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Light-Cone Distribution Amplitudes of Light Baryons in Large-momentum Effective Theory

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Light-cone distribution amplitudes (LCDAs) of baryons encode the nonperturbative internal structure of hadrons and play an essential role in understanding exclusive processes in QCD. However, their direct calculation from first principles remains a major challenge due to their inherently light-cone definition. In this work, we explore the framework of large-momentum effective theory (LaMET), which enables the extraction of baryon LCDAs from lattice-calculable quasi-distribution amplitudes (quasi-DAs). We present the one-loop matching between quasi-DAs and LCDAs for both octet and decuplet baryons. The study demonstrates that merely four distinct matching kernels are sufficient to characterize all leading-twist LCDAs in both octet and decuplet baryon configurations. These insights fundamentally support lattice QCD investigations targeting the derivation of light baryon LCDAs directly from first principle of QCD. The content of this conference paper is primarily derived from the work presented in our works [1, 2].

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1. Introduction

Light-cone distribution amplitudes (LCDAs) of light baryons play a fundamental role in understanding the partonic structure of hadrons in quantum chromodynamics (QCD). They encode the momentum distributions of valence quarks and gluons in a baryon along the light-cone, and serve as essential non-perturbative inputs in QCD factorization for exclusive processes involving large momentum transfer [3]. A prominent example is the weak decays of bottom baryons, which are important for extracting Cabibbo–Kobayashi–Maskawa (CKM) matrix elements such as $|V_{ub}|$ [4], as well as probing potential new physics beyond the Standard Model via flavor-changing neutral current processes [5–7]. Compared to parton distribution functions (PDFs), which describe the probability densities of parton momentum fractions in inclusive processes, LCDAs characterize the momentum-fraction dependence of hadronic wave functions projected on the light-cone.

Despite their critical importance, our understanding of baryon LCDAs remains incomplete. Most theoretical efforts to date have been restricted to extracting a few low-order moments using methods such as QCD sum rules [8–12] or lattice QCD [13–16]. However, such moment-based approaches are insufficient to reconstruct the full shape of the LCDAs, leading to the use of model parameterizations in phenomenological analyses [17–19], which introduce sizable and often uncontrollable uncertainties in predictions for decay observables.

Theoretically, direct calculation of LCDAs on the lattice is difficult because they involve light-cone separated operators, which are inaccessible in Euclidean spacetime. To overcome this challenge, the large-momentum effective theory (LaMET) framework was proposed [20, 21]. In LaMET, instead of directly computing light-cone correlation functions, one evaluates equal-time spatial correlators in a boosted hadron state, known as quasi-distributions. These quasi-DAs share the same infrared structure as LCDAs and can be matched to them using perturbative QCD. The LaMET framework has seen remarkable developments over the past decade. Extensive lattice simulations have been performed to extract quasi-PDFs and meson LCDAs [22–30], with encouraging results in both the theoretical formulation and practical implementation [31–39]. Alternative methods for accessing light-cone physics have also been explored [40–44].

Applications of LaMET to baryon LCDAs are relatively recent. In a series of works [1, 2, 45–47], the vacuum-to-hadron spatial correlator approach was used to define quasi-DAs for the Λ baryon and to derive one-loop matching to LCDAs. A key advance in these works is the introduction of a hybrid renormalization scheme, which combines self-renormalization (to remove UV and linear divergences at large separations) with a ratio scheme (to eliminate residual divergences at short distances). This scheme ensures multiplicative renormalizability and avoids introducing additional nonperturbative parameters.

While earlier efforts focused on the Λ baryon [1], a complete study of the light baryon multiplets has remained missing. In this paper, we extend the LaMET program to systematically investigate the leading-twist LCDAs of all lowest-lying octet and decuplet baryons. At leading twist, the octet baryons possess three independent LCDAs, while the decuplet baryons have four due to their spin-3/2 structure. We define appropriate quasi-DAs for each channel, perform one-loop perturbative matching calculations, and demonstrate the factorization.

