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Narrow Spectra in Heavy Quark Systems

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Opportunities to observe new hadrons with very narrow widths are discussed. Two areas of focus are the lowest P-states of heavy-light mesons and doubly heavy baryons.

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1. Overview

In the recent years many heavy quark states with very narrow widths (≤ MeV) have been observed. The properties of such states have given detailed insight into the dynamics of QCD. Such narrow states can't be far about the threshold for Zweig allowed strong decays. We can classify the opportunities for such narrow states by their heavy quark content as follows:

- $c\bar{c}$, $b\bar{b}$, $b\bar{c}$, ccc, ccb, cbb and bbb systems. Very narrow states are possible below the respective thresholds the Zweig allowed decays (e.g, $D\bar{D}$ in the $c\bar{c}$ system, etc). Some resonance states very near threshold might be narrow. Many such states have already been observed [1] and the predictions for the rest are well established.

- Heavy-light mesons ($\bar{c}q, \bar{b}q$) q = u,d,s. Generally strong decays above ground states and possibly some of the lowest P-states.

- Doubly heavy baryons : $ccq, bbq, \{bc\}q$ and $[bc]q$. For unequal mass heavy quarks, two set of states exist depending on whether the heavy quarks are in a symmetric ({}) or an antisymmetric ([]) spatial configuration. Can any excited states be narrow?

- Tetraquark systems with heavy quarks (c or b) plus light degrees of freedom. Here surprisingly some ground states are actually stable against strong decays[2].

- Tetraquark systems with all heavy quarks. Near threshold wide $c\bar{c}c\bar{c}$ resonance states have been observed at the LHC[3]. There might be related narrow tetraquark states.

How many more such states remain to be found? In the remaining sections the prospects for discovery are examined.

2. Heavy Light Mesons

Consider a system consisting of both heavy and light quarks. The total angular momentum of the light subsystem $\mathbf{j_l}$ is just the sum of the total orbital angular momentum, $\mathbf{l}$, and the total spin, $\mathbf{s}$ of that subsystem, ie ($\mathbf{j_l} = \mathbf{l} + \mathbf{s}$). Similarly, the total angular momentum of the heavy subsystem is just $\mathbf{J_h} = \mathbf{L_h} + \mathbf{S_h}$. Thus the total angular momentum is $\mathbf{J} = \mathbf{j_l} + \mathbf{J_h}$. In the case of heavy-light mesons considered in this section, the heavy quark is nearly static, so $\mathbf{J_h} = \mathbf{S_h}$.

The two systems of heavy-light mesons observed to date are the $D_{u,d}$, D_s and $B_{u,d}$, B_s mesons. All the ground states ($L = 0$) $J^P = 0^-, 1^-$ are stable against strong decays and are very narrow. All the excited states above the lowest P states have strong decays.

The lowest P states ($L = 1$) have $\mathbf{j_l} = \frac{1}{2}$ or $\frac{3}{2}$ and positive parity. The two $\mathbf{j_1} = \frac{1}{2}$ states are $\mathbf{J^P} = \mathbf{0}^+, \mathbf{1}^+$ and the $\mathbf{j_l} = \frac{3}{2}$ states are $\mathbf{J^P} = \mathbf{1}^+, \mathbf{2}^+$. These P states are a special case because strong decay to the associated two ground states $\mathbf{j_1} = \frac{1}{2}$ with $\mathbf{J^P} = \mathbf{0}^-, \mathbf{1}^-$ S waves might be kinematically forbiden.

