

Electromagnetic and axial-vector structure of singly heavy baryons

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In this talk, we present a series of recent works on the electromagnetic and axial-vector structures of low-lying singly heavy baryons. We first explain the pion mean-field approach, in which light and singly heavy baryons can be considered on an equal footing. We then discuss the results for the electromagnetic and radiative transition form factors of the singly heavy baryons. We also demonstrate the results for the strong decay rates and quark spin content of the singly heavy baryons. Finally, we propose a consistent way of dealing with the $1/m_Q$ corrections in the pion mean-field approach.

*The XVIth Quark Confinement and the Hadron Spectrum Conference (QCHSC24)
19-24 August, 2024 Cairns Convention Centre, Cairns, Queensland, Australia*

*Speaker

1. Introduction

A singly heavy baryon contains two light quarks and one heavy quark. If we take the limit of the infinitely heavy quark mass, i.e., $m_Q \rightarrow \infty$, the heavy-quark spin, S_Q can not be flipped, which leads to the conservation of S_Q . So, the spin of the light quarks is also conserved: $S_L \equiv S - S_Q$. This is called “heavy-quark spin symmetry” [1–3], which makes the total spin of the light quarks a good quantum number. Moreover, in the limit of $m_Q \rightarrow \infty$, the heavy quark is independent of flavor, which results in “heavy-quark flavor symmetry.” Thus, the heavy quark inside a singly heavy baryon plays a mere role of a static color source. It only contributes to the total spin of the singly heavy baryon. This indicates that dynamics inside the singly heavy baryon is governed by the light quark degrees of freedom. Consequently, the low-lying singly heavy baryons can be classified by flavor $SU(3)_f$ symmetry: $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$. The total spin of the light quarks is given by $\frac{1}{2} \otimes \frac{1}{2} = \mathbf{0} \oplus \mathbf{1}$. When it is coupled to heavy quark, one can get the total spins of the singly heavy baryons as $J' = 1/2$ and $J' = 3/2$. The baryon antitriplet ($\bar{\mathbf{3}}$) and sextet are depicted in the left and right panels of Fig. 1, respectively. It is natural for the baryon antitriplet to have $J' = 1/2$, and for the baryon sextet to have either $J' = 1/2$ and $J' = 3/2$. The baryon sextet are degenerate. The degeneracy can only be lifted by considering the $1/m_Q$ corrections. We will later discuss how the baryon antitriplet has $J' = 1/2$ whereas the baryon sextet has either $J' = 1/2$ and $J' = 3/2$ within the chiral quark-soliton model (χ QSM).

