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Heavy-Strange Mesons from Lattice QCD and HEFT

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In this study, we investigate the four P-wave D_s states mesons within a Hamiltonian effective field theory framework, where both the $c\bar{s}$ and $D^{(*)}K$ components, as well as their interactions, are taken into account. The coupling between the $c\bar{s}$ and $D^{(*)}K$ is modeled via the quark-antiquark pair creation mechanism, while the interaction within $D^{(*)}K$ is described using the one-boson exchange model. The resulting coupled-channel system is discretized in a finite volume, and the model parameters are constrained by fitting to lattice QCD energy levels. Our analysis shows that the lower states, $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ are strong mixtures of $c\bar{s}$ and $D^{(*)}K$ components, while the higher states, $D^*_{s1}(2536)$ and $D^*_{s2}(2573)$ are predominantly conventional P-wave $c\bar{s}$ meson.

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1. Introduction

Since 2003, a large number of new hadronic states have been observed that cannot be directly explained by the conventional quark model, where the mesons are described as $q\bar{q}$ and baryons as qqq. By analyzing their properties, such as the masses, decay widths, electric charges, the J^{PC} quantum numbers, the decay channels and production mechanisms, many of these states are good candidates for the multi-quark states, such as tetraquarks $qq\bar{q}\bar{q}$ or pentaquarks $q\bar{q}qqq$. Among these exotic candidates, a special class of states stands out: those with spin-parity J^{PC} quantum numbers allowed in the quark model, but with masses and decay widths that deviate from its predictions. In such cases, the creation of light quark-antiquark pair from the vacuum lead to a natural and strong coupled channel effect between the excited heavy system $q'\bar{q}'$ and the nearby hadronic channels $q'\bar{q}-q\bar{q}'$ states. This coupled channel effects are especially important and may significantly modify the mass spectrum predicted by the conventional quark model.

A representative example is $D^*_{s0}(2317)$ as shown in Fig 1. For a P-wave $c\bar{s}$ state, the quark model predicts four states with the spin-parity $J^P = 1^+$ ($S_{c\bar{c}} = 0$), $J^P = 0^+, 1^+, 2^+$ ($S_{c\bar{c}} = 1$). Experimentally, four D_s states with the same J^P quantum numbers, namely $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, $D^*_{S1}(2536)$, $D^*_{s2}(2573)$, have been observed and naturally may be the candidates for the four P-wave states. However, while the higher states $D^*_{S1}(2536)$and $D^*_{s2}(2573)$ are consistent with the quark model predictions, the lower states $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ appear significantly lighter than expected. Given their proximity to the DK and D^*K thresholds, the coupled channel effect may be critical in explaining their anomalous masses.

Many theoretical approaches have been proposed to explain the nature of the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, including quenched and unquenched $c\bar{s}$ quark models [1–6], hadronic molecular models [7–23], tetraquark models [24–28], and hybrid configurations including $c\bar{s}$ and tetraquark model [29–32] (see review articles [33–36] for more details). Nevertheless, their inner structures are still elusive and the debating continues.

Given that the quark model works well for most D_s states, particularly for the two higher P-wave states, we expect that P-wave $c\bar{s}$ state should also contribute to the lower $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, but strongly coupled to nearby $D^{(*)}K$ channels. To study this scenario quantitatively, we construct a coupled-channel framework that incorporates both $c\bar{s}$ and the relevant $D^{(*)}K$ to study the four D-wave channels [37]. A major obstacle is the absence of experimental data for the scattering amplitude of the $D^{(*)}K \to D^{(*)}K$ process. To overcome this, we use lattice QCD energy levels in a finite volume as a substitute. To connect these finite-volume spectra to infinite-volume observables, we employ Hamiltonian Effective Field Theory (HEFT) [38–41], which allows us to discretize the Hamiltonian appropriately for comparison with lattice data. Once the parameters are determined by fitting to these levels, we use them to construct the scattering T-matrix which can then be analytically continued into the complex energy plane to extract pole positions.