2.1 D mesons

The mass splittings in the D_s system between the $\mathbf{j_l} = \frac{1}{2}$ and $\mathbf{j_l} = \frac{3}{2}$ P states was very surprising when the $\mathbf{j_l} = \frac{1}{2}$ states were first observed experimentally. These $\mathbf{j_l} = \frac{1}{2}$ masses violated the expectations of simple potential models. Various theoretical approaches were proposed to understand this unexpected behaviour. These approaches generally followed the directions: (1) Treat the ground $\mathbf{j_l} = \frac{1}{2}(\mathbf{P} = \mathbf{-1})$ S states and $\mathbf{j_l} = \frac{1}{2}(\mathbf{P} = \mathbf{+1})$ P states as a chiral multiplet (e.g. [4]), (2) Treat the $\mathbf{j_l} = \frac{1}{2}(\mathbf{P} = \mathbf{+1})$ P states as bound states in KD and KD^* scattering (e.g. [5–7]), or (3) Consider these states as a mixture with both a quarkonium core and molecular component. For a recent effort in this dirrection see e.g. [8]).

For $D_{u,d}$ states, the $\mathbf{j_l} = \frac{1}{2}$ P states have total widths [1] $\Gamma[D_0^*(2343)] = 229 \pm 16$ MeV and $\Gamma[D_1(2412))] = 314 \pm 29$ MeV and have S wave pion transitions and $\Gamma[D_1(2422)] = 31.3 \pm 1.9$ MeV and $\Gamma[D_1(2412)] = 314 \pm 29$ MeV and $\Gamma[D_2^*(2461)] = 47.3 \pm 0.8$ MeV for the $\mathbf{j_l} = \frac{3}{2}$ states which have D wave transitions. For D_s state, $\Gamma[D_{s1}(2535] = 0.92 \pm 0.05$ MeV and $\Gamma[D_{s2}^*(2569)] = 16.0 \pm 0.7$ MeV for the $\mathbf{j_l} = \frac{3}{2}$ states which have D wave kaon transitions. The $\mathbf{j_l} = \frac{1}{2}$ states are below the kinematic threshold for Zweig allowed strong decays. Their transitions are shown in Figure 1 Detailed calculations of the various transition rates for these states are model

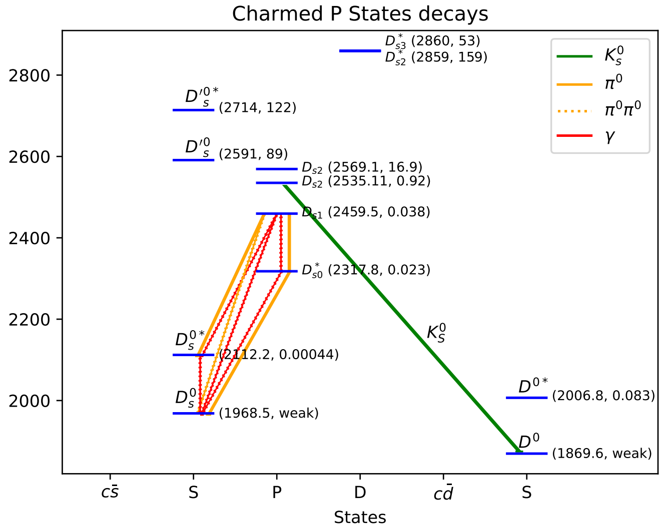


Figure 1: The predicted hadronic and electromagnetic transition rates for narrow $1S$ $j_l^P = 1/2^-$ and $1P$ $j_l^P = 1/2^+$ in $(c\bar{s})$ heavy-light states.

dependent. Measurements of the rates can distinguish between theoretical approaches. Table 1 shows some examples.

Table 1: The predicted hadronic and electromagnetic transition rates for narrow $1Sj_l^P = 1/2^-$ and $1Pj_l^P = 1/2^+$ heavy-light states. The original model of Bardeen et al.[4] $c\bar{s}$ model along with a more recent relativistic model [9]and a model including coupling to decay channels [10] (using $\Lambda' = 1.0$ (GeV)) are compared with 2024 PDG decay rates for the D_s system [1].