Importantly, we show that the renormalization only depends on the operator structure and is independent of the specific baryon species, which significantly simplifies the analysis. The resulting hard kernels are computed at next-to-leading order in α_s, and will serve as a critical input for future lattice QCD extractions of baryon LCDAs. These results provide a solid theoretical foundation for systematically accessing baryon LCDAs from first principles.

2. Definitions of LCDAs and Quasi-Distribution Amplitudes

The leading-twist LCDA of a light baryon B is defined through the vacuum-to-hadron matrix element of a three-quark operator with light-like separations:

$$\langle 0|\epsilon^{ijk} f^i_\alpha(a_1 n)\, g^j_\beta(a_2 n)\, h^k_\gamma(a_3 n)|B(P,\lambda)\rangle, \tag{1}$$

where f, g, and h denote the quark fields corresponding to the flavor content of baryon B, n^μ is a light-cone vector satisfying $n^2 = 0$, and λ is the helicity. The Lorentz indices (α, β, γ) and appropriate Dirac matrices are inserted to project the desired spinor structures. The momentum fractions x_i are related to the coordinate variables a_i through Fourier transforms.

2.1 Octet Baryons

The leading-twist LCDAs for octet baryons decompose into three distinct components [8]:

$$\begin{aligned}
&\left\langle 0\left|f_\alpha\left(z_1 n\right) g_\beta\left(z_2 n\right) h_\gamma\left(z_3 n\right)\right| B\left(P_B, \lambda\right)\right\rangle \\
&= \frac{1}{4} f_V\left[(\not{P}_B C)_{\alpha\beta}\left(\gamma_5 u_B\right)_\gamma V^B\left(z_i n\cdot P_B\right)+(\not{P}_B\gamma_5 C)_{\alpha\beta}\left(u_B\right)_\gamma A^B\left(z_i n\cdot P_B\right)\right] \\
&+\frac{1}{4} f_T\left(i\sigma_{\mu\nu} P_B^\nu C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 u_B\right)_\gamma T^B\left(z_i n\cdot P_B\right),
\end{aligned} \tag{2}$$

where $C \equiv i\gamma^2\gamma^0$ represents the charge conjugation matrix, u_B denotes the baryon spinor, and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. The decay constants $f_{V/A/T}$ correspond to distinct LCDA components. Isospin symmetry enforces $f_T = f_V$ for nucleons.

2.2 Decuplet Baryons

For decuplet baryons [11], the decomposition contains four terms:

$$\begin{aligned}
&\langle 0|f_\alpha\left(z_1 n\right) g_\beta\left(z_2 n\right) h_\gamma\left(z_3 n\right)|B\left(P_B, \lambda\right)\rangle \\
&= \frac{1}{4}\lambda_V\left[\left(\gamma_\mu C\right)_{\alpha\beta}\Delta^\mu_\gamma V^B\left(z_i n\cdot P_B\right)+\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(\gamma_5\Delta^\mu\right)_\gamma A^B\left(z_i n\cdot P_B\right)\right] \\
&-\frac{1}{8}\lambda_T\left(i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma^\mu\Delta^\nu\right)_\gamma T^B\left(z_i n\cdot P_B\right) \\
&-\frac{1}{4}\lambda_\varphi\left[\left(i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(P_B^\mu\Delta^\nu-\frac{1}{2} M_B\gamma^\mu\Delta^\nu\right)_\gamma\varphi^B\left(z_i n\cdot P_B\right)\right],
\end{aligned} \tag{3}$$

where $\Delta^\mu_\gamma(p,\lambda)$ satisfies the Rarita-Schwinger conditions:

$$\begin{aligned}
(\not{p} - M_B)\Delta^\mu_\gamma(p,\lambda) &= 0, \\
\bar{\Delta}^\gamma_\mu(p,\lambda)\Delta^\mu_\gamma(p,\lambda) &= -2M_B, \\
\gamma_\mu\Delta^\mu_\gamma(p,\lambda) &= 0, \\
p_\mu\Delta^\mu_\gamma(p,\lambda) &= 0.
\end{aligned}$$

