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Heavy hadrons in a chiral-diquark picture

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A chiral effective model including scalar, pseudoscalar, axial-vector, and vector diquarks is reviewed. The resulting diquark mass formulas can describe the relationship between the diquark masses and spontaneous chiral symmetry breaking or the $U(1)_A$ anomaly effect. Based on the two-body picture with one heavy quark and one light diquark for singly heavy baryons, we discuss the baryon spectra, decay widths, and their dependence on chiral symmetry restoration.

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1. Introduction

Diquarks are objects composed of two quarks, and its concept was first suggested in the well-known paper by Gell-Mann in 1964 [1] (see Refs. [2, 3] for reviews). In general, it is relevant in the context of two-quark correlations expected inside baryons (or exotic hadrons having more quarks) or the color superconductivity. In particular, singly heavy baryons may be understood as two-body systems with one heavy (charm or bottom) quark and one light diquark composed of only up, down, and strange quarks, so that its mass spectrum may provide a promising platform to elucidate the property of diquarks (see Refs. [4–6] for early studies). In addition, nowadays, many properties of diquarks, such as their masses and sizes, can be extracted from lattice QCD simulations [7–15]. The matching between such numerical simulations and phenomenological models will be more precise in the future.

A key question here is how the effects of spontaneous chiral symmetry breaking [16, 17] and the $U(1)_A$ anomaly [18–22] contribute to the properties of diquarks, and furthermore, how these contributions are modified in finite-temperature or finite-density environments. These questions are expected to be addressed by future lattice QCD simulations and experiments, but in these proceedings, we focus on an intuitive effective-model approach: we briefly review our recent studies based on a chiral effective Lagrangian of diquarks (see Refs. [23–29] for more details).

2. Model

The scalar diquark is known to be the lightest diquark in many theoretical studies. A chiral effective Lagrangian for the scalar (S, $J^P = 0^+$) and pseudoscalar (P, $J^P = 0^-$) diquarks is [23, 29]

$$
\begin{aligned}
\mathcal{L}_S =& \mathcal{D}_\mu d_{R,i} (\mathcal{D}^\mu d_{R,i})^\dagger + \mathcal{D}_\mu d_{L,i} (\mathcal{D}^\mu d_{L,i})^\dagger \\
&- m_{S0}^2 (d_{R,i} d_{R,i}^\dagger + d_{L,i} d_{L,i}^\dagger) \\
&- \frac{m_{S1}^2}{f_\pi} (d_{R,i} \Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i} \Sigma_{ij} d_{R,j}^\dagger) \\
&- \frac{m_{S2}^2}{2 f_\pi^2} \epsilon_{ijk} \epsilon_{lmn} (d_{R,k} \Sigma_{li} \Sigma_{mj} d_{L,n}^\dagger + d_{L,k} \Sigma_{li}^\dagger \Sigma_{m,j}^\dagger d_{R,n}^\dagger) \\
&+ \frac{\mu_0^2}{f_\pi^2} \left(d_R \{ \Sigma^\dagger \Sigma - \frac{1}{3} \mathrm{Tr}[\Sigma^\dagger \Sigma] \} d_R^\dagger + d_L \{ \Sigma \Sigma^\dagger - \frac{1}{3} \mathrm{Tr}[\Sigma \Sigma^\dagger] \} d_L^\dagger \right). \qquad (1)
\end{aligned}
$$

The details of this Lagrangian are as follows:

- This Lagrangian is invariant under the $SU(3)_R \times SU(3)_L$ chiral transformation.
- $d_{R,i} = \epsilon_{ijk} q_{R,j}^T C q_{R,k}$ and $d_{L,i} = \epsilon_{ijk} q_{L,j}^T C q_{L,k}$ are the right-handed and left-handed diquark fields in the chiral $(\bar{3}, 1)$ and $(1, \bar{3})$ representations, respectively. In the current work, we assumed that these diquarks belong to the color $\bar{3}$ and the flavor $\bar{3}$ representation. For the color $\bar{3}$, we omitted the color index for simplifying the notation. The flavor-$\bar{3}$ diquark fields are represented as a vector labeled by the flavor index $i = 1, 2, 3$ (physically, $i = ds, su, ud$) and composed of the antisymmetric tensor ϵ_{ijk} and the chirality-projected quark fields, $q_{R,i}$ or $q_{L,i}$ labeled by $i = 1, 2, 3$ (physically, $i = u, d, s$).

- This Lagrangian is written in the basis of the right-handed and left-handed diquarks. Because the parity eigenstates are represented as their linear combinations, $S_i = \frac{1}{\sqrt{2}}(d_{R,i} - d_{L,i})$ and $P_i = \frac{1}{\sqrt{2}}(d_{R,i} + d_{L,i})$, it is straightforward to rewrite this Lagrangian as the basis of the scalar and pseudoscalar diquarks [23].

- Since diquarks are not color-singlets, the kinetic term is introduced using a covariant derivative $\mathcal{D}_\mu$. Its explicit form is neglected within our study.

- The term with m_{S0}^2 is the mass term of diquarks, where m_{S0} is called the *chiral invariant mass*. The chiral invariant mass gives a survival mass in the chiral limit after the chiral symmetry breaking is fully restored.

- The term with m_{S1}^2 represents the interaction between the diquarks and one meson field. Σ_{ij} is the meson nonet and belongs to the chiral $(\bar{3}, 3)$ representation. As a characteristic feature, this term violates the $U(1)_A$ symmetry [23] and can be interpreted as an instanton-induced six-point quark interaction or the Kobayashi-Maskawa't Hooft term. f_π is the pion decay constant.