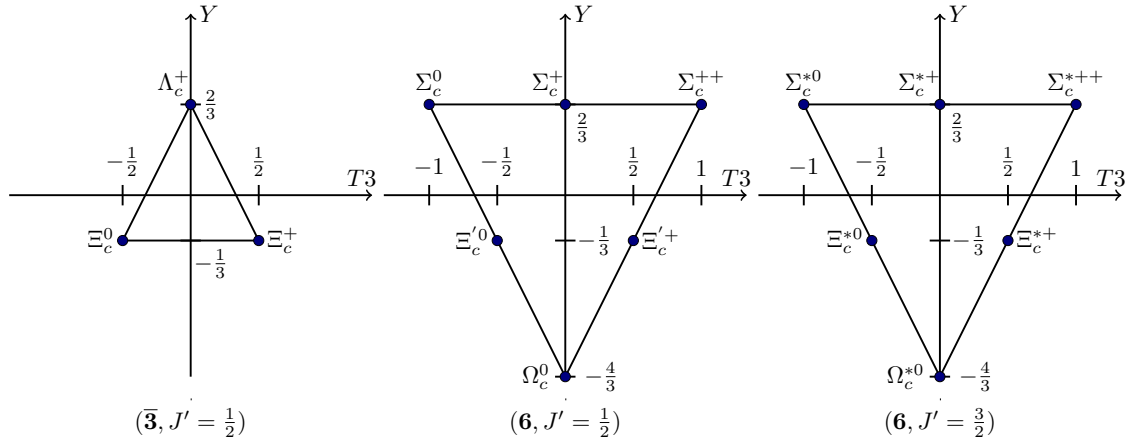


Figure 1: $SU(3)$ representation of lowest-lying singly heavy baryons

Witten proposed a seminal idea that an ordinary light baryon can be viewed as the bound state of N_c valence quarks in a meson mean field, if one considers the large N_c (the number of colors) limit. The presence of the N_c valence quarks will polarize the vacuum, and generate the pion mean field. Then, the N_c valence quarks are bound by the pion mean field in a self-consistent manner. The χ QSM can be considered as a realization of this Witten’s idea [4–6]. The very same idea can be applied to the singly heavy baryon. If we strip off one light quark and replace it with a heavy quark, then the singly heavy baryon can be regarded as a state of the $N_c - 1$ valence quarks bound by the pion mean field, where have been produced by the presence of the $N_c - 1$ valence quarks [7, 8, 11]. Thus, the pion mean-field approach or the χ QSM allows one to describe both the light and singly heavy baryons in an equal footing (see also a review [9]).

The mass spectrum of the singly heavy baryons was successfully reproduced [7, 8, 11], compared with the experimental data. We found that the pion mean field for the singly heavy baryons becomes weaker than that for the light baryons. Since the baryon sextet are degenerate, we introduced the $1/m_Q$ chromomagnetic hyperfine interactions to remove the degeneracy. Using the same theoretical framework, we extended to compute various properties of the singly heavy baryons. In this talk, we summarize a series of recent results for the electromagnetic (EM) and axial-vector observables and discuss them.

2. Pion mean-field approach

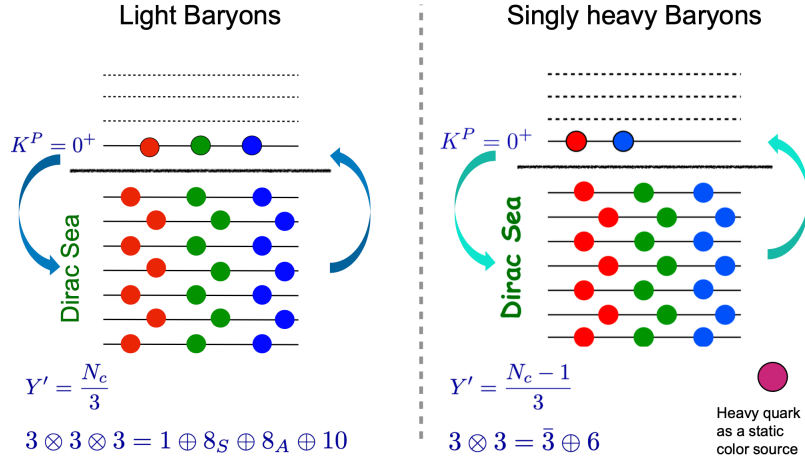


Figure 2: Pion mean-field picture: Light baryons and singly heavy baryons

Figure 2 depicts schematically the mean-field picture of both light and singly heavy baryons. The pion mean field for the singly heavy baryons is created by the presence of the $N_c - 1$ valence quarks, whereas the heavy quark remains as a static color source. Thus, the low-lying singly heavy baryons can be classified as representations of flavor $SU_f(3)$ symmetry. For example, a singly heavy baryon Σ_c^+ can be identified as the state with $J = 1/2$, $T = 1$, $T_3 = 0$, and $Y = 2/3$. The right hypercharge Y' for singly heavy baryons, which will later be explained in detail, is constrained by the number of the valence quarks. As shown in Fig. 2, the right hypercharges for the light and heavy baryons are given as $Y' = N_c/3$ and $Y' = (N_c - 1)/3$, respectively. They allow one to have the lowest-lying representations: the baryon octet (**8**) and decuplet (**10**) for the light baryons, and the baryon antitriplet ($\bar{\mathbf{3}}$), sextet (**6**) and so on [7, 9].