system	transition	partial widths (kev)			exp BR
		[4](2003)	[9](2023)	[10](2024)	PDG [1] (2024)
$(c\bar{u})$	$1^- \to 0^- + \gamma$	33.5			$(38.1 \pm 2.9)\%$
	$1^- \to 0^- + \pi^0$	43.6			$(61.9 \pm 2.9)\%$
	total	76.9			
$(c\bar{d})$	$1^- \to 0^- + \gamma$	1.63			$(1.6 \pm 0.4)\%$
	$1^- \to 0^- + \pi^0$	30.1			$(30.7 \pm 0.5)\%$
	$1^- \to 0^- + \pi^+$	65.1			$(67.7 \pm 0.5)\%$
	total	95.8			96 ± 22
$(c\bar{s})$	$1^- \to 0^- + \gamma$	0.43			$(93.6 \pm 0.4)\%$
	$1^- \to 0^- + \pi^0$	0.0079			$(5.77 \pm 0.35)\%$
	total	0.44			
$(c\bar{s})$	$0^+ \to 1^- + \gamma$	1.74	$(2.55 \pm^{+0.37}_{-0.45}$	1.41	$(< 5)\%$
	$0^+ \to 0^- + \pi^0$	21.5	$(7.83^{+1.97}_{-1.55})$		$(100\ ^{+0}_{20})\%$
	total	23.2			
$(c\bar{s})$	$1^+ \to 0^+ + \gamma$	0.43			$(3.7^{+5.0}_{-2.4})\%$
	$1^+ \to 1^- + \gamma$	3.49		2.19	$(< 8)\%$
	$1^+ \to 0^- + \gamma$	7.62		6.41	$(18 \pm 4)\%$
	$1^+ \to 1^- + \pi^0$	21.5			$(48 \pm 11)\%$
	$1^+ \to 1^- + 2\pi$	9.7			$(4.3 \pm 1.3)\%$
	total	42.7			
$(b\bar{u})$	$1^- \to 0^- + \gamma$	0.78			(seen)
	total	0.78			
$(b\bar{d})$	$1^- \to 0^- + \gamma$	0.24			(seen)
	total	0.24			
$(b\bar{s})$	$1^- \to 0^- + \gamma$	0.15			
	total	0.15			
$(b\bar{s})$	$0^+ \to 1^- + \gamma$	58.3		17.56	
	$0^+ \to 0^- + \pi^0$	21.5			
	total	79.8			
$(b\bar{s})$	$1^+ \to 0^+ + \gamma$	0.15			
	$1^+ \to 1^- + \gamma$	42.3		10.62	
	$1^+ \to 0^- + \gamma$	58.3		17.27	
	$1^+ \to 1^- + \pi^0$	21.5			
	$1^+ \to 1^- + 2\pi$	0.24			
	total	123.8			

2.2 B Mesons

The 1P $j^P = \frac{1}{2}^+$ states have not yet been observed. If we knew the splitting in the limit of the infinitely heavy quark, then we could use HQET to interpolate between the splittings in theD_s system to predict the splitting in the B_s system. From a theoretical point of view what can we say about mass difference between the 1P ($j^P = 3/2^+$) and ($j^P = 1/2^+$) meson states in QCD? If one considers a light quark moving in a funnel potential about a static source the splitting of the form $A\mathbf{s} \cdot \mathbf{L}$. u Using a nonrelativistic Schrodinger eqution we have ($A > 0$) while using a Dirac equation [11] we have ($A < 0$). Hence even the sign of the splitting is not known apriori.

Lattice QCD results in the heavy quark limit have been presented by Green et el.[12]. The results for the interpolation between $\frac{1}{m_Q} = 0$ and the charm-strange system is shown in Figure 2. Even more precise lattice. results in the HQET limit could be useful. Hence assuming linear