- The term with m_{S2}^2 is the interaction between the diquarks and two mesons, which originates from an eight-point quark interaction. Unlike the m_{S1}^2 term, this term is $U(1)_A$ symmetric.

- The last term with the parameter μ_0 was neglected in Ref. [23] but introduced in Ref. [29], which comes from an eight-point quark interaction. This term is also $U(1)_A$ symmetric.

A chiral effective Lagrangian for the axial-vector (A, $J^P = 1^+$) and vector (V, $J^P = 1^-$) diquarks is [26]

$$
\begin{aligned}
\mathcal{L}_V =& -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F^\dagger_{\mu\nu}] + m_{V0}^2\mathrm{Tr}[d^\mu d^\dagger_\mu] \\
&+ \frac{m_{V1}^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d^{\dagger T}_\mu] \\
&+ \frac{m_{V2}^2}{f_\pi^2}\left\{\mathrm{Tr}[\Sigma^T\Sigma^{\dagger T} d^\dagger_\mu d^\mu] + \mathrm{Tr}[\Sigma\Sigma^\dagger d^\mu d^\dagger_\mu]\right\}, \qquad (2)
\end{aligned}
$$

where $F^{\mu\nu} = \mathcal{D}^\mu d^\nu - \mathcal{D}^\nu d^\mu$ and $d^\mu_{ij} = q^T_{L,i} C\gamma^\mu q_{R,j}$ in the chiral $(3, 3)$ representation. We assumed that these diquarks belong to the color $\bar{3}$ and the flavor 6 for the axial-vector and the flavor $\bar{3}$ for the vector. The parity eigenstates are given as $A^\mu_{ij} = \frac{1}{\sqrt{2}}(d^\mu_{ij} + d^\mu_{ji})$ and $V^\mu_{ij} = \frac{1}{\sqrt{2}}(d^\mu_{ij} - d^\mu_{ji})$. We find that A^μ_{ij} is symmetric under the exchange of $i \leftrightarrow j$. V^μ_{ij} is antisymmetric, and also $V^\mu_{ii} = 0$ (i.e., uu, dd, ss are forbidden).

In addition, an interaction Lagrangian between the scalar and vector diquarks is [28]

$$
\begin{aligned}
\mathcal{L}_{SV} =& g_1\epsilon_{ijk}\left[d^\mu_{ni}(\partial_\mu\Sigma^\dagger)_{jn}d^\dagger_{R,k} + d^\mu_{in}(\partial_\mu\Sigma)_{jn}d^\dagger_{L,k}\right] \\
&+ \frac{g_2}{f_\pi}\epsilon_{ijk}\left[d^\mu_{in}\{\Sigma_{jn}(\partial_\mu\Sigma)_{km} - (\partial_\mu\Sigma)_{jn}\Sigma_{km}\}d^\dagger_{R,m} + d^\mu_{ni}\{\Sigma^\dagger_{jn}(\partial_\mu\Sigma^\dagger)_{km} - (\partial_\mu\Sigma^\dagger)_{jn}\Sigma^\dagger_{km}\}d^\dagger_{L,m}\right], \qquad (3)
\end{aligned}
$$

where g_1 and g_2 are dimensionless coupling constants. The first term keeps the $U(1)_A$ symmetry, while the second term breaks it.

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3. Diquark spectrum

In order to obtain the diquark spectrum from the effective Lagrangian, we introduce a mean-field approximation for the meson field matrix Σ in the flavor space:

$$\langle \Sigma \rangle = f_\pi \mathrm{diag}(1, 1, A), \tag{4}$$

$$A = \frac{f_s}{f_\pi} + \frac{m_s}{g_s f_\pi}, \tag{5}$$

where A is a flavor SU(3) breaking parameter with $f_s = 2f_K - f_\pi$, the bare strange-quark mass m_s, and the quark-meson coupling constant $g_s > 0$. The flavor SU(3) symmetry limit corresponds to $A = 1$. In the realistic situation, $f_K > f_\pi$, leading to $f_s > f_\pi$ and $A > 1$.

By substituting Eq. (4) into the Lagrangian (1), we obtain the mean-field Lagrangian. By diagonalizing the matrix of the terms proportional to m_{S0}^2, m_{S1}^2, m_{S2}^2, and μ_0^2 in the $(d_{R,i}, d_{L,i})$ basis, we can derive the diquark mass formulas. For the scalar diquarks [23, 29],

$$M_{ns}^2(0^+) = m_{S0}^2 + \frac{1}{3}(A^2 - 1)\mu_0^2 - m_{S1}^2 - Am_{S2}^2, \tag{6}$$

$$M_{ud}^2(0^+) = m_{S0}^2 - \frac{2}{3}(A^2 - 1)\mu_0^2 - Am_{S1}^2 - m_{S2}^2, \tag{7}$$

$$M_{ns}^2(0^-) = m_{S0}^2 + \frac{1}{3}(A^2 - 1)\mu_0^2 + m_{S1}^2 + Am_{S2}^2, \tag{8}$$

$$M_{ud}^2(0^-) = m_{S0}^2 - \frac{2}{3}(A^2 - 1)\mu_0^2 + Am_{S1}^2 + m_{S2}^2, \tag{9}$$

where the subscript is $n = u, d$. The formulas excluding the μ_0^2 term were derived in Ref. [23], and those including the μ_0^2 term were in Ref. [29]. In the flavor SU(3) symmetry limit ($A = 1$), we can find that $M_{ns}(0^+) = M_{ud}(0^+)$ and $M_{ns}(0^-) = M_{ud}(0^-)$. In addition, this limit removes the μ_0^2 term, whereas its breaking leads to a μ_0^2 effect.