The χ QSM starts with the effective chiral action defined by

$$S_{\text{eff}} := -N_c \text{Tr} \ln [i\partial + iMU^{\gamma_5} + i\hat{m}], \quad (1)$$

where Tr consists of the functional trace over four-dimensional space-time, and the traces over flavor and spin spaces. M is the dynamical quark mass that arises from the spontaneous breakdown of chiral symmetry. The U^{γ_5} represents the chiral field defined by

$$U^{\gamma_5}(z) = \frac{1 - \gamma_5}{2} U(z) + U^\dagger(z) \frac{1 + \gamma_5}{2} \quad (2)$$

with

$$U(z) = \exp[i\pi^a(z)\lambda^a], \quad (3)$$

where $\pi^a(z)$ stands for the pseudo-Nambu-Goldstone (pNG) fields and λ^a are the flavor Gell-Mann matrices. \hat{m} designates the mass matrix of current quarks $\hat{m} = \text{diag}(m_u, m_d, m_s)$. Note that we deal with the strange current quark mass m_s perturbatively. Thus, we will consider it when we make a zero-mode quantization for a collective baryon state.

The one-particle Dirac Hamiltonian is given as

$$H = \gamma_4 \gamma_i \partial_i + \gamma_4 M U^{\gamma^5} + \gamma_4 \bar{m} \mathbf{1}, \quad (4)$$

where \bar{m} is the average value of the current up and down quark masses. Solving the following eigenvalue equation

$$H\psi_n(\mathbf{x}) = E_n\psi_n(\mathbf{x}), \quad (5)$$

we obtain the energy eigenvalues of the single-quark state, E_n , and the corresponding eigenfunctions $\psi_n(\mathbf{x})$. Then, we can compute the baryon correlation function $\langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x) \rangle_0$ as follows [11]:

$$\langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x) \rangle_0 \sim \exp[-\{(N_c - 1)E_{\text{val}} + E_{\text{sea}} + m_Q\}T] = \exp[-M_B T], \quad (6)$$

where the classical mass of a singly heavy baryon is obtained to be

$$M_B = (N_c - 1)E_{\text{val}} + E_{\text{sea}} + m_Q. \quad (7)$$

To assign the quantum numbers to the classical baryon, we need to quantize them. This can be done by considering the zero-mode quantization, which will preserve the mean-field solution [5, 7]. Having carried out the quantization, we arrive at the collective Hamiltonian for singly heavy baryons

$$H = H_{\text{sym}} + H_{\text{sb}}^{(1)}, \quad (8)$$

where H_{sym} is the flavor SU(3) symmetric part

$$H_{\text{sym}} = M_{\text{cl}} + \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i^2 + \frac{1}{2I_2} \sum_{a=4}^7 \hat{J}_a^2. \quad (9)$$

Here, I_1 and I_2 are the moments of inertia of the soliton. The explicit expressions for $I_{1,2}$ are given in Appendix A in Ref. [8]. The operators \hat{J}_i and \hat{J}_a are the spin generators in SU(3). In the (p, q) representation of the SU(3) group, we obtain the eigenvalue of the SU(3) quadratic Casimir operator $\sum_{i=1}^8 J_i^2$ as

$$C_2(p, q) = \frac{1}{3} [p^2 + q^2 + pq + 3(p + q)]. \quad (10)$$

Thus, the eigenvalues of H_{sym} are obtained as

$$E_{\text{sym}}(p, q) = M_{\text{cl}} + \frac{1}{2I_1} J(J + 1) + \frac{1}{2I_2} [C_2(p, q) - J(J + 1)] - \frac{3}{8I_2} Y'^2. \quad (11)$$

The right hypercharge Y' is constrained to be $(N_c - 1)/3$, which is imposed by the $N_c - 1$ valence quarks inside a singly heavy baryon. In the Skyrme model the right hypercharge is constrained by the Wess-Zumino term. The right hypercharge is given by $Y' = 2/3$, so that allowed representations are the baryon antitriplet ($\bar{\mathbf{3}}$), sextet ($\mathbf{6}$) with $J = 1/2$ and $J = 3/2$, antidecapentaplet ($\bar{\mathbf{15}}$) with $J = 1/2$ and $J = 3/2$, and so on.

The pion mean-field solution must be coupled to the heavy quark. Thus, the collective wave functions of the baryons are obtained to be

$$\psi_B^{(\mathcal{R})}(J' J'_3, J; A) = \sum_{m_s = \pm 1/2} C_{J_Q m_3 J J_3}^{J' J'_3} \sqrt{\dim(p, q)} (-1)^{-\frac{\bar{Y}}{2} + J_3} D_{(Y, T, T_3)(\bar{Y}, J, -J_3)}^{(\mathcal{R})*}(A), \quad (12)$$

where

$$\dim(p, q) = (p + 1)(q + 1) \left(1 + \frac{p + q}{2}\right). \quad (13)$$

J and J_3 are the spin and its third component of the light quark degree of freedom, respectively.

