk, we will mainly focus on the twist-2 EMT operator.

PoS(QCHSC24)074

2. Pion mean-field approach

We will use the pion mean-field approach, also known as the chiral quark-soliton model (χQSM), to investigate the GFFs. The χQSM was developed based on large N_c QCD [11]. In the large N_c limit of QCD, a classical baryon can be regarded as N_c valence quarks bound by a mesonic mean field that arises as a classical solution of the saddle point equation in a self-consistent manner, while the quantum fluctuations are suppressed and of order $1/N_c$. Mean-field theories have been successful in many different areas of physics such as nuclear shell models, Ginzburg-Landau theory for superconductivity, quark potential models, etc. Main idea for a mean field can schematically be

Figure 1: Schematic view on a mean field

illustrated as Fig. 1. Many particles produce a mean field, which governs a single particle in it.

In quantum field theory, a mean field is just the solution of the classical equation of motion, i.e., $\delta S/\delta\phi|_{\phi=\phi_0} = 0$, given an action S. The χQSM starts from the effective chiral action given by

$$S_{\rm eff}[U] = -N_c \mathrm{Tr}\log\left[i\partial\!\!\!/ + i\hat{m} + iMU^{\gamma_5}\right], \tag{7}$$

where $\hat{m}$ denotes the mass matrix of the current quark masses, M is the dynamical quark mass, and U^{γ_5} is the pseudo-Nambu-Goldstone boson field. For details, we refer to Refs. [12, 13].

To derive the classical mass of the nucleon, we first calculate the two-point nucleon correlation function, which consists of N_c valence quarks as shown in Fig. 2. Taking the large Euclidean time,

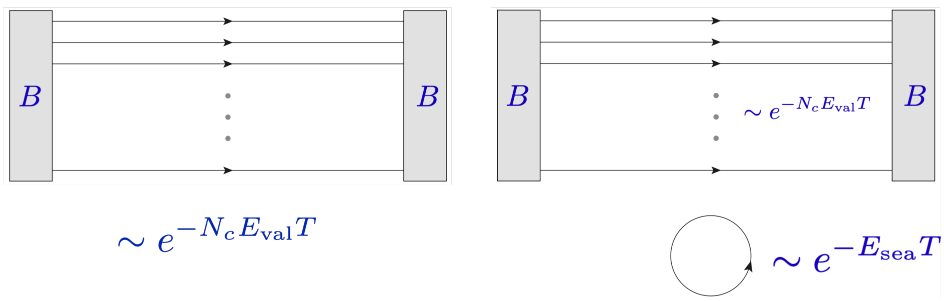

Figure 2: Nucleon correlation function.

we obtain the discrete-level (valence-quark) energy $E_{\rm val}$ and Dirac-continuum (sea-quark) energy $E_{\rm sea}$. Having minimized the sum of $E_{\rm val}$ and $E_{\rm sea}$ by using the classical equation of motion, we obtain the classical mass of the nucleon

$$M_{\rm cl} = \min[N_c E_{\rm val} + E_{\rm sea}]_{U=U_{\rm cl}} \tag{8}$$

and the pion mean field, $U_{\rm cl}$. Having performed the zero-mode quantization [12, 13], we can obtain the masses and the spin-flavor quantum numbers of the low-lying SU(3) baryons. Then we can compute the GFFs by computing the three-point correlation function with the effective EMT operators [8–10].

3. Results and discussion

In this section, we will briefly present the results for the GFFs, mainly focusing on the twist-2 contributions from $\bar{T}^{\mu\nu}$. In Fig. 3, we draw the results for each flavor contribution to the mass distribution (left panel) and corresponding form factor with the twist-2 EMT current considered only. The form factor $\bar{\mathcal{E}}(t)$ is defined as the monopole contribution to the matrix element of the temporal component of the EMT operator $\bar{T}_q^{00}$ in the three-dimensional (3D) multipole expansion, and the mass distribution $\bar{\varepsilon}(r)$ is given by the 3D Fourier transform of $\bar{\mathcal{E}}(t)$. Note that all the flavor-