From the above mass formulas, the difference between the squared masses of the singly-strange and ud diquarks are easily obtained as

$$[M_{ns}(0^+)]^2 - [M_{ud}(0^+)]^2 = (A - 1)(m_{S1}^2 - m_{S2}^2) + (A^2 - 1)\mu_0^2, \tag{10}$$

$$[M_{ns}(0^-)]^2 - [M_{ud}(0^-)]^2 = -(A - 1)(m_{S1}^2 - m_{S2}^2) + (A^2 - 1)\mu_0^2. \tag{11}$$

Here, a naive constraint is $M_{ns}(0^+) > M_{ud}(0^+)$, which can be expected in the charmed-baryon mass ordering of the ground-state Λ_c and Ξ_c. Under this constraint, if ${\mu_0}^2 = 0$, then we can conclude $(A - 1)(m_{S1}^2 - m_{S2}^2) > 0$ and $[M_{ns}(0^-)]^2 - [M_{ud}(0^-)]^2 < 0$: the pseudoscalar ud diquark is heavier than its singly strange partner, $M_{ns}(0^-) < M_{ud}(0^-)$. This ordering is called the *inverse mass hierarchy* [23], which is a characteristic prediction within our model. This behavior is induced by not only the m_{S1}^2 term (including the $U(1)_A$ anomaly effect) but also the m_{S2}^2 term (with no anomaly). On the other hand, if ${\mu_0}^2 \neq 0$, this ordering depends on the parameter choice [29]. In this case, the *normal mass hierarchy*, $M_{ud}(0^-) < M_{ns}(0^-)$, is also possible.

For the axialvector and vector diquarks, by substituting Eq. (4) into the Lagrangian (2), the mass formulas are [26]

$$M_{nn}^2(1^+) = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2, \tag{12}$$
$$M_{ns}^2(1^+) = m_{V0}^2 + A(m_{V1}^2 + 2m_{V2}^2), \tag{13}$$
$$M_{ss}^2(1^+) = m_{V0}^2 + (2A-1)(m_{V1}^2 + 2m_{V2}^2), \tag{14}$$
$$M_{ud}^2(1^-) = m_{V0}^2 - m_{V1}^2 + 2m_{V2}^2, \tag{15}$$
$$M_{ns}^2(1^-) = m_{V0}^2 + A(-m_{V1}^2 + 2m_{V2}^2). \tag{16}$$

Here, we can see that $M_{nn}(1^+)$ and $M_{ud}(1^-)$ do not depend on the SU(3) breaking parameter A: the ud, uu, and dd diquarks do not feel the strange-quark mass effect. Due to this property, unlike the scalar and pseduoscalar diquarks, the axial-vector and vector diquarks *do not* exhibit the conclusive structure of inverse mass hierarchy:

$$[M_{ns}(1^+)]^2 - [M_{nn}(1^+)]^2 = (A-1)(m_{V1}^2 + 2m_{V2}^2), \tag{17}$$
$$[M_{ns}(1^-)]^2 - [M_{ud}(1^-)]^2 = (A-1)(-m_{V1}^2 + 2m_{V2}^2). \tag{18}$$

From a realistic parameter fitting, we can get $m_{V1}^2 < 0$ and $m_{V2}^2 > 0$. Then, recalling $A > 1$, we can find that the mass difference (18) between the singly-strange and ud vector diquarks is relatively enhanced, compared to Eq. (17) for the axial-vector diquarks.

4. Diquark spectrum under chiral symmetry restoration

In the previous section, we discussed the diquark mass formulas at zero temperature and density. Since these formulas connect the diquark masses and the spontaneous chiral symmetry breaking due to the chiral condensates, it is interesting to discuss their behaviors in chiral-symmetry restored environments such as finite temperature and density.

To investigate the masse shifts of diquarks under a chiral symmetry restoration, as a naive estimate, the mean-field ansatz (4) can be replaced by the following form:

$$\langle \Sigma \rangle = \mathrm{diag}(xf_\pi, xf_\pi, xf_s + \frac{m_s}{g_s}), \tag{19}$$

where the parameter $0 \leq x \leq 1$ is a changeable parameter introduced to characterize the strength of spontaneous chiral symmetry breaking. The normal (i.e., chiral-symmetry-broken) vacuum is at $x = 1$, and at $x = 0$ the chiral-symmetry breaking due to the chiral condensates is fully restored. Note that the $\frac{m_s}{g_s}$ term describes the bare strange-quark mass effect and is independent of x.