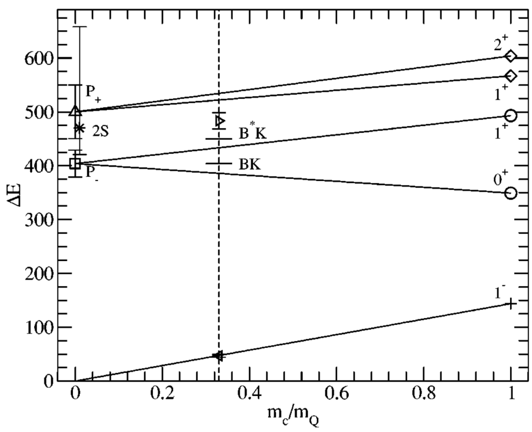


Figure 2: The energies in MeV of P-wave excited states relative to the ground state heavy-light meson with a heavy quark mass m_Q and a light quark which is strange. Data from experiment are plotted for charm and for b quarks while our lattice results are shown for static quarks. The 2S excitation from their larger volume results is also shown. The dotted vertical line gives the interpolated value appropriate for b quarks. The BK and B*K thresholds are also shown. These are the lightest isospin-conserving decay modes allowed by strong interactions. From Green et al. [12]

behaviour in $1/m_Q$ and knowing M(1/2+) - M(1/2-) for $1/m_Q \to 0$ we can predict the value $M(B_{s1})$ and $M(B^*_{s0})$. However, including the effects of coupling to the strong decay channels or in molecular model the distance of the state from the two body strong decay threshold is critical. So one might not expect this simple behavior.

These masses can also be calculated directly in Lattice QCD. Recently Hudspith and Mohler have presented a detailed LQCD calculation [13]. Their compare their results with earlier LQCD results and the results of other approaches. The masses obtained by various models are shown in Figure 3. Essentially all the models show that the $j^+\frac{1}{2}$ states are below the associated threshold for strong decays. If the masses of the $j^P = \frac{1}{2}^+$ P wave b mesons are as expected above, the allowed transitions would be given by Figure 4.

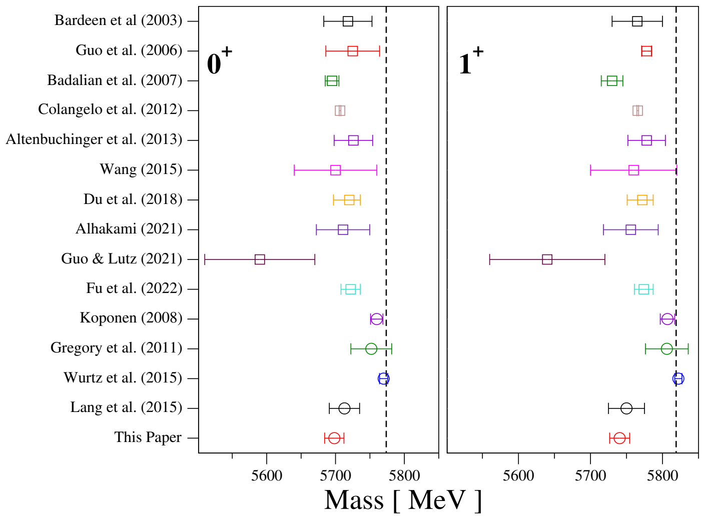


Figure 3: Below Threshold Comparison of Hudspith and Mohler results [13] or the B^*_{s0} (left pane) and B_{s1} (right pane) ground-state masses to various other results. Circles denote the results of lattice calculations, while squares denote the results from model/EFT calculations. The vertical line denotes the respective threshold. To translate their result for the binding energy to the result displayed in this plot an iso-symmetric kaon mass of 494.2 MeV has been used. Adapted from[13].