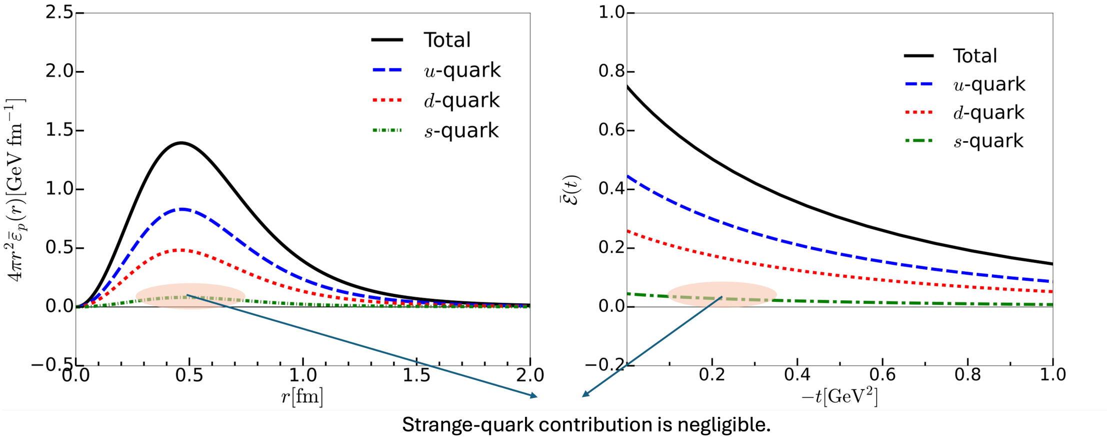

Figure 3: Flavor-decomposed mass distributions (left panel) and corresponding form factors (right panel) of the nucleon.

decomposed mass distributions are positive definite at any given r, i.e., $\bar{\varepsilon}_p^{u,d,s}(r) > 0$. The u-quark contributions to the mass distribution are approximately as twice as those of the d-quark for the proton. This can be understood in terms of the number of valence quarks inside the proton. The s-quark contribution is about 10% of the u-quark contribution.

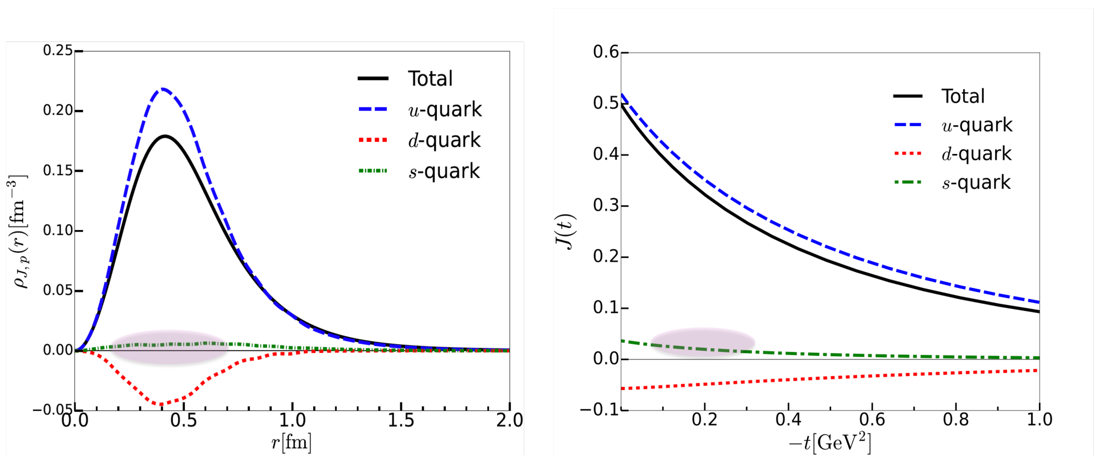

Figure 4: Flavor-decomposed angular-momentum distributions (left panel) and corresponding form factors (right panel).

In Fig. 4, we find that the angular-momentum distribution is governed by the up-quark contribution. The down-quark contribution is small but negative. So, it is compensated by that of the strange-quark contribution. While it is of great importance to decompose the total angular momentum into the orbital angular momentum and quark spin, it is a very nontrivial problem. Ji's sum rule [3] is expressed as $J = \frac{1}{2}\sum_q \Delta q + \sum_q L^q + J_g$. The gluon contributions are parametrically suppressed in the QCD instanton vacuum [14, 15], i.e., $J_g \approx 0$. Then, we can perform the decomposition of J, and estimate each contribution as follows: $\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L^q = 0.23 + 0.27$. Thus, 54% of the nucleon spin arises from the orbital angular momenta of the quarks within the χQSM.