The symmetry-breaking part of the collective Hamiltonian is given by

$$H_{\text{sb}}^{(1)} = \frac{\Sigma \pi N}{m_0} \frac{m_s}{3} + \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{f}_i, \quad (14)$$

where

$$\alpha = \left(-\frac{\Sigma \pi N}{3m_0} + \frac{K_2}{I_2} Y\right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right) m_s. \quad (15)$$

We refer to Ref. [8] for the explicit expressions for the moments of inertia and the πN sigma term.

3. Electromagnetic observables of singly heavy baryons

The EM current is written as

$$J_\mu(x) = \bar{\psi}(x) \gamma_\mu \hat{Q} \psi(x) + e_Q \bar{\Psi} \gamma_\mu \Psi, \quad (16)$$

where \hat{Q} denotes the charge operator in $\text{SU}_f(3)$, defined by

$$\hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2} \left(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right). \quad (17)$$

Here, λ_3 and λ_8 are the flavor $\text{SU}(3)$ Gell-Mann matrices. The e_Q in the second part of the EM current in Eq. (16) denotes the heavy-quark charge, which is $e_c = 2/3$ for the charm quark or $e_b = -1/3$ for the bottom quark. Since the magnetic form factor of a heavy quark is proportional to the inverse of the corresponding heavy-quark mass, i.e., $\mu \sim (e_Q/m_Q)\sigma$, the heavy-quark term can be ignored in the limit of $m_Q \rightarrow \infty$. Note that a heavy quark inside a singly heavy baryon only gives a constant contribution to its electric form factor in the $m_Q \rightarrow \infty$ limit.

To compute EM and axial-vector observables of baryons, we have to compute the three-point correlation functions, which are schematically shown in Fig. 3. For a detailed formalism, we refer to Ref. [12, 13].

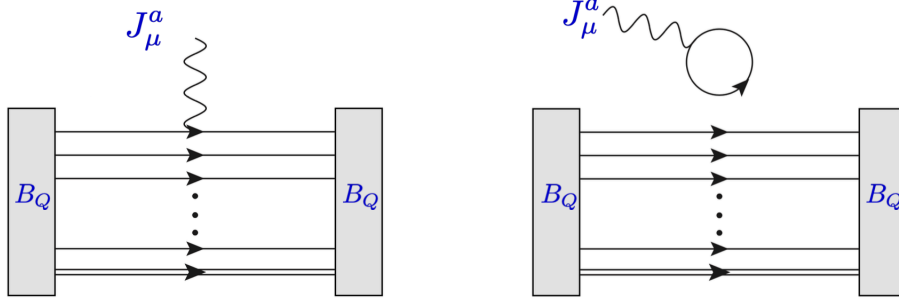
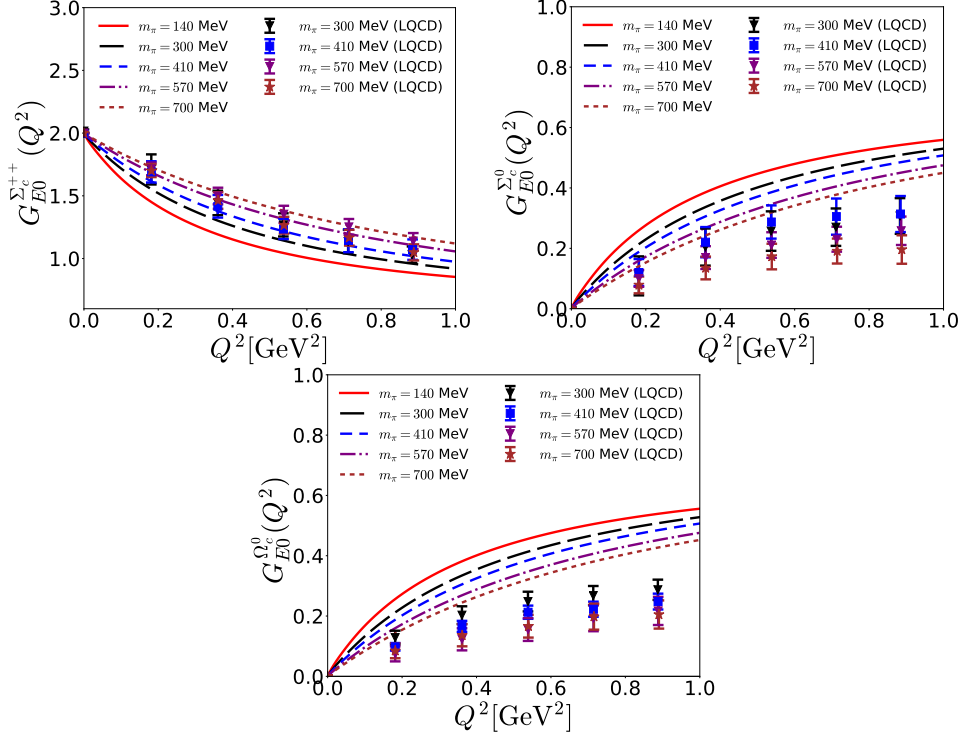
Figure 3: Three-point correlation function in the χ QSM.