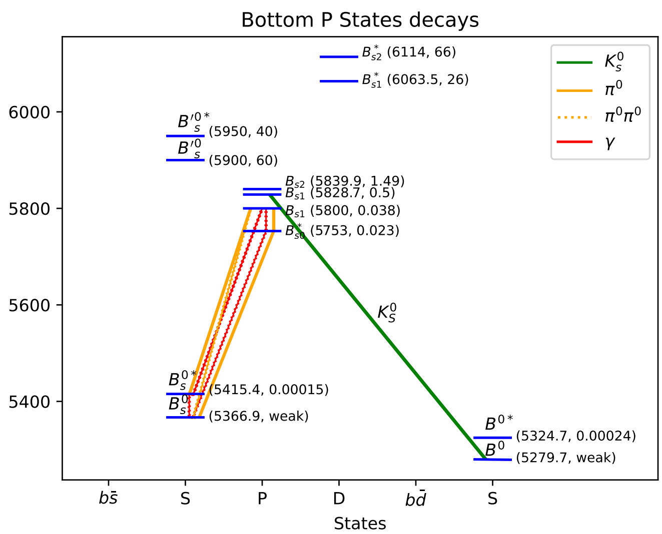
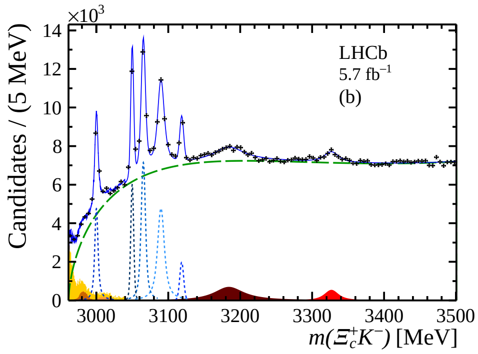


Figure 4: The predicted hadronic and electromagnetic transition rates for narrow $1S$ $j_l^P = 1/2^-$ and $1P$ $j_l^P = 1/2^+$ in $(b\bar{s})$ heavy-light states.

3. Singly Heavy Baryons

The lowest P states for heavy baryons have quantum number $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$. In fact the Ω_c lowest P wave states Figure 5 have been observed by LHCb [14] [15]. All the P state widths are fairly narrow which suggests that lowest J^P state is not far about threshold for the strong decays $\Omega_c \to \Xi_c^+ K^-$. To date the quantum numbers have been determined for only the $\Omega_c(3050)$ state [16]. It is possible that for the still unobserved Ω_b analogies the J^P P state might be below threshold



Resonance	m (MeV)	Γ (MeV)
$\Omega_c(3000)^0$	$3000.44 \pm 0.07\ ^{+0.07}_{-0.13} \pm 0.23$	$3.83 \pm 0.23\ ^{+1.59}_{-0.29}$
$\Omega_c(3050)^0$	$3050.18 \pm 0.04\ ^{+0.06}_{-0.07} \pm 0.23$	$0.67 \pm 0.17\ ^{+0.64}_{-0.72}$
		< 1.8 MeV, 95% C.L.
$\Omega_c(3065)^0$	$3065.63 \pm 0.06\ ^{+0.06}_{-0.06} \pm 0.23$	$3.79 \pm 0.20\ ^{+0.38}_{-0.47}$
$\Omega_c(3090)^0$	$3090.16 \pm 0.11\ ^{+0.06}_{-0.10} \pm 0.23$	$8.48 \pm 0.44\ ^{+0.61}_{-1.62}$
$\Omega_c(3119)^0$	$3118.98 \pm 0.12\ ^{+0.09}_{-0.23} \pm 0.23$	$0.60 \pm 0.63\ ^{+0.90}_{-1.05}$
		< 2.5 MeV, 95% C.L.
$\Omega_c(3185)^0$	$3185.1 \pm 1.7\ ^{+7.4}_{-0.9} \pm 0.2$	$50 \pm 7\ ^{+10}_{-20}$
$\Omega_c(3327)^0$	$3327.1 \pm 1.2\ ^{+0.1}_{-1.3} \pm 0.2$	$20 \pm 5\ ^{+13}_{-1}$

Figure 5: The $\Xi_c^+ K^-$ invariant mass distributions and measured masses and natural width of the $\Omega_c(X)^0$ candidates [16].

for strong decays.

4. Heavy Heavy Light Baryons

For heavy-heavy-light baryons the only possibility for very narrow states above the ground states are the lowest P-wave states. Defining $Q_1 = c, Q_2 = b$ and $q = u, d, s$ the baryons: $Q_i Q_j q$ the lowest P states for $q = u$ and $q = d$ have strong pion transitions to the associated ground states. while the $Q_i Q_j s$ P-waves masses may be low enough that K transitions to singly heavy baryons are kinematically forbidden. These states would then be the analogy of the lowest heavy-light meson P states.