The spin-$\frac{3}{2}$ vectors decompose as:

$$\begin{aligned}\Delta_{\mu\gamma}(p,\lambda=1/2) &= \sqrt{\frac{2}{3}}e^{0}_{\mu}(p)u^{\uparrow}_{\gamma}(p)+\sqrt{\frac{1}{3}}e^{+1}_{\mu}(p)u^{\downarrow}_{\gamma}(p),\\ \Delta^{\mu}_{\gamma}(p,\lambda=3/2) &= e^{+1}_{\mu}(p)u^{\uparrow}_{\gamma}(p).\end{aligned} \tag{4}$$

At leading twist and $M_B \to 0$, polarization vectors reduce to $e^0_\mu \simeq \bar{n}_\mu$, $e^{\pm}_\mu = (0,1,\mp i,0)$. The helicity structure follows: V, A, T describe helicity-$\frac{1}{2}$ states, while φ corresponds to helicity-$\frac{3}{2}$. For octet baryons, the normalization of the LCDAs are defined as [1]:

$$\begin{aligned}&V^{B\neq\Lambda}(0,0,0)=T^{B\neq\Lambda}(0,0,0)=1, \quad A^{B\neq\Lambda}(0,0,0)=0,\\ &V^{\Lambda}(0,0,0)=T^{\Lambda}(0,0,0)=0, \quad A^{\Lambda}(0,0,0)=1.\end{aligned} \tag{5}$$

For decuplet baryons:

$$V^{\Lambda}(0,0,0)=T^{\Lambda}(0,0,0)=\varphi^{B}(0,0,0)=1, \quad A^{\Lambda}(0,0,0)=0. \tag{6}$$

The momentum-space representation is defined through:

$$\begin{aligned}\Phi^{B}\left(x_1,x_2,x_3,\mu\right) = &\int_{-\infty}^{+\infty}\frac{n\cdot P dz_1}{2\pi}\frac{n\cdot P dz_2}{2\pi}\\ &\times e^{ix_1 n\cdot P z_1 + ix_2 n\cdot P z_2}\times \Phi^{B}_{R}\left(z_1 n\cdot P, z_2 n\cdot P, 0,\mu\right),\end{aligned} \tag{7}$$

where x_i represent longitudinal momentum fractions with $x_3 = 1 - x_1 - x_2$.

The quasi-DAs for octet baryons B are defined through spatial correlators:

$$\begin{aligned}\widetilde{M}^{B}_{V} &= \left\langle 0\left| f^{T}(z_1 n_z)(C\gamma^{z})g(z_2 n_z)h(z_3 n_z)\right| B(P_B,\lambda=1/2)\right\rangle = -f_V\tilde{V}^{B}P^{z}\gamma_5 u_B,\\ \widetilde{M}^{B}_{A} &= \left\langle 0\left| f^{T}(z_1 n_z)(C\gamma_5\gamma^{z})g(z_2 n_z)h(z_3 n_z)\right| B(P_B,\lambda=1/2)\right\rangle = f_A\tilde{A}^{B}P^{z}u_B,\\ \widetilde{M}^{B}_{T} &= \left\langle 0\left| f^{T}(z_1 n_z)(\frac{1}{2}C[\gamma^{z},\gamma^{\mu}])g(z_2 n_z)\gamma_\mu h(z_3 n_z)\right| B(P_B,\lambda=1/2)\right\rangle = 2f_T\tilde{T}^{B}P^{z}\gamma_5 u_B.\end{aligned} \tag{8}$$

For decuplet baryons and spin-3/2 states, the corresponding quasi-DAs are given as:

$$\begin{aligned}\widetilde{M}^{B}_{V} &= \left\langle 0\left| f^{T}(C\gamma^{z})gh\right| B(P_B,\lambda=1/2)\right\rangle = -\lambda_V\tilde{V}^{B}\gamma_5(n_z\cdot\Delta),\\ \widetilde{M}^{B}_{A} &= \left\langle 0\left| f^{T}(C\gamma_5\gamma^{z})gh\right| B(P_B,\lambda=1/2)\right\rangle = \lambda_A\tilde{A}^{B}(n_z\cdot\Delta),\\ \widetilde{M}^{B}_{T} &= \left\langle 0\left| f^{T}(\frac{1}{2}C[\gamma^{z},\gamma^{\mu}])g\gamma_\mu h\right| B(P_B,\lambda=1/2)\right\rangle = -\lambda_T\tilde{T}^{B}(n_z\cdot\Delta),\\ \widetilde{M}^{B}_{\varphi} &= \left\langle 0\left| f^{T}\left(\frac{1}{2}C[\gamma^{\nu},\gamma^{z}]\right)gh\right| B(P_B,\lambda=3/2)\right\rangle = -\lambda_\varphi\tilde{\varphi}^{B}\Delta^{\nu}.\end{aligned} \tag{9}$$