Figure 4: Results for the electric form factors of the singly heavy baryons.

While there are no experimental data on the EM form factors of the singly heavy baryons, the results from lattice QCD exist [14, 18]. However, Can et al. [14] employed the pion mass deviated from the physical one, of which the value lies in the range of $300 \leq m_\pi \leq 700$ MeV. Thus, to compare the numerical results with those from lattice data, we need to compute the EM form factors with m_π varied. The results for the electric and magnetic form factors of the singly heavy baryons with spin 1/2 are given in Figs. 4 and 5, respectively.

When we increase the values of the pion mass, the results of the electric form factors fall off more slowly than those with the physical pion mass. Note that this is a well-known behavior of the lattice results. On the other hand, as the pion mass becomes larger, the electric form factors of the neutral heavy baryons increase more slowly. If we increase the pion mass larger than 140 MeV, the Yukawa tail of the pion mean field is suppressed stronger, which indicates that the size of the

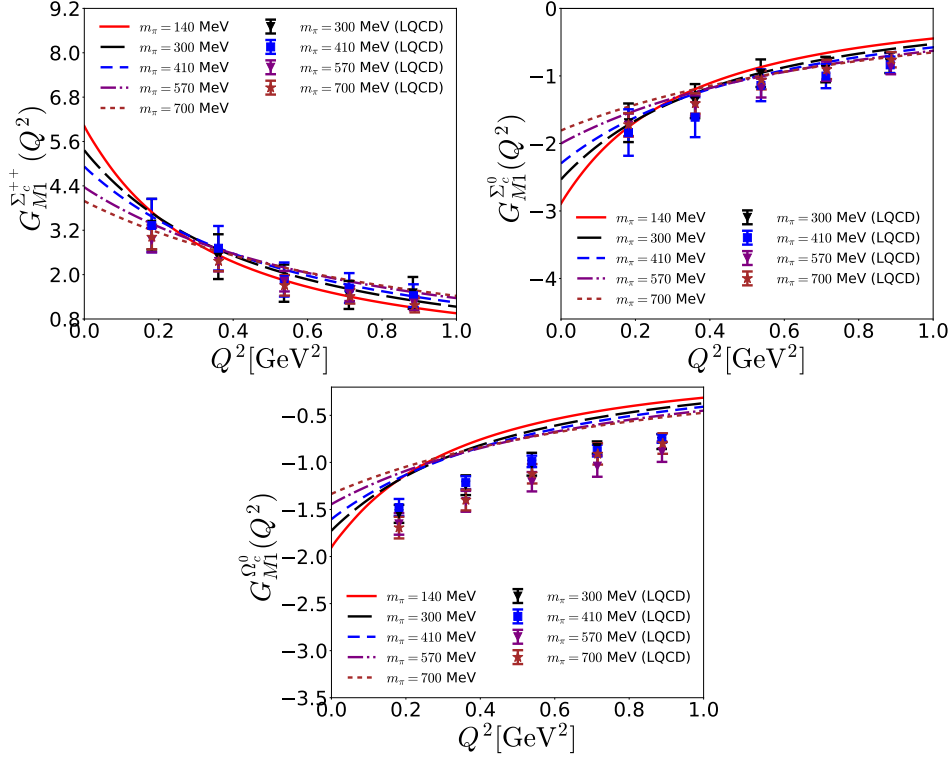


Figure 5: Results for the magnetic form factors of the singly heavy baryons.

baryon gets more compact than the physical one. The dependence of the electric form factors with unphysical pion masses reflect this fact. The numerical results for the Σ_c^{++} electric form factor are in good agreement with the lattice data. Those for Σ_c^0 and Ω_c^0 get closer to the data with the pion mass increased.