Under the assumption (19), the mass formulas of the scalar and pseduscocalar ud diquarks are [29]

$$M_{ud}^2(0^+) = m_{S0}^2 - \frac{2}{3}\left\{\left(x\frac{f_s}{f_\pi} + \frac{m_s}{g_s f_\pi}\right)^2 - x^2\right\}\mu_0^2 - \left(x\frac{f_s}{f_\pi} + \frac{m_s}{g_s f_\pi}\right)m_{S1}^2 - x^2 m_{S2}^2, \tag{20}$$

$$M_{ud}^2(0^-) = m_{S0}^2 - \frac{2}{3}\left\{\left(x\frac{f_s}{f_\pi} + \frac{m_s}{g_s f_\pi}\right)^2 - x^2\right\}\mu_0^2 + \left(x\frac{f_s}{f_\pi} + \frac{m_s}{g_s f_\pi}\right)m_{S1}^2 + x^2 m_{S2}^2. \tag{21}$$

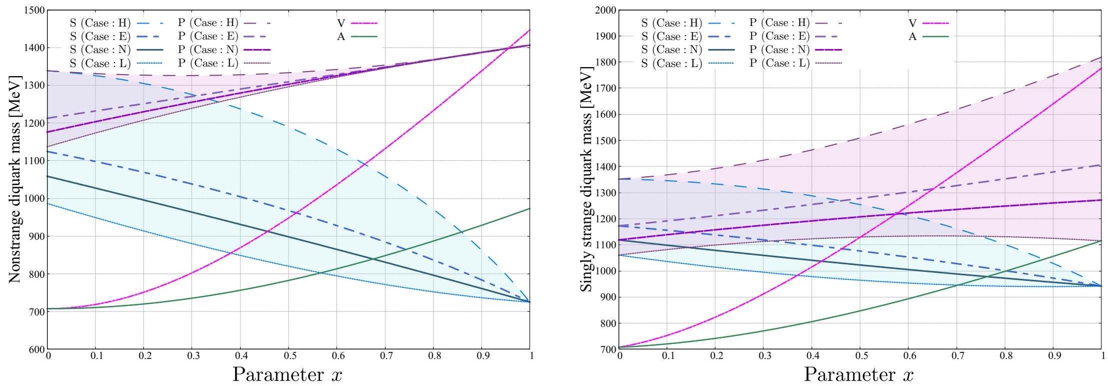

Figure 1: Dependence of diquark masses on the chiral symmetry breaking parameter x [29] (Ref. [26] for older data). Left: the nonstrange (ud, uu, or dd) diquarks. Right: the singly strange (us, ds) diquarks. The shaded regions cover the parameter dependence, where the some parameter choices, Case:L, Case:N, Case:E, and Case:H, are given in Ref. [29].

In these formulas, we can see the roles of each term in the diquark masses. For example, the m_{S1}^2 term can be interpreted as the combination of the chiral condensate x, the strange quark mass m_s, and the anomaly effect m_{S1}. Similarly, the m_{S2}^2 term contains only the contribution from the chiral condensate, and the μ_0^2 term includes the chiral condensate and the strange quark mass. When the chiral condensate vanishes ($x = 0$), the diquark masses are determined by the m_{S0}^2, μ_0^2, and m_{S1}^2 terms, and the m_{S2}^2 vanishes. Then, the mass splitting between the scalar and pseudoscalar diquarks is induced by only the m_{S1}^2 term.

For the axial-vector and vector diquarks [26],

$$M_{nn}^2(1^+) = m_{V0}^2 + x^2(m_{V1}^2 + 2m_{V2}^2), \tag{22}$$

$$M_{ud}^2(1^-) = m_{V0}^2 + x^2(-m_{V1}^2 + 2m_{V2}^2). \tag{23}$$

In this case, at $x = 0$, the diquark masses are determined by only the m_{V0}^2 term. Note that, for the mass formulas for the singly strange diquarks, see Ref. [29].

In Fig. 1, we show the results of the x-dependent diquark masses, using the model parameters determined in Ref. [29]. In this figure, we showed parameter-dependent uncertainties as the shaded regions. The left panel of Fig. 1 shows the results of the nonstrange diquarks. Within our parameters, at $x = 1$, the mass ordering is $M_{ud}(0^+) < M_{nn}(1^+) < M_{ud}(0^-) < M_{ud}(1^-)$. As x decreases, the mass of the scalar diquark increases, while that of the pseudoscalar diquark decreases. This tendency reflects the chiral-partner structure of two diquarks. Eventually, at $x = 0$, the two masses approach each other closely, but they are not degenerate. This survival splitting is caused by the m_{S1}^2 term, particularly the combination of the bare strange quark mass and the $U(1)_A$ anomaly effect [see Eqs. (20) and (21)]. If we neglect this effect, the chiral partners are exactly degenerate at $x = 0$, which corresponds to Case:H in Fig. 1. Also, both the masses of the axial-vector and vector diquarks decrease and finally are degenerate. This degeneracy is also based on the chiral-partner structure of two diquarks.

The right panel of Fig. 1 shows the results of the singly strange diquarks. Because we choose a quite wide parameter range for the pseudoscalar diquark mass at $x = 1$, the mass hierarchy at $x = 1$

is parameter-dependent. However, the x-dependence of the diquark mass is qualitatively robust: the scalar-diquark mass increases, while the pseudoscalar diquark mass decreases, as x decreases. At $x = 0$, all the chiral partners are degenerate, which is different from the case of nonstrange diquarks This is because, for the singly strange diquarks, the m_{S1}^2 term is proportional to x, and the anomaly effect does not affect the diquark masses at $x = 0$.

5. Heavy baryon spectrum

A singly heavy baryon (Qqq, where $Q = c, b$ and $q = u, d, s$) may be regarded as a two-body system consisting of one heavy quark (Q) and one light diquark (qq). In particular, in heavy-light-light three-body systems, their P-wave excited states are classified as the ρ-mode (the excitation between q and q) and the λ-mode (the excitation between Q and qq) [30, 31]. In the singly-heavy-baryon spectrum, the λ-modes of Λ_c, Ξ_c, Λ_b, and Ξ_b are experimentally identified, whereas their ρ-modes are not so far. This is one of the problems in the study of singly-heavy-baryon spectroscopy.