The quasi-DA in momentum space is then defined via:

$$\widetilde{\Phi}^{B}(x_1,x_2,x_3,P^z,\mu) = \int\frac{P^z dz_1}{2\pi}\frac{P^z dz_2}{2\pi}e^{-ix_1P^z z_1 - ix_2 P^z z_2}\widetilde{\Phi}^{B}_{R}(z_1,z_2,0,P^z,\mu). \tag{10}$$

[1] In Eq. (2.7) of our work Ref. [2], the normalization conditions $V^{\Lambda}(0,0,0) = T^{\Lambda}(0,0,0) = 1$, $A^{\Lambda}(0,0,0) = 0$ contains a typo. The correct values for the corresponding part should be $V^{\Lambda}(0,0,0) = T^{\Lambda}(0,0,0) = 0$, $A^{\Lambda}(0,0,0) = 1$.

2.3 Quasi-Distribution Amplitudes (Quasi-DAs)

Within the LaMET framework, the extraction of LCDAs starts from the construction of spatial correlators, whose Fourier transforms are referred to as quasi-distribution amplitudes (quasi-DAs). Quasi-DAs share the same infrared (IR) structure as LCDAs and can be directly computed using lattice QCD. Their differences arise in the ultraviolet (UV) regime, where quasi-DAs differ from LCDAs due to the finite momentum and equal-time operator insertions. In the large-momentum regime, the key difference between quasi-distribution amplitudes (quasi-DAs) and LCDAs manifests in their ultraviolet (UV) behavior—a regime governed by perturbative QCD. The collinear component of a quasi-DA converges to its light-cone counterpart in this limit, establishing a leading-order factorization framework where the quasi-DA's infrared (IR) physics aligns with the LCDA, while its UV divergence is systematically renormalized. This underpins the formal relationship:

$$\begin{aligned}&\tilde{\Phi}^B(x_1,x_2,P^z,\mu)\\ =&\int dy_1dy_2C(x_1,x_2,y_1,y_2,\mu,P^z)\,\Phi^B(y_1,y_2,\mu)+O\left(\frac{\Lambda^2_{\rm QCD}}{(x_1P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(x_2P^z)^2},\frac{\Lambda^2_{\rm QCD}}{((1-x_1-x_2)P^z)^2}\right).\end{aligned} \tag{11}$$

Here, C is the perturbative matching kernel accounting for UV differences. The quasi-DAs for octet baryons B are defined through spatial correlators:

$$\begin{aligned}\widetilde{M}^B_V&=\left\langle 0\left|f^T(z_1n_z)(C\gamma^z)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=1/2)\right\rangle=-f_V\tilde{V}^BP^z\gamma_5u_B,\\ \widetilde{M}^B_A&=\left\langle 0\left|f^T(z_1n_z)(C\gamma_5\gamma^z)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=1/2)\right\rangle=f_A\tilde{A}^BP^zu_B,\\ \widetilde{M}^B_T&=\left\langle 0\left|f^T(z_1n_z)(\frac{1}{2}C[\gamma^z,\gamma^\mu])g(z_2n_z)\gamma_\mu h(z_3n_z)\right|B(P_B,\lambda=1/2)\right\rangle=2f_T\tilde{T}^BP^z\gamma_5u_B.\end{aligned} \tag{12}$$