2. The coupled-channel framework

In this work, we construct a coupled-channel framework that incorporates both the bare charmonium state $c\bar{s}$ and the two-meson state $D^{(*)}K$

$$H = H_0 + H_I, \tag{1}$$

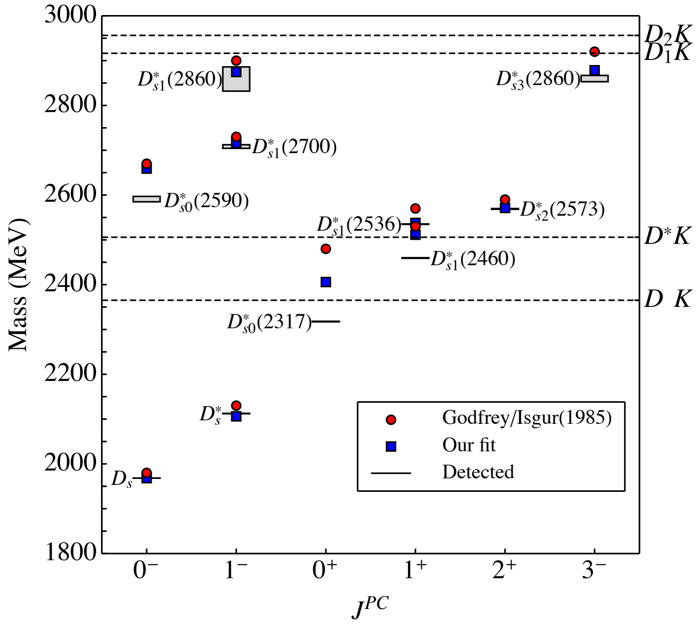

Figure 1: Comparison of the mass spectrum of bare $c\bar{s}$ mesons as predicted by the relativized quark model (GI) [1] and the experimental results. We have refitted the parameters of the original GI model using the latest experimental data. The original and refitted predictions are denoted by the circles and squares, respectively. The shaded areas correspond to the experimental masses and their uncertainties [42, 43].

where H_0 and H_I are the free and interacting Hamiltonian, respectively.

The H_0 is given by

$$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|. \tag{2}$$

Here, $|B\rangle$ denotes the $c\bar{s}$ state, and $|\alpha(\vec{k})\rangle$ represents the relative wavefunction in the two-meson channel. The m_B and $E_\alpha(\vec{k})$ are the $c\bar{s}$ mass and kinematic energy of the $D^{(*)}K$ channels, which are given by the well-established quark model [1].

The interaction Hamiltonian consists of two parts:

$$H_I = g + v, \tag{3}$$

where g denotes the coupling strength between the bare $\bar{c}s$ and $D^{(*)}K$ channels, while v represents the direct interaction within the two-meson sector. The coupling g is described using the $^3P_0$ model and the corresponding transition amplitude is given by [44, 45]

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}, \tag{4}$$

where γ and Λ are model parameters to be fitted, and $I_{\alpha B}$ is the overlap integral. The potentials between the two-meson channels $D^{(*)}K - D^{(*)}K$ are described using a one-boson-exchange (OBE) model. The interaction Lagrangian is

$$\mathcal{L} = \mathcal{L}_{PPV} + \mathcal{L}_{VVV} = i g_v \,\mathrm{Tr}\left(\partial^\mu P \left[P, V_\mu\right]\right) + i g_v \,\mathrm{Tr}\left(\partial^\mu V^\nu \left[V_\mu, V_\nu\right]\right), \tag{5}$$

We also introduce a phenomenological form factor to ensure renormalizability

$$v \rightarrow v \left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2, \tag{6}$$

where p_i and p_f are the initial and final momenta, respectively. The dependence of the cut-off parameter Λ can be absorbed into the redefinition of the coupling constants.

we construct the full coupled-channel Hamiltonian and discretize it within the Hamiltonian Effective Field Theory (HEFT) formalism. In a finite cubic box of size L, the allowed momenta are quantized as $k_n = 2\pi\sqrt{n}/L$ with $n = n_x^2 + n_y^2 + n_z^2$, $n = 0, 1, 2, \ldots$. Correspondingly, the discretized Hamiltonian reads [38–41]

$$H_0 = \sum_{i=1,n} |B_i\rangle\, m_i\, \langle B_i| + \sum_{\alpha,\mathrm{i}} \left|\vec{k}_\mathrm{i}, -\vec{k}_\mathrm{i}\right\rangle_\alpha \left[\sqrt{m_{\alpha B}^2 + k_\alpha^2} + \sqrt{m_{\alpha M}^2 + k_\alpha^2}\right]_\alpha \left\langle \vec{k}_\mathrm{i}, -\vec{k}_\mathrm{i}\right| \tag{7}$$