To compare the Q^2 dependence of the magnetic form factors, we have normalized the magnitudes of the magnetic form factors at $Q^2 = 0$ to be the same as the lattice ones. Can et al. [14] carried out the chiral extrapolation to the physical mass of the pion and obtained the values of the quadratic fitting: 4.12 for Σ_c^{++} , 3.80 for Σ_c^0 , and 2.71 for Ω_c . We use them for normalization. The current results on the Q^2 dependence of the $M1$ form factors are generally in agreement with the lattice data. We also observe that the lattice results lessen more slowly, compared to the present ones.

We also computed the radiative transitions for the singly heavy baryons with spin 3/2 [16, 17]. We will only show the Ω_c^* transitions in this talk. As shown in Fig. 6, the sea-quark contribution is dominant over that of the valence quarks. As is well known from the $N\gamma \rightarrow \Delta$, the electric quadrupole form factor measures how the Δ is deformed from a spherical shape. This implies that the outer part of the Δ isobar comes into essential play to explain the $E2$ transition. This is a typical feature of the spin-3/2 baryon. On the other hand, the sea-quark effects are marginal in the case of the $M1$ form factor. Though the lattice data on the EM transition form factors of the singly heavy baryons exist [18], they suffer from large uncertainty.

Table 1 lists the present results in the second column in comparison with those from other

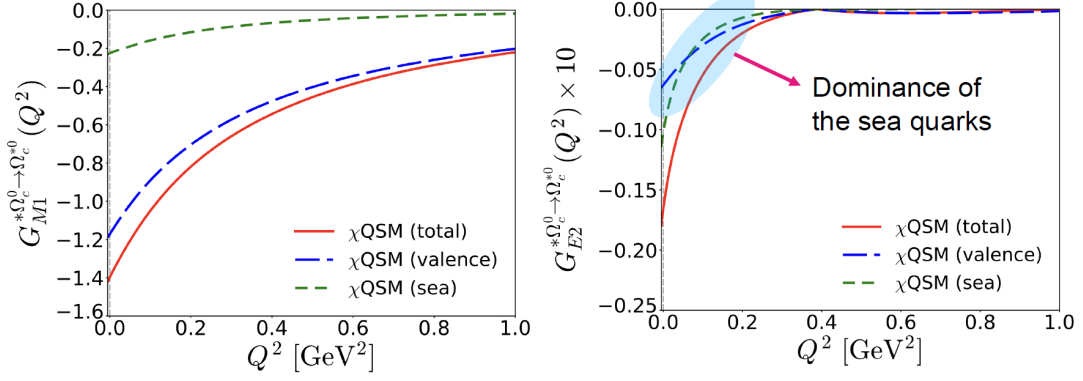


Figure 6: Results for the radiative $M1$ and $E2$ transition form factors for $\Omega_c \gamma \rightarrow \Omega_c^*$.

Table 1: Results for the radiative decay widths of $B\gamma \rightarrow B^*$.

$\Gamma(B_c \gamma \rightarrow B_c^*)$	χ QSM	χ SM [19]	LQCD [20]	Bag [21]	χ PT [22]	QCDSR [23, 24]	QM [25]
$\Lambda_c^+ \gamma \rightarrow \Sigma_c^{*+}$	69.76	191.13 ± 15.15	–	126	161.8	130(45)	151(4)
$\Xi_c^+ \gamma \rightarrow \Xi_c^{*+}$	31.97	55.77 ± 5.22	–	44.3	21.6	52(25)	54(3)
$\Xi_c^0 \gamma \rightarrow \Xi_c^{*0}$	0.08	1.61 ± 0.42	–	0.908	1.84	0.66(32)	0.68(4)
$\Sigma_c^{++} \gamma \rightarrow \Sigma_c^{*++}$	1.08	2.41 ± 0.22	–	0.826	1.20	2.65(1.20)	–
$\Sigma_c^+ \gamma \rightarrow \Sigma_c^{*+}$	0.06	0.11 ± 0.02	–	0.004	0.04	0.40(16)	0.140(4)
$\Sigma_c^0 \gamma \rightarrow \Sigma_c^{*0}$	0.30	0.80 ± 0.06	–	1.08	0.49	0.08(3)	–
$\Xi_c'^+ \gamma \rightarrow \Xi_c^{*+}$	0.09	0.21 ± 0.02	–	0.011	0.07	0.274	–
$\Xi_c'^0 \gamma \rightarrow \Xi_c^{*0}$	0.34	0.64 ± 0.05	–	1.03	0.42	2.142	–
$\Omega_c^0 \gamma \rightarrow \Omega_c^{*0}$	0.34	0.49 ± 0.08	0.074	1.07	0.32	0.932	–