For decuplet baryons and spin-3/2 states, the corresponding quasi-DAs are given as:

$$\begin{aligned}\widetilde{M}^B_V&=\left\langle 0\left|f^T(C\gamma^z)gh\right|B(P_B,\lambda=1/2)\right\rangle=-\lambda_V\tilde{V}^B\gamma_5(n_z\cdot\Delta),\\ \widetilde{M}^B_A&=\left\langle 0\left|f^T(C\gamma_5\gamma^z)gh\right|B(P_B,\lambda=1/2)\right\rangle=\lambda_A\tilde{A}^B(n_z\cdot\Delta),\\ \widetilde{M}^B_T&=\left\langle 0\left|f^T(\frac{1}{2}C[\gamma^z,\gamma^\mu])g\gamma_\mu h\right|B(P_B,\lambda=1/2)\right\rangle=-\lambda_T\tilde{T}^B(n_z\cdot\Delta),\\ \widetilde{M}^B_\varphi&=\left\langle 0\left|f^T\left(\frac{1}{2}C[\gamma^\nu,\gamma^z]\right)gh\right|B(P_B,\lambda=3/2)\right\rangle=-\lambda_\varphi\tilde{\varphi}^B\Delta^\nu.\end{aligned} \tag{13}$$

The quasi-DA in momentum space is then defined via:

$$\widetilde{\Phi}^B(x_1,x_2,x_3,P^z,\mu)=\int\frac{P^zdz_1}{2\pi}\frac{P^zdz_2}{2\pi}e^{-ix_1P^zz_1-ix_2P^zz_2}\widetilde{\Phi}^B_R(z_1,z_2,0,P^z,\mu). \tag{14}$$

2.4 General Structure of the Matching Kernel

At one-loop, the matching kernel $C(x_1,x_2,y_1,y_2,\mu)$ takes the form

$$C(x_1,x_2,y_1,y_2,\mu,P^z)=\delta(x_1-y_1)\delta(x_2-y_2)+\frac{\alpha_sC_F}{4\pi}\mathcal{H}(x_1,x_2,y_1,y_2,\mu,P^z), \tag{15}$$

where $\mathcal{H}$ [1, 2] represents the hard kernel containing one-loop corrections. It consists of terms with logarithmic dependence on the renormalization scale and hadron momentum, such as

$$\mathcal{H}(x_1, x_2, y_1, y_2, \mu, P^z) \supset \ln\left(\frac{\mu^2}{(P^z)^2}\right) P(x_1, x_2; y_1, y_2) + \text{finite terms.} \tag{16}$$

The function P can be viewed as an analog of the ERBL kernel generalized for baryonic systems, and the full structure of $\mathcal{H}$ depends on the specific Dirac structure of the operator.

Because the matching is performed at the operator level, the kernels are universal for a given operator type. That is, the same C applies to all baryons (octet or decuplet) that share the same interpolating operator structure. This universality simplifies the extraction of multiple baryon LCDAs using a common set of matching inputs. A key result is that the matching kernels are *universal*—they do not depend on the hadron state, but only on the Dirac structure of the nonlocal operator. This allows the same kernel to be applied across different baryons (e.g., proton, neutron, Λ, Σ, etc.) as long as the quasi-DA is constructed using the same operator. This universality is crucial for future lattice QCD applications, as it avoids the need to recalculate the matching for every baryon species.

3. Summary and Outlook

In this work, we have presented a systematic framework to compute the leading-twist LCDAs of light baryons from lattice QCD using large-momentum effective theory (LaMET). We introduced appropriate definitions of LCDAs and quasi-distribution amplitudes for SU(3) octet and decuplet baryons, including the momentum fraction structure and symmetry properties. A perturbative matching framework was established, with matching kernels computed at one-loop level. The factorization between quasi- and light-cone distributions was explicitly verified. The resulting matching kernels can be applied across all light baryons, paving the way for future lattice QCD calculations of baryon LCDAs from first principles. This theoretical setup provides a practical and universal toolset for studying the internal structure of light baryons.

Future developments may include lattice implementations of quasi-DAs for individual baryons (such as the proton, Λ, and Σ), two-loop matching calculations to reduce theoretical uncertainties, and phenomenological applications in heavy baryon decays and hadronic form factors. These efforts will ultimately enable a precise and model-independent understanding of baryon structure and dynamics in QCD.

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