Within the two-body picture, the singly-heavy-baryon spectrum can be calculated by inputting the diquark masses predicted from the chiral effective Lagrangian. In particular, we numerically solved the Schrödinger equation for a nonrelativistic two-body potential model and obtained the baryon spectra [24, 26, 29], where the linear, Coulomb, spin-spin, spin-orbit, and tensor potentials are included.

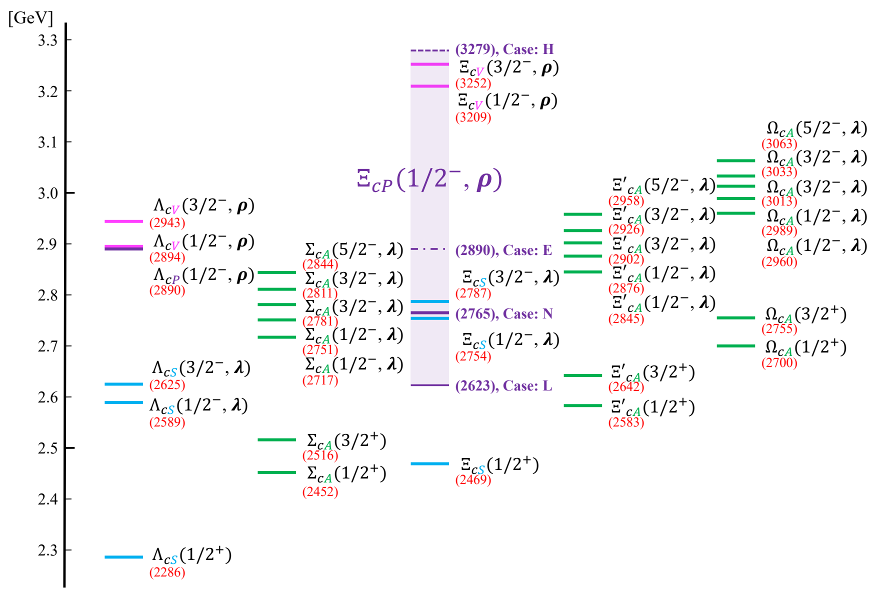

Figure 2: Mass spectra of singly charmed baryons [26, 29]. The colors of lines correspond to the four types of constituent diquarks, S (cyan), P (purple), V (magenta), and A (green). The shaded band covers the parameter dependence in Ref. [29].

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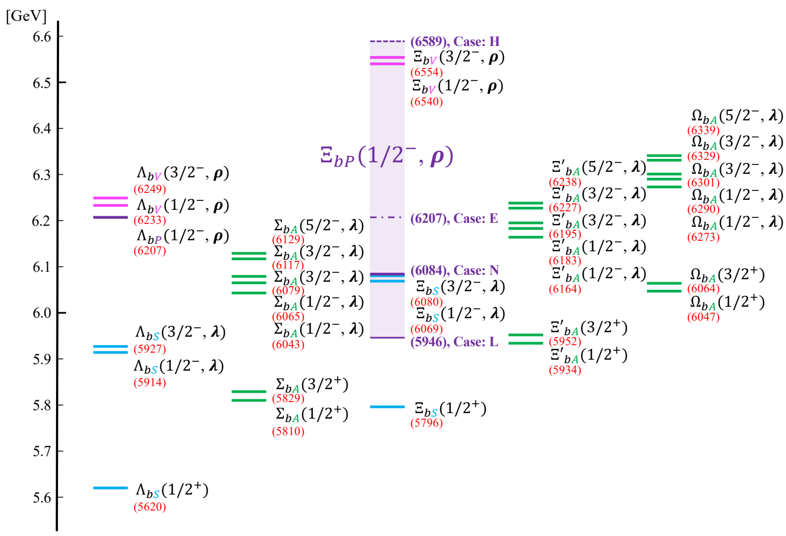

Figure 3: Mass spectra of singly bottom baryons [26, 29]. The notations are the same as those in Fig. 2.

In Fig. 2, we show the mass spectra of singly charmed baryons. Since our parameter dependence comes from the singly strange pseudoscalar diquark, we showed the uncertainty for $\Xi_c(1/2^-, \rho)$ as a colored band. In Case:L and Case:N, we can see that the inverse mass hierarchy for the diquark masses, $M_{ns}(0^-) < M_{ud}(0^-)$, leads to that for the corresponding charmed baryons $\Xi_{cP}(1/2^-, \rho) < \Lambda_{cP}(1/2^-, \rho)$. However, this hierarchy is parameter-dependent, as shown in $\Xi_{cP}(1/2^-, \rho) \sim \Lambda_{cP}(1/2^-, \rho)$ in Case:E, and $\Xi_{cP}(1/2^-, \rho) > \Lambda_{cP}(1/2^-, \rho)$ in Case:H. The hierarchy for the axial-vector and vector diquarks, $M_{nn}(1^+) < M_{ns}(1^+) < M_{ss}(1^+) < M_{ud}(1^-) < M_{ns}(1^-)$, results in $\Xi_{cA}(1/2^+, 3/2^+) < \Xi'_{cA}(1/2^+, 3/2^+) < \Omega_{cA}(1/2^+, 1/3^+) < \Lambda_{cV}(1/2^-, 3/2^-, \rho) < \Xi_{cV}(1/2^-, 3/2^-, \rho)$.

