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Form Factors of Light Pseudoscalar Mesons – A Schwinger-Dyson Equations Perspective

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Understanding the internal structure of pseudoscalar mesons is at least as important in unraveling the working of quantum chromodynamics as hydrogen atom and its spectrum were to comprehend quantum electrodynamics. The Q^2 evolution of their electromagnetic and two-photon transition form factors from $Q^2 \to 0$ to its asymptotically large values helps us explore the infrared and ultraviolet behavior of quantum chromodynamics within one single observable. Schwinger-Dyson equations provide an ideal framework to study the internal (fundamental) degrees of a meson, *i.e.*, quarks and gluons, as their derivation and structure requires no recourse to the coupling strength being small or large. We adopt a coupled formalism based on Schwinger-Dyson and Bethe-Salpeter equations to investigate light pseudoscalar meson form factors, making comparison with the experimental results whenever available and also discussing their implications for the tests of the Standard Model of particle physics.

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1. Introduction

From rather naive perspective, pions (π) and kaons (K) might be regarded as the simplest composite systems produced by the strong interactions described by quantum chromodynamics (QCD) – after all, these light hadrons are merely two-body bound-states. Unsurprisingly, such a simplistic characterization turns out to be largely inadequate [1]. Pions and kaons are the lightest Nambu-Goldstone bosons, emerging from the dynamical chiral symmetry breaking (DCSB), one of the realizations of the emergent hadronic mass (EHM). Arguably, EHM is intimnately connected with another emergent phenomenom in QCD: color confinement [2]. Understanding the $\pi - K$ properties is thus essential for grasping the fundamental nature of QCD interactions.

In addressing the structural characteristics of pions and kaons (mesons in general), the combined framework of Schwinger-Dyson (SD) and Bethe-Salpeter (BS) equations has achieved remarkable success. Within this formalism, gravitational and electromagnetic form factors [3–6], parton distribution amplitudes and distribution functions [7–9], as well as generalized parton functions, [10–12], have been reliably studied. In this manuscript, we focus on the $\pi - K$ electromagnetic form factors (EFFs), which provide insights into the evolution of QCD features across different energy scales and are connected to precision observables like the muon's anomalous magnetic moment (a_μ), [13].

We begin by outlining key aspects of the calculation of EFFs, within the SD-BS formalism and through a computationally amicable *algebraic model* framework. This section is followed by the corresponding results, and its implications for a_μ. Finally, we discuss the scope of the employed framework and its relevance to current and future experimental efforts.

2. Electromagnetic form factors: SD-BS formalism and algebraic model

Let us consider a pseudoscalar meson $\mathbf{P}$ composed of an $f(\bar{g})$-flavored valence-quark(antiquark). The electromagnetic process $\gamma^*\mathbf{P} \to \mathbf{P}$ is described by a single form factor, defined through the matrix element:

$$\Lambda_\mu^{\mathbf{P}\to\mathbf{P}} := 2K_\mu F_{\mathbf{P}}(Q^2) \equiv K_\mu[e_f F_{\mathbf{P}}^f(Q^2) + e_{\bar{g}} F_{\mathbf{P}}^{\bar{g}}(Q^2)]\,, \tag{1}$$

where the two terms correspond to the valence-quark(antiquark) contributions, with electric charges $e_{f,\bar{g}}$. Herein, $Q^2 = (p_f - p_i)^2$ is the momentum squared transferred by the electromagnetic probe and $K = (p_f + p_i)/2$, with $p_f(p_i)$ denoting the momenta of the outoing (incoming) meson. The on-shell condition entails $p_{f,i}^2 = -m_{\mathbf{P}}^2$, where $m_{\mathbf{P}}$ is the mass of the pseudoscalar meson.