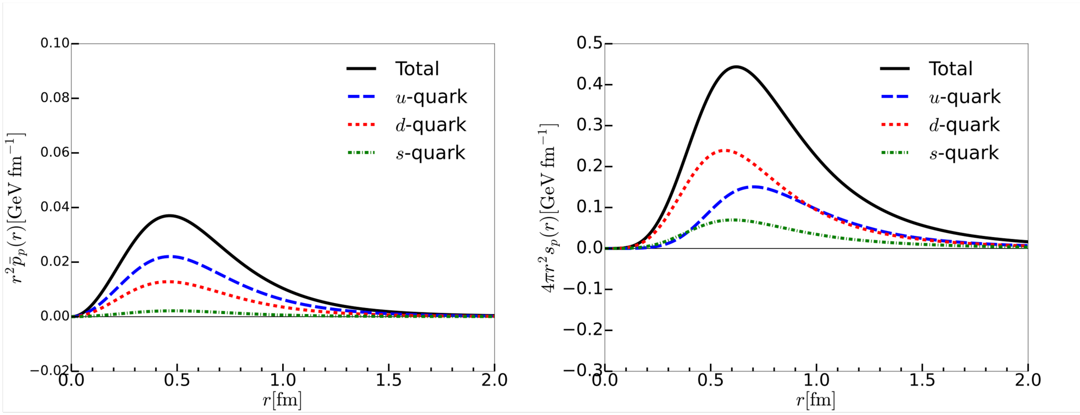

Figure 5: Flavor-decomposed twist-2 pressure distributions (left panel) and shear-force distributions (right panel) with the twist-2 contributions.

In Fig. 5, we draw the flavor-decomposed pressure and shear-force distributions of the nucleon with the twist-2 EMT operators considered only. The results indicate that both the pressure and shear-force distributions from the twist-2 EMT operator are repulsive. Thus, we expect that the twist-4 contributions must be negative, so that the stability of the nucleon is secured.

As shown in Fig. 6, the s-quark's influences on the D form factor is found to be non-negligible. Consequently, the s-quark plays an important role in the mechanical interpretation of the proton. For additional insights into the contributions of valence and sea quarks to the GFFs, refer to Ref. [9].

It is also interesting to compare various radii with each other. Figure 7 illustrates the comparison of the mass radius, mechanical radius, and charge radius. Within the χQSM, we find the following relation:

$$\langle r^2\rangle_{\text{mass}} < \langle r^2\rangle_{\text{mech}} < \langle r^2\rangle_{\text{ch}}. \tag{9}$$

4. Conclusions and outlook

In the current talk, we have presented the results from a series of recent works on the flavor decomposition of the nucleon gravitational form factors obtained by using the chiral quark-soliton model and emphasizing the role of twist-2 EMT contributions. We summarize the conclusions as follows:

- The mass and angular-momentum distributions are dominated by the up quark, with the down and strange quarks playing relatively smaller but distinct roles.

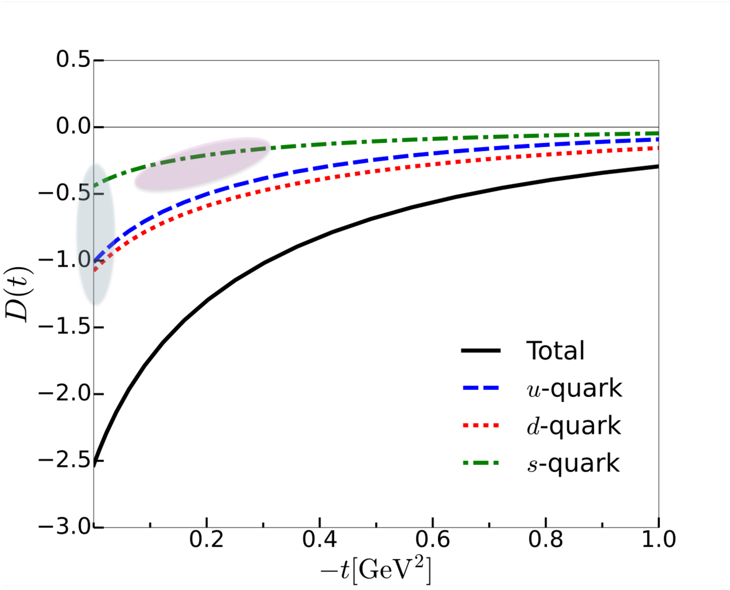

Figure 6: Flavor-decomposed D-term form factors.