The mass spectra of bottom baryons are shown in Fig. 3. These spectra are similar to the case of charmed baryons. Since the bottom quark is heavier than the charm quark, it is useful to check the behavior of the heavy-quark spin (HQS) symmetry [32]. The HQS doublet structure is seen in $\Lambda_{QS}(1/2^-, 3/2^-, \lambda)$, $\Lambda_{QV}(1/2^-, 3/2^-, \rho)$, $\Xi_{QS}(1/2^-, 3/2^-, \lambda)$, $\Xi_{QV}(1/2^-, 3/2^-, \rho)$, $\Sigma_{QA}(1/2^+, 3/2^+)$, $\Xi'_{QA}(1/2^+, 3/2^+)$, and $\Omega_{QA}(1/2^+, 3/2^+)$, where the heavy-quark limit degenerates these doublets. Also, the five λ modes of Σ_{QA}, Ξ'_{QA}, and Ω_{QA} consists of two HQS doubles and one HQS singlet. By comparing the charmed and bottom baryons, we can see that HQS structure becomes more apparent in the bottom sector.

In Fig. 4, we show the x-dependence of the singly-heavy-baryon masses, which was obtained by inputting the diquark masses in Fig. 1. The chiral partner structures are seen in the pairs of $[\Lambda_Q(1/2^+), \Lambda_{QP}(1/2^-, \rho)]$, $[\Sigma_Q(1/2^+, 3/2^+), \Lambda_{QV}(1/2^-, 3/2^-, \rho)]$, $[\Xi_Q(1/2^+), \Xi_{QP}(1/2^-, \rho)]$, and $[\Xi'_Q(1/2^+, 3/2^+), \Xi_{QV}(1/2^-, 3/2^-, \rho)]$. For example, as x decreases, the mass of $\Lambda_Q(1/2^+)$ increases, while that of $\Lambda_Q(1/2^-, \rho)$ decreases. Both $\Sigma_Q(1/2^+, 3/2^+)$ and $\Lambda_{QV}(1/2^-, 3/2^-, \rho)$ also decreases. As a result, we can find that when x is small enough, the lowest state is $\Sigma_Q(1/2^+)$ [26].

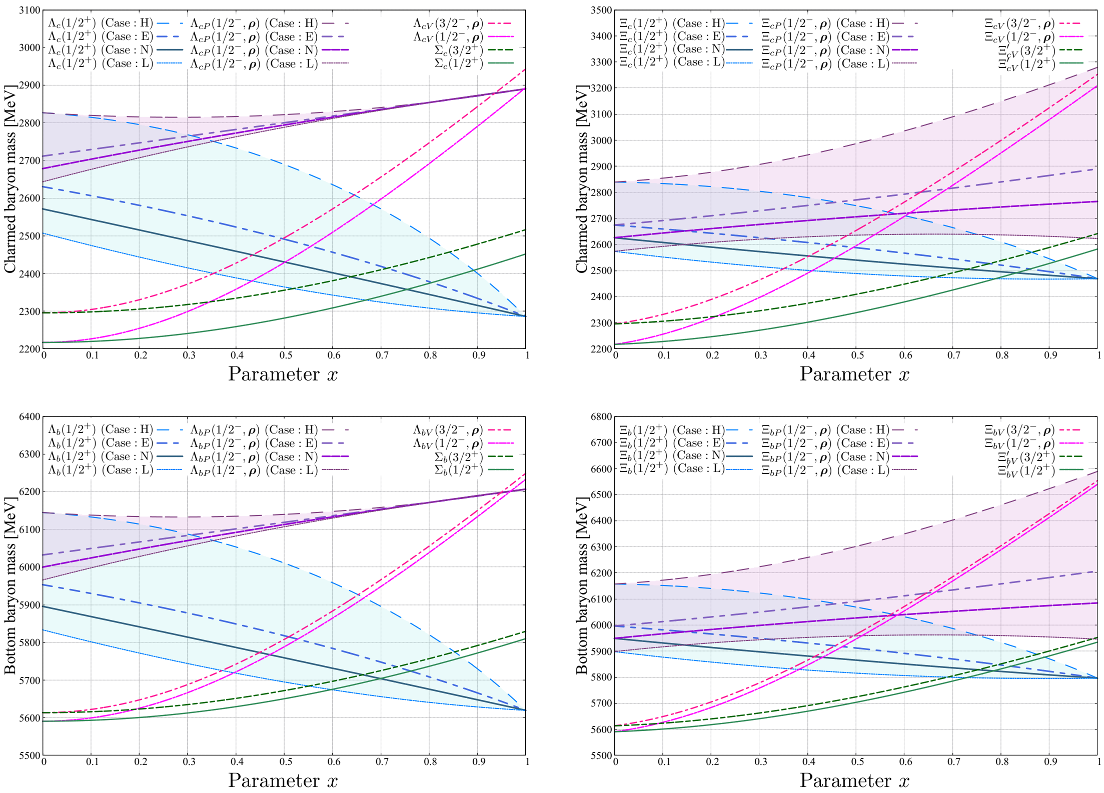

Figure 4: Dependence of singly-heavy-baryon masses on the chiral symmetry breaking parameter x [29]. Top left: Λ_c, Σ_c. Top right: Ξ_c, Ξ'_c. Bottom left: Λ_b, Σ_b. Bottom right: Ξ_b, Ξ'_b. The shaded regions cover the parameter dependence, where the four parameter choices, Case:L, Case:N, Case:E, and Case:H, are given in Ref. [29].