theoretical works. Note that the results from Ref. [19] is based on the same theoretical framework, but all the dynamical parameters were fixed in a *model-independent* way by using the experimental data on magnetic moments of hyperons [27].

4. Axial-vector structure of singly heavy baryons

The strong decay rates for the singly heavy baryons can be derived by considering the following collective operator [26]:

$$O_\varphi = \frac{3}{M_1 + M_2} \sum_{i=1,2,3} \left[G_0 D_{\varphi i}^{(8)} - G_1 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c - G_2 \frac{1}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i \right] p_i, \quad (18)$$

where p_i denotes the momentum of the outgoing meson of mass m . Three coupling constants G_i can be related to the dynamical parameters a_i for the axial-vector operator [27, 28] via the Goldberger-Treiman relation

$$\{G_0, G_1, G_2\} = \frac{M_1 + M_2}{2f_\varphi} \frac{1}{3} \{-a_1, a_2, a_3\}, \quad (19)$$

where F_φ stands for the meson decay constant ($f_\pi = 93$ MeV or $f_K = 112$ MeV). The numerical values of a_i can be determined by using the experimental data on hyperon semileptonic decays [27]:

$$a_1 = -3.509 \pm 0.011, \quad a_2 = 3.437 \pm 0.028, \quad a_3 = 0.604 \pm 0.030. \quad (20)$$

The strong decay rate is expressed by

$$\Gamma_{B_1 \rightarrow B_2 + \varphi} = \frac{1}{2\pi} \overline{\langle B_2 | O_\varphi | B_1 \rangle^2} \frac{M_2}{M_1} p, \quad (21)$$

where $\overline{\langle \dots \rangle}$ denotes the integration over phase space. Thus, we obtain the results for the strong decay rates. Tables 2 and 3 list the results for the strong decay rates of the singly charmed and bottom baryons, respectively, which are in remarkable agreement with the experimental data [29]. We want to emphasize that all the dynamical parameters have already been fixed in the light baryon sector.

Table 2: Decay widths for the charm baryon sextet in MeV.

Decay	χ QSM	Exp.[29]
$\Sigma_c^{++}(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	1.93	$1.89^{+0.09}_{-0.18}$
$\Sigma_c^+(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	2.24	< 4.6
$\Sigma_c^0(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	1.90	$1.83^{+0.11}_{-0.19}$
$\Sigma_c^{++}(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	14.47	$14.78^{+0.30}_{-0.19}$
$\Sigma_c^+(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	15.02	< 17
$\Sigma_c^0(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	14.49	$15.3^{+0.4}_{-0.5}$
$\Xi_c^+(\mathbf{6}_1, 3/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	2.35	2.14 ± 0.19
$\Xi_c^0(\mathbf{6}_1, 3/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	2.53	2.35 ± 0.22

Table 3: Decay widths for the bottom baryon sextet in MeV.