It is typical to evaluate Eq. (1) in the impulse approximation [3]. This corresponds to a triangle diagram and can be mathematically expressed as:

$$K_\mu F_{\mathbf{P}}^f(Q^2) = \mathrm{tr}\int_q S_f(q+p_f)\Gamma_\mu^f(q+p_f, q+p_i)S_f(q+p_i)\Gamma_{\mathbf{P}}(q_i;p_i)S_{\bar{g}}(q)\Gamma_{\mathbf{P}}(q_f;-p_f)\,. \tag{2}$$

Here tr indicates trace over Dirac and color indices, whereas $\int_q$ denotes a four-momentum integration on the variable q. The rest of the elements carry their usual meanings: $\Gamma_{\mathbf{P}}$ denotes the meson's BS amplitude; $S_{f,\bar{g}}$ the fully-dressed quark(antiquark) propagators; and, Γ_μ^f the dressed quark-photon vertex (QPV). Each component obeys either a SD or BS equation of its own. These are briefly described in the next section.

2.1 Schwinger-Dyson and Bethe-Salpeter equations

Fundamentally, the SD equations are the equations of motion in a quantum field theory. Each Green function satisfies a SD equation, which, in turn, depends on at least one higher-order Green function. It thus forms an infinite system of coupled equations that must be decoupled in order to arrive at a tractable problem [14, 15]. The SD equation for the quark propagator reads:

$$S^{-1}(p) \;=\; Z_2[S^{(0)}(p)]^{-1} + \int_q \mathcal{K}^{(1)}(q,p)S(q)\,, \tag{3}$$

where $\mathcal{K}^{(1)}$ denotes the one-body interaction kernel that encodes all the posible self-interactions; Z_2 is a renormalization constant; $S^{(0)}(p) = [i\gamma\cdot p + m^{\text{bm}}]^{-1}$ denotes the bare quark propagator with bare mass m^{bm}. The fully-dressed quark propagator is represented in analogy to its bare counterpart:

$$S(p) = Z(p^2)(i\gamma\cdot p + M(p^2))^{-1}\,. \tag{4}$$

Expressed in this way, $M(p^2)$ denotes the mass function, which encodes the non-perturbative effects. The relativistic two-body bound-state equation, the BS equation, can be compactly expressed as:

$$\Gamma_{\mathbf{P}}(p;P) = \int_q \mathcal{K}^{(2)}(q,p;P)S(q_+)\Gamma_{\mathbf{P}}(q;P)S(q_-)\,, \tag{5}$$

with $q_+ = q + \eta P$ and $q_- = q - (1-\eta)P$ defining the relative momentum between the quarks (here $\eta \in [0,1]$ and P denotes the meson's total momentum). As with the SD for the quark propagator, Eq. (4), $\mathcal{K}^{(2)}$ refers to a two-body interaction kernel that accounts for all possible interactions taking between the dressed valence quark and antiquark. The matrix structure characterizing the BSA depends on the meson's quantum numbers. In particular, for a pseudoscalar meson [16]:

$$\Gamma_{\mathbf{P}}(q;P) = \gamma_5[iE_{\mathbf{P}}(q;P) + \gamma\cdot PF_{\mathbf{P}}(q;P) + \gamma\cdot qG_{\mathbf{P}}(q;P) + q_\mu\sigma_{\mu\nu}P_\nu H_{\mathbf{P}}(q;P)]\,. \tag{6}$$

It is worth noting that the one-body and two-body kernels, $\mathcal{K}^{(1,2)}$, are connected by symmetry principles. Among them, charge conservation and the emergence of pions and kaons as Nambu-Goldstone bosons in the chiral limit [17] are particularly important. For this reason, the rainbow-ladder (RL) approximation, which respects these symmetries, is typically used. Other truncations use more sophisticated schemes, which include contributions from the anomalous chromomagnetic moment [18], or meson cloud effects [19, 20].

The remaining component in Eq. (2) is the QPV, Γ_μ. In principle, it also satisfies a SD equation (see Ref. [19]), but its determination is often guided by mathematical principles and phenomenological considerations [4, 21–23]. The computational difficulty of each of the elements described above depends on the complexity of the truncation used. Below, we detail a computationally accessible framework that captures the essential features of the SD-BS formalism.