Such a *lowest-state inversion* could be a significant evidence to check our model in experiments or lattice QCD simulations.

6. Decay widths

From the chiral effective Lagrangian, we can predict also the decay properties of singly heavy baryons. For example, in Ref [25], $\Sigma_Q(1/2^-, \rho) \to \Lambda_Q(1/2^+) + \eta$ was calculated from the m^2_{S1} and m^2_{S1} terms of the Lagrangian (1). Also, in Refs. [28, 29], $\Sigma_Q(1/2^+, 3/2^+) \to \Lambda_Q(1/2^+) + \pi$ and $\Xi'_Q(1/2^+, 3/2^+) \to \Xi_Q(1/2^+) + \pi$, were calculated based on the Lagrangian (3).

In Fig. 5, we show the results for the x-dependent decay widths of $\Sigma_Q(1/2^+, 3/2^+) \to \Lambda_Q(1/2^+) + \pi^\pm$. As shown in the previous section, when x is small enough, the mass of $\Lambda_Q(1/2^+)$ increases, and $\Sigma_Q(1/2^+, 3/2^+)$ becomes the lowest states. Due to this behavior, the decay widths of $\Sigma_Q(1/2^+, 3/2^+)$ also decrease and eventually vanish under a threshold. Such suppressed or missing decays could be a signal of the partial restoration of chiral symmetry.

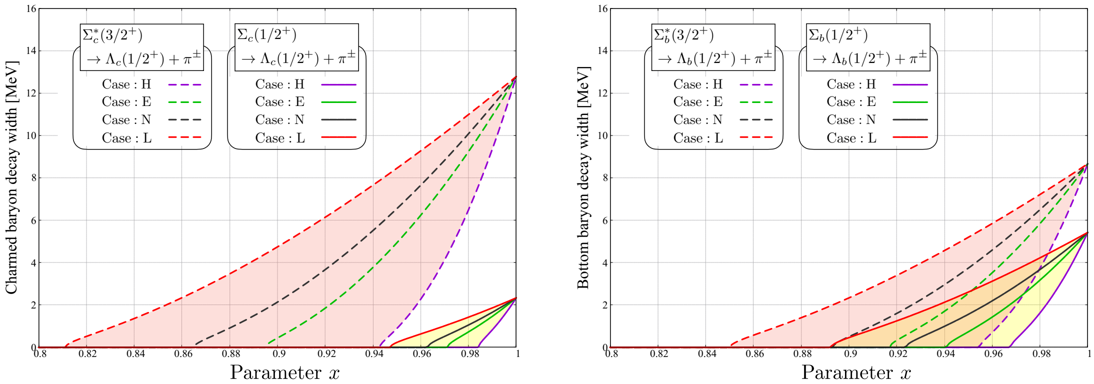

Figure 5: Dependence of decay widths of $\Sigma_Q^{(*)} \to \Lambda_Q + \pi^\pm$ on the chiral symmetry breaking parameter x [29] (Ref. [28] for older data). Left: $\Sigma_c^{(*)} \to \Lambda_c + \pi^\pm$. Right: $\Sigma_b^{(*)} \to \Lambda_b + \pi^\pm$. The shaded regions cover the parameter dependence, where the four parameter choices, Case:L, Case:N, Case:E, and Case:H, are given in Ref. [29].

7. Conclusions

In these proceedings, we reviewed some results predicted from the chiral effective Lagrangian. Here, we summarize some features among our predictions:

(i) *Inverse mass hierarchy*: a singly strange diquark is lighter than its nonstrange partner. This situation holds for the pseudoscalar diquarks within a certain parameter region. In the language of heavy baryons, we can expect the inverse mass hierarchy of $\Xi_{QP}(1/2^-, \rho) < \Lambda_{QP}(1/2^-, \rho)$.

(ii) *Lowest-state inversion*: the ground state of diquarks changes when the chiral symmetry is sufficiently restored. In the language of heavy baryons, we can expect the hierarchy such as $\Sigma_Q(1/2^+, 3/2^+) < \Lambda_Q(1/2^+)$ and $\Xi'_Q(1/2^+, 3/2^+) < \Xi_Q(1/2^+)$.

(iii) *Missing/Suppressed decays*: the decay width to the ground state is suppressed (or eventually missing) when the chiral symmetry is sufficiently restored. In particular, we pointed out the suppression/missing of the decay widths of $\Sigma_Q(1/2^+, 3/2^+) \to \Lambda_Q(1/2^+)\pi^\pm$.

(iv) *Nondegeneracy of chiral partners*: the mass splitting between chiral partners survives even when the chiral symmetry due to the chiral condensate is fully restored. This behavior arises in the chiral partners of the nonstrange scalar and pseudoscalar diquarks, which is caused by the combination of the $U(1)_A$ anomaly and strange-quark mass effect. In the language of heavy baryons, it is seen in the mass difference of the partners $[\Lambda_Q(1/2^+), \Lambda_{QP}(1/2^-, \rho)]$.

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References

[1] M. Gell-Mann, *A schematic model of baryons and mesons*, *Phys. Lett.* **8** (1964) 214.

[2] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson and D.B. Lichtenberg, *Diquarks*, *Rev. Mod. Phys.* **65** (1993) 1199.

[3] R.L. Jaffe, *Exotica*, *Phys. Rep.* **409** (2005) 1.