Decay	χ QSM	Exp. [29]
$\Sigma_b^+(\mathbf{6}_1, 1/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	6.12	$9.7^{+4.0}_{-3.0}$
$\Sigma_b^-(\mathbf{6}_1, 1/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	6.12	$4.9^{+3.3}_{-2.4}$
$\Xi_b'(\mathbf{6}_1, 1/2) \rightarrow \Xi_b(\bar{\mathbf{3}}_0, 1/2) + \pi$	0.07	< 0.08
$\Sigma_b^+(\mathbf{6}_1, 3/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	10.96	11.5 ± 2.8
$\Sigma_b^-(\mathbf{6}_1, 3/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	11.77	7.5 ± 2.3
$\Xi_b^0(\mathbf{6}_1, 3/2) \rightarrow \Xi_b(\bar{\mathbf{3}}_0, 1/2) + \pi$	0.80	0.90 ± 0.18
$\Xi_b^-(\mathbf{6}_1, 3/2) \rightarrow \Xi_b(\bar{\mathbf{3}}_0, 1/2) + \pi$	1.28	1.65 ± 0.33

Using the parameters given in Eq. (20), we can also study the quark spin content of the baryon sextet [30]. Since the antitriplet baryons consist of the $J = 0$ pion mean field and a heavy quark, no light-quark contributions exists for them. In this talk, we only present the results for the quark spin content of the baryon sextet with both spin $J' = 1/2$ and $J' = 3/2$. Tables 4 and 5 list the corresponding results, respectively. Compared with the lattice data [31], the results are in agreement with them. For detailed formalism and discussion, we refer to Ref. [30].

Table 4: Quark spin content of the baryon sextet with $J' = 1/2$

$J'_3 = 1/2$	$g_A^{(0)}$	Δu	Δd	Δs	Δc
Σ_c^{++}	0.566	0.991	-0.064	-0.028	-0.333
Σ_c [31]	0.4094 ± 0.0199	0.7055 ± 0.0191	–	–	-0.2970 ± 0.0113
$\Xi_c'^{+}$	0.531	0.505	-0.087	0.447	-0.333
Ξ_c' [31]	0.4872 ± 0.0127	0.3433 ± 0.0085	–	0.4539 ± 0.0055	-0.3133 ± 0.0069
Ω_c^0	0.497	-0.069	-0.069	0.968	-0.333
Ω_c^0 [31]	0.5428 ± 0.0118	–	–	0.8554 ± 0.0117	-0.3125 ± 0.0054

Table 5: Quark spin content of the baryon sextet with $J' = 3/2$

$J'_3 = 3/2$	$g_A^{(0)}$	Δu	Δd	Δs	Δc
Σ_c^{*++}	2.349	1.487	-0.096	-0.042	1.000
Σ_c^* [31]	2.0004 ± 0.0346	1.0899 ± 0.0308	–	–	0.9043 ± 0.0090
Ξ_c^{*+}	2.297	0.889	-0.131	0.670	1.000
Ξ_c^* [31]	2.1192 ± 0.0254	0.5466 ± 0.0150	–	0.6587 ± 0.0104	0.9103 ± 0.0075
Ω_c^{*0}	2.245	-0.104	-0.104	1.452	1.000
Ω_c^{*0} [31]	2.1961 ± 0.0261	–	–	1.2904 ± 0.0204	0.9026 ± 0.0090

5. Summary and conclusions

In the current talk, we first discussed a pion mean-field approach or the chiral quark-soliton model, which describe both the low-lying light baryons and singly heavy baryons on an equal footing. We first showed the results for the electromagnetic form factors of the singly heavy baryons and compared them with the lattice data. Once we introduce large unphysical values of the pion mass, which correspond to those used in lattice QCD, the results are in agreement with the lattice data. We also presented the results for the radiative and strong decay widths for the singly heavy baryons. Finally, we showed the results for the quark spin content of the baryon sextet with both spin $J' = 1/2$ and $J' = 3/2$, which are in qualitative agreement with the lattice data.

We considered the infinitely large mass of the heavy quark in the current talk. we want to mention that to introduce the $1/m_Q$ corrections one has to deal with gluonic degrees of freedom. Very recently, we constructed a new effective theory for the baryon from the instanton vacuum [32], within which we can derive an effective operator for a gluonic operator. We will soon compute various observables for the singly heavy baryons with the $1/m_Q$ corrections.

Acknowledgments

The current work was a summary of a series of the works done together with Y.-S. Jun, J. Y. Kim, M. Oka, J. M. Suh, and G. S. Yang. I want to express my gratitude to them for fruitful collaborations. The present work was supported by the Basic Science Research Program through the National Research Foundation of Korea funded by the Korean government (Ministry of Education, Science and Technology, MEST), Grant-No. 2021R1A2C2093368 and 2018R1A5A1025563.

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