$$\begin{aligned} H_I = & \sum_{\mathrm{j}} (2\pi/\mathrm{L})^{3/2} \sum_\alpha \sum_{i=1,n} \left[\left|\vec{k}_\mathrm{j}, -\vec{k}_\mathrm{j}\right\rangle_\alpha g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle\, g_{i,\alpha} \left\langle \vec{k}_\mathrm{j}, -\vec{k}_\mathrm{j}\right|\right] \\ & + \sum_{\mathrm{ij}} (2\pi/\mathrm{L})^3 \sum_{\alpha\beta} \left|\vec{k}_\mathrm{i}, -\vec{k}_\mathrm{i}\right\rangle_\alpha v_{\alpha,\beta\beta} \left\langle \vec{k}_\mathrm{j}, -\vec{k}_\mathrm{j}\right|. \end{aligned} \tag{8}$$

The finite-volume Schrödinger equation is then given by

$$(H_0 + H_\mathrm{I})\,|\Psi\rangle = E|\Psi\rangle, \tag{9}$$

where the eigenvalues E correspond to the discrete energy levels obtained from lattice QCD simulations. By fitting the lattice spectra in the 0^+ and 1^+ sectors, all the model parameters can be determined. The fitting results are shown in Fig. 2.

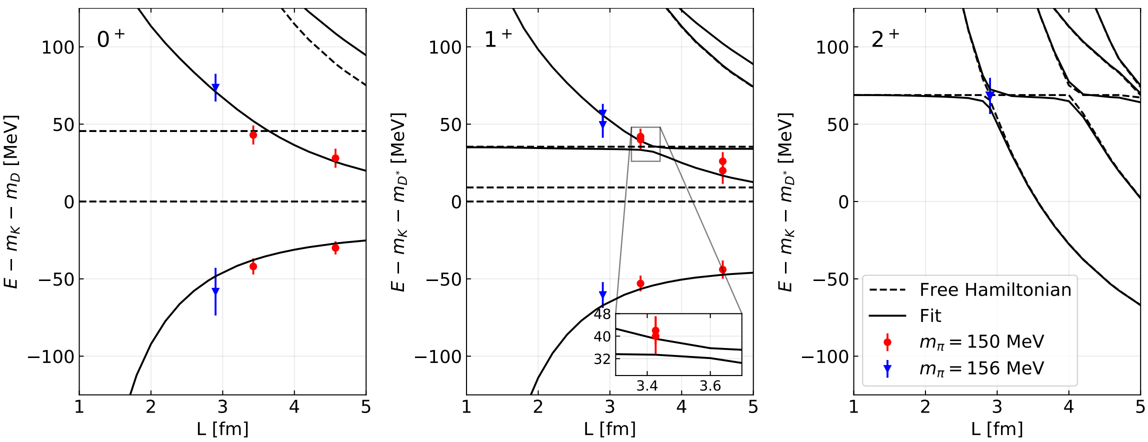

Figure 2: The fitted binding energy with the length L for the $D_{s0}^*(2317)$ (left), the $D_{s1}^*(2460/2536)$ (middle) states with the pion mass $m_\pi = 150$ MeV [46] and $m_\pi = 156$ MeV [47]. The comparison of the lattice binding energies and our predicted ones for $D_{s2}^*(2573)$ is shown in the right panel. The black curves are the results using finite-volume Hamiltonian H, while the dashed lines represent the $c\bar{s}$ masses and $D^{(*)}K$ thresholds obtained with the free Hamiltonian H_0.

Table 1: The pole masses (MeV) of D_s states (ours) with the coupled-channel effect compared with the experimental results (exp), along with the corresponding contents of different components at $L = 4.57$ fm.

	$P(c\bar{s})$ [%]	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

3. Results and discussions

In $J^P = 0^+$ and 1^+ sectors, three D_s states have been observed: $D_{s0}^*(2317)$, $D_{s1}^*(2460)$ and $D_{s1}^*(2536)$. Their pole masses and the internal compositions obtained by our coupled-channel analysis, are summarized in Table 1. The predicted masses are very well consistent with the experimental data.