[4] D.B. Lichtenberg, *Charmed baryons in a quark-diquark model*, *Nuovo Cimento A* **28** (1975) 563.

[5] D.B. Lichtenberg, W. Namgung, E. Predazzi and J.G. Wills, *Baryon Masses in a Relativistic Quark-Diquark Model*, *Phys. Rev. Lett.* **48** (1982) 1653.

[6] D.B. Lichtenberg, W. Namgung, J.G. Wills and E. Predazzi, *Light and heavy hadron masses in a relativistic quark potential model with diquark clustering*, *Z. Phys. C* **19** (1983) 19.

[7] M. Hess, F. Karsch, E. Laermann and I. Wetzorke, *Diquark masses from lattice QCD*, *Phys. Rev. D* **58** (1998) 111502 [hep-lat/9804023].

[8] K. Orginos, *Diquark properties from lattice QCD*, *PoS* **LAT2005** (2006) 054 [hep-lat/0510082].

[9] C. Alexandrou, P. de Forcrand and B. Lucini, *Evidence for Diquarks in Lattice QCD*, *Phys. Rev. Lett.* **97** (2006) 222002 [hep-lat/0609004].

[10] R. Babich, N. Garron, C. Hoelbling, J. Howard, L. Lellouch and C. Rebbi, *Diquark correlations in baryons on the lattice with overlap quarks*, *Phys. Rev. D* **76** (2007) 074021 [hep-lat/0701023].

[11] T. DeGrand, Z. Liu and S. Schaefer, *Diquark effects in light baryon correlators from lattice QCD*, *Phys. Rev. D* **77** (2008) 034505 [0712.0254].

[12] J. Green, J. Negele, M. Engelhardt and P. Varilly, *Spatial diquark correlations in a hadron*, *PoS* **LATTICE2010** (2010) 140 [1012.2353].

[13] Y. Bi, H. Cai, Y. Chen, M. Gong, Z. Liu, H.-X. Qiao et al., *Diquark mass differences from unquenched lattice QCD*, *Chin. Phys. C* **40** (2016) 073106 [1510.07354].

[14] A. Francis, P. de Forcrand, R. Lewis and K. Maltman, *Diquark properties from full QCD lattice simulations*, *JHEP* **05** (2022) 062 [2106.09080].

[15] K. Watanabe, *Quark-diquark potential and diquark mass from lattice QCD*, *Phys. Rev. D* **105** (2022) 074510 [2111.15167].

[16] Y. Nambu and G. Jona-Lasinio, *Dynamical model of elementary particles based on an analogy with superconductivity. I.*, *Phys. Rev.* **122** (1961) 345.

[17] Y. Nambu and G. Jona-Lasinio, *Dynamical model of elementary particles based on an analogy with superconductivity. II*, Phys. Rev. **124** (1961) 246.

[18] J.S. Bell and R. Jackiw, *A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ model*, Nuovo Cimento A **60** (1969) 47.

[19] S.L. Adler, *Axial vector vertex in spinor electrodynamics*, Phys. Rev. **177** (1969) 2426.

[20] G. 't Hooft, *Symmetry Breaking Through Bell-Jackiw Anomalies*, Phys. Rev. Lett. **37** (1976) 8.

[21] G. 't Hooft, *Computation of the quantum effects due to a four-dimensional pseudoparticle*, Phys. Rev. D **14** (1976) 3432.

[22] G. 't Hooft, *How Instantons Solve the $U(1)$ Problem*, Phys. Rep. **142** (1986) 357.

[23] M. Harada, Y.-R. Liu, M. Oka and K. Suzuki, *Chiral effective theory of diquarks and the $U_A(1)$ anomaly*, Phys. Rev. D **101** (2020) 054038 [1912.09659].

[24] Y. Kim, E. Hiyama, M. Oka and K. Suzuki, *Spectrum of singly heavy baryons from a chiral effective theory of diquarks*, Phys. Rev. D **102** (2020) 014004 [2003.03525].

[25] Y. Kawakami, M. Harada, M. Oka and K. Suzuki, *Suppression of decay widths in singly heavy baryons induced by the $U_A(1)$ anomaly*, Phys. Rev. D **102** (2020) 114004 [2009.06243].

[26] Y. Kim, Y.-R. Liu, M. Oka and K. Suzuki, *Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*, Phys. Rev. D **104** (2021) 054012 [2105.09087].

[27] Y. Kim, M. Oka and K. Suzuki, *Doubly heavy tetraquarks in a chiral-diquark picture*, Phys. Rev. D **105** (2022) 074021 [2202.06520].

[28] Y. Kim, M. Oka, D. Suenaga and K. Suzuki, *Strong decays of singly heavy baryons from a chiral effective theory of diquarks*, Phys. Rev. D **107** (2023) 074015 [2212.08338].

[29] Y. Kim, M. Oka and K. Suzuki, *Chiral effective theory of scalar and vector diquarks revisited*, Phys. Rev. D **111** (2025) 034014 [2411.17803].

[30] L.A. Copley, N. Isgur and G. Karl, *Charmed baryons in a quark model with hyperfine interactions*, Phys. Rev. D **20** (1979) 768.

[31] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka and K. Sadato, *Spectrum of heavy baryons in the quark model*, Phys. Rev. D **92** (2015) 114029 [1510.01067].

[32] N. Isgur and M.B. Wise, *Weak decays of heavy mesons in the static quark approximation*, Phys. Lett. B **232** (1989) 113.