2.2 Algebraic Model

As a simpler alternative to the complex numerical methods of the SD-BS framework, the contact interaction model is often used (*e.g.* Refs. [25, 26]). While this approach preserves key QCD features like dynamical chiral symmetry breaking (DCSB) and confinement, it does so at, *e.g.*,

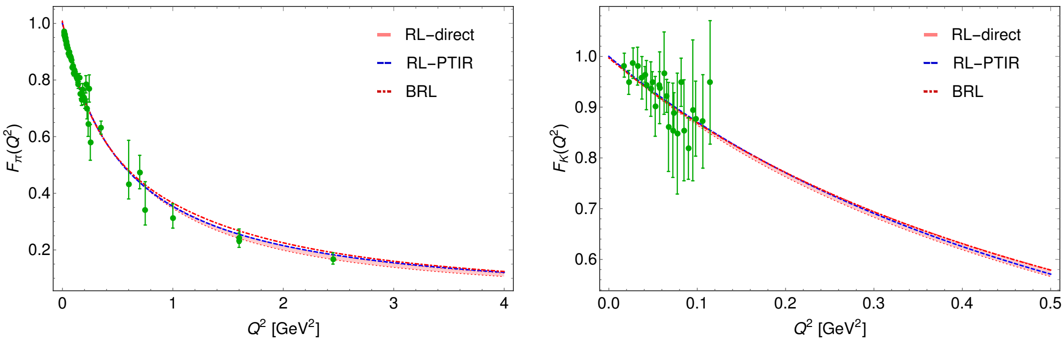

Figure 1: Pion and kaon EFFs, as derived from the SD-BS equations framework [24].

the cost of neglecting momentum-dependent effects in form factors. On the other hand a family of algebraic models (AM) have been designed to better reflect the momentum dependence of the observables studied while maintaining computational efficiency *e.g.* Refs. [12, 27–29].

We shall adopt the model proposed in Ref. [28]. This model has been employed to study the internal structure of light pseudoscalars ($\pi - K$), heavy quarkonia and heavy-light systems [28, 30, 31], with explorations also conducted in the context of vector mesons [32]. In this case, the quark propagator takes the form:

$$S_{f(\bar{g})}(k) = \left[-i\gamma \cdot k + M_{f(\bar{g})}\right] \Delta\left(k^2, M^2_{f(\bar{g})}\right) , \tag{7}$$

where $M_{f(\bar{g})}$ is interpreted as the dynamically generated constituent quark (antiquark) mass and $\Delta(s,t) = (s+t)^{-1}$. For pseudoscalar mesons, BSA is expressed as:

$$n_{\mathbf{P}}\Gamma_{\mathbf{P}}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho(w) \left[\hat{\Delta}\left(k_w^2, \Lambda_w^2\right)\right]^{\nu}. \tag{8}$$

This choice reflects the consideration of only the leading BSA, which is sufficient for many applications, provided an appropriate spectral density, $\rho(w)$, is employed [12]. We define $\hat{\Delta}(s,t) = t\Delta(s,t)$ and $k_w = k + (w/2)P$. Note $n_{\mathbf{P}}$ serves as a canonical normalization constant, while the exponent ν governs the asymptotic behavior of the BSA. The last defining component of the model, $\Lambda_w^2 := \Lambda^2(w)$, encapsulates the interplay between the relevant mass scales ($M_{f,(\bar{g})}$, $m_{\mathbf{P}}$) and the spectral density function (via w-dependence), as expressed by:

$$\Lambda^2(w) = M_f^2 - \frac{1}{4}\left(1-w^2\right) m_{\mathbf{P}}^2 + \frac{1}{2}(1-w)\left(M_{\bar{g}}^2 - M_f^2\right) . \tag{9}$$

For the QPV, the Ansatz from Ref. [4] is adopted. In the context of the AM, it yields a relatively simple structure for the vertex, dominated by the tree-level γ_μ term. More details on the derivation and application of the AM are found through Refs. [28, 30, 31].