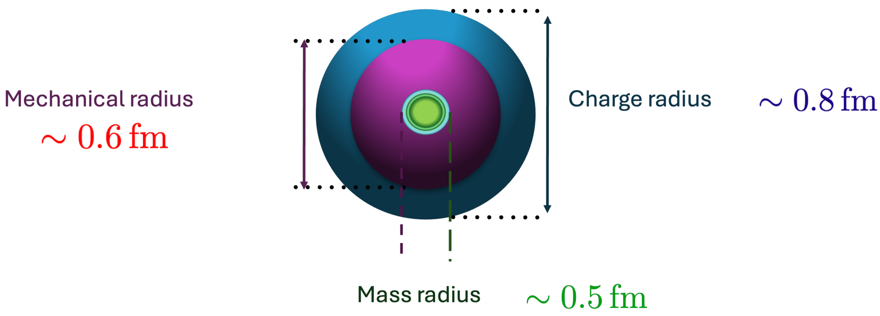

Figure 7: Comparison of the mass radius, mechanical radius, and charge radius.

- The strange quark, while minor in the mass and angular momentum sectors, significantly impacts the D-term form factor.
- The flavor-blindness of the D-term (i.e., $D^{u-d} \approx 0$) holds only when the strange quark is included.
- The twist-4 contributions, although beyond the scope of this investigation, are expected to play a crucial role in ensuring the stability of the nucleon due to their attractive nature.

In future work, it will be essential to extend the previous work [10] by incorporating the twist-4 contributions explicitly and by comparing the results with those from lattice QCD and those derived from generalized parton distributions.

Acknowledgments

The present work was supported by the Basic Science Research Program through the National Research Foundation of Korea funded by the Korean government (Ministry of Education, Science and Technology, MEST), Grant-No. 2021R1A2C2093368 and 2018R1A5A1025563. This work was also supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177 (JYK) and by the France Excellence scholarship through Campus France funded by the French government (Ministère de l'Europe et des Affaires Étrangères), 141295X (HYW).

References

[1] I. Y. Kobzarev and L. B. Okun, Zh. Eksp. Teor. Fiz. **43**, 1904-1909 (1962)

[2] H. Pagels, Phys. Rev. **144**, 1250-1260 (1966)

[3] X. D. Ji, Phys. Rev. Lett. **78**, 610-613 (1997) [arXiv:hep-ph/9603249 [hep-ph]].

[4] M. V. Polyakov, Phys. Lett. B **555**, 57-62 (2003) [arXiv:hep-ph/0210165 [hep-ph]].

[5] X. D. Ji, Phys. Rev. D **52**, 271-281 (1995) [arXiv:hep-ph/9502213 [hep-ph]].

[6] Y. Hatta, A. Rajan and K. Tanaka, JHEP **12**, 008 (2018) [arXiv:1810.05116 [hep-ph]].

[7] M. Diehl, Phys. Rept. **388**, 41-277 (2003) [arXiv:hep-ph/0307382 [hep-ph]].

[8] H. Y. Won, H.-Ch. Kim and J. Y. Kim, Phys. Lett. B **850**, 138489 (2024) [arXiv:2302.02974 [hep-ph]].

[9] H. Y. Won, H.-Ch. Kim and J. Y. Kim, Phys. Rev. D **108**, no.9, 094018 (2023) [arXiv:2307.00740 [hep-ph]].

[10] H. Y. Won, H.-Ch. Kim and J. Y. Kim, JHEP **05**, 173 (2024) [arXiv:2310.04670 [hep-ph]].

[11] E. Witten, Nucl. Phys. B **160**, 57-115 (1979)

[12] C. V. Christov, A. Blotz, H.-Ch. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola and K. Goeke, Prog. Part. Nucl. Phys. **37**, 91-191 (1996) [arXiv:hep-ph/9604441 [hep-ph]].

[13] D. Diakonov, [arXiv:hep-ph/9802298 [hep-ph]].

[14] D. Diakonov, M. V. Polyakov and C. Weiss, Nucl. Phys. B **461**, 539-580 (1996) [arXiv:hep-ph/9510232 [hep-ph]].

[15] J. Balla, M. V. Polyakov and C. Weiss, Nucl. Phys. B **510**, 327-364 (1998) [arXiv:hep-ph/9707515 [hep-ph]].

[16] H. Y. Won, J. Y. Kim and H. C. Kim, Phys. Rev. D **106**, no.11, 114009 (2022) [arXiv:2210.03320 [hep-ph]].