The $D_{s0}^*(2317)$ with $J^P = 0^+$ is interpreted as a strong mixture of a bare $c\bar{s}$ and an S-wave DK component. While our predicted mass is slightly higher than the experimental one, it is consistent with Lattice QCD simulations at the physical m_π, which also predicts a higher mass compared to experiment. We also studied the composition of the $D_{s0}^*(2317)$. For large enough box sizes (e.g., $L = 4.57$ fm), the component fractions become stable and can be considered as good approximations to the infinite-volume limit. At $L = 4.57$ fm, we find the DK component dominates, accounting for approximately 68% of the wavefunction.

The m_π dependence of the $D_{s0}^*(2317)$ provides further insight into its structure. As m_π increases, the $c\bar{s}$ mass remains stable [46], whereas the DK threshold rises due to increasing light quark mass [18]. Consequently, the physical mass of the $D_{s0}^*(2317)$, being a mixture of the two components, initially increases with the threshold. However, once the DK threshold becomes much larger than the bare $c\bar{s}$ mass, the DK component decouples, and the $D_{s0}^*(2317)$ becomes dominated by the $c\bar{s}$ core, with its mass stabilizing. The lattice QCD has been observed the qualitative behavior in various lattice QCD simulations [46, 48], and future studies of m_π dependence are expected in the future.

In the $J^P = 1^+$ sector, the quark model predicts two $c\bar{s}$ configurations: ${}^1P_1$ and ${}^3P_1$, which mix with through tensor interactions. Theses can also be expressed in the heavy quark spin base: $\left|\frac{1}{2}_l \otimes \frac{1}{2}_H\right\rangle_1$ and $\left|\frac{3}{2}_l \otimes \frac{1}{2}_H\right\rangle_1$. One finds that the lower- and higher- $c\bar{s}$ closely resemble these heavy-quark symmetry eigenstates. The lower state, $D_{s1}^*(2460)$, couples predominantly to the S-wave D^*K channel. This strong coupling induces the large mass shift and a substantial D^*K component. In contrast, the higher state, $D_{s1}^*(2536)$, couple primarily to the D-wave D^*K, where interactions are expected to be kinematically suppressed. Thus, the coupled-channel effect is negligible, and this state is almost a pure $c\bar{s}$. In summary, the $D_{s1}^*(2460)$ is a strongly mixed state composed of $c\bar{s}$ and D^*K components, whereas the $D_{s2}^*(2573)$ is mainly a pure $c\bar{s}$ state.

As a prediction, we calculate the energy levels of $D_{s2}^*(2573)$ with $J^P = 2^+$, as shown in Fig. 2. Our results show that the lattice QCD result is located on our energy levels. In the quark model, this state corresponds to a single $c\bar{s}$ and coupes with D-wave DK and D^*K channels.The coupled-channel effect is expected to be suppressed. Our result confirms this expectation and find

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that the $D^*_{s2}(2573)$ is dominated by the $c\bar{s}$ component.

4. Summary

In this work, we have constructed a coupled-channel framework to study the four P-wave D_s states: $D^*_{s0}(2317)$, $D^*_{s1}(2460)$, $D^*_{s1}(2536)$, and $D^*_{s2}(2573)$. Both the bare $c\bar{s}$ and the nearby two-meson channels $D^{(*)}K$, along with their possible interactions have been considered. To connect our model with lattice QCD results, we employ Hamiltonian Effective Field Theory (HEFT), which enables a finite-volume Hamiltonian direct comparison with lattice spectra.

Our results show that the lower two states, $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, couple strongly to the S-wave $D^{(*)}K$ channels. The strong coupling induces significant mixing between the $D^{(*)}K$ and $c\bar{s}$ components, resulting in the notably lower physical masses compared to the quark model predictions. In contrast, the two higher two states, $D^*_{s1}(2536)$ and $D^*_{s2}(2573)$, couples mainly to the D-wave $D^{(*)}K$ channels. The coupling is small and the two states remain predominantly pure $c\bar{s}$, with the masses in good agreement with the quark model predictions.

This framework provides a consistent description for the masses and inner structures of the P-wave D_s states. It can also be extended to study the other near-threshold exotic states, such as the B_s [49], the $X(3872)$ [50] and so on.

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