3. Predictions on the electromagnetic form factors

The $\pi - K$ EFFs obtained using the SD-BS formalism, [24], are shown in Fig. 1. For both the pion and kaon, there is excellent agreement with the available experimental data, although the data

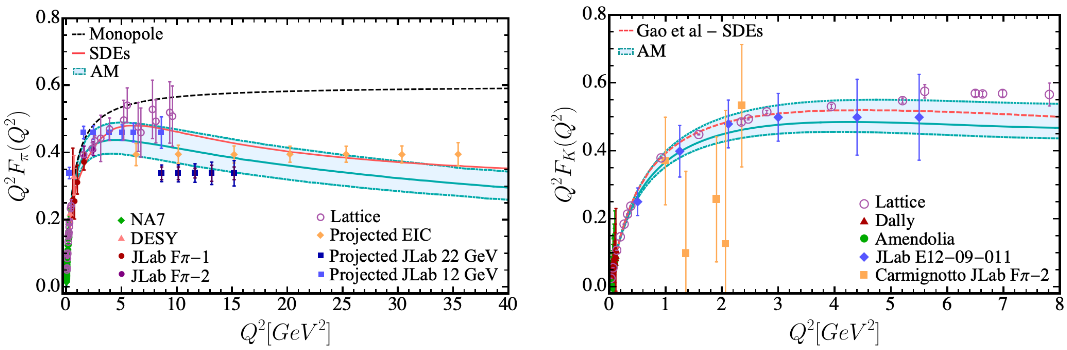

Figure 2: Pion and kaon EFFs obtained from the AM in Ref. [30]. The SD-BS equations large-Q^2 evaluations (SDEs) are taken from Refs. [3, 5]. For lattice QCD and experimental data, see [30].

is more limited in the case of the kaon. Notably, the EFFs obtained from RL truncation, whether through direct evaluation or using perturbation theory integral representation (PTIR), as well as the beyond-RL results (which account for meson cloud effects), converge to the same outcome. On one hand, this validates the PTIR approach. On the other hand, it suggests that the meson cloud effects are significantly suppressed in the large space-like regime for these systems.

These form factors are employed in the direct evaluation of the hadron light-by-light scattering contribution of the pion and kaon box diagrams to the a_μ, yielding the following estimates:

$$a_\mu^{\pi^\pm-\text{box}} = -(5.6 \pm 0.2) \times 10^{-11}\,,\; a_\mu^{K^\pm-\text{box}} = -(0.48 \pm 0.2) \times 10^{-11}\,. \tag{10}$$

The produced values are entirely compatible with previous SD-BS determinations [33], falling within typical ranges, [13], and ensuring the robustness of our approach. Analogous results have been derived recently for the corresponding first excited-states [34]:

$$a_\mu^{\pi_1^\pm-\text{box}} = -(2.02 \pm 0.10) \times 10^{-13}\,,\; a_\mu^{K_1^\pm-\text{box}} = -(1.38 \pm 0.20) \times 10^{-13}\,. \tag{11}$$

In this case, the kaon result is merely exploratory, due to subtleties related to the $SU_F(3)$ flavor symmetry breaking. The way to overcome this obstacle is currently being studied.

Finally, the $\pi - K$ EFFs obtained from the AM, [30], are depicted in Fig. 2 over a wide range of Q^2. Clearly, the SD-BS predictions are well-reproduced across the entire spectrum of photon virtualities. Furthermore, in the high-energy regime, the form factors follow the trends predicted by perturbative QCD [35]. This, together with its applications to distribution functions and generalized parton functions, [28, 31, 32], highlight the robustness of the AM under consideration and its ability to compute hadronic observables.

4. Summary

In this manuscript, we have revisited the EFFs of the pion and kaon through the SD-BS formalism, which exhibits a traceable connection to QCD and subsequently through an AM capable of reproducing the expected momentum dependence. Both approaches show consistency with the experimental observations to date, and manifest the expected trends. The validity of our results is further supported by the calculated contributions to a_μ, which fall within sensible ranges. Since the $\pi - K$ properties are closely linked to the emergent phenomena in QCD, the results shown here are of interest to ongoing and planned global efforts. Specifically, these form factors are expected to be within reach of JLab and the future Electron-Ion Collider [36, 37].

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