4.1 Models (in collaboration with Chris Quigg)

One theoretically attractive picture of doubly heavy baryons is to consider the two heavy quarks form a subsystem similar to the NRQCD system that exists for $Q\bar{Q}$ quarkonium systems. Heavy quarks $(Q_i Q_j)$ bound in a color $\bar{\mathbf{3}}$ by an effective potential of the "Cornell" Coulomb + linear form at half strength for both components. The strength of the Coulomb contribution is fixed by the color Casimir. Lattice studies indicate that the effective string tension for the color $\bar{\mathbf{3}}$ is half that for the singlet configuration [17]. Other lattices studies of the three body potential reach the same conclusion. For example, for baryons fitted with the Δ string configuration the string tension is $\sigma_{Q_i Q_j Q_k} \approx 0.53\ \sigma_{Q_i \bar{Q}_i}$ [18]. For sufficiently heavy Q_i, Q_j, it makes sense to regard the doubly heavy diquark and the nucleus of a diquark–light-quark atom, small in extent relative to overall size of the "atom" determined by the motion of the light (strange) quark. This system closely resembles a heavy–light meson with the same light degrees of freedom for the quantum states of the light quark [19], with the important added element that the core can be excited.

In the heavy quark limit the average separation between the two heavy quarks (denoted r) is much smaller than the separation between the center of mass to the two heavy quark system and the light quark (denoted R) here. In this limit the wavefunction for the baryon system the ground state can be factorized into the product of wavefunctions

$$\Phi(r, R) = \phi(r)\psi(R) \tag{1}$$

where $\phi(r)$ is a solution of the NR Schroedinger Equation for two color 3 quarks combined into a total color $\bar{3}$. The solution of the system has excitation energies to is approximate one half the excitation energy of the corresonding state in $Q\bar{Q}$ as the solutions are not very dependent of m_Q in charmonium to bottomonium region. For applications involving b and c heavy quarks, we will have to check that the planetary light quark orbiting a tiny diquark is a plausible approximation. The low-lying excitation spectrum is given in Table 2.

Table 2: The excitation energies (in MeV) of the low-lying excited states in the QQ systems compared with the $Q\bar{Q}$ systems. Only states in a limited excitation energy range (below 600MeV for the bb system) are shown.

state	cc	bc	bb	$\bar{c}c$	$\bar{b}c$	$\bar{b}b$
1P	226	217	208	428	436	467
2S	337	311	278	591	570	563
1D	393	369	340	713	702	710
2P	499	470	409	871	838	815
1F	537	498	448	951	919	898
3S	598	545	472	1015	957	902
2D	635	585	514	1098	1046	980
1G	666	615	544	1164	1110	1077
3P	732	669	577	1242	1170	1095

It is important to note that the splittings between the 1S and 1P state in the $[QQ]_{\bar{3}}$ subsystem are comparable to the splittings between the ground states and first excited states in a heavy-light meson. Thus the spectrum of heavy-heavy-light baryons is not well approximated by two separate scales usually assumed for these baryons. In particular we can compare the splittings in Table 2.

4.2 A Simple Model

One theoretically attractive picture of doubly heavy baryons is to consider the two heavy quarks form a subsystem similar to the NRQCD system that exists for $Q\bar{Q}$ quarkonium systems. Heavy quarks (Q_iQ_j) bound in a color $\bar{\mathbf{3}}$ by an effective potential of the "Cornell" Coulomb + linear form at half strength for both components. The strength of the Coulomb contribution is fixed by the color Casimir. Lattice studies indicate that the effective string tension for the color $\bar{\mathbf{3}}$ is half that for the singlet configuration [17]. Other lattices studies of the three body potential reach the same conclusion. For example, for baryons fitted with the Δ string configuration the string tension is $\sigma_{Q_iQ_jQ_k} \approx 0.53\ \sigma_{Q_i\bar{Q}_i}$ [18]. For sufficiently heavy Q_i, Q_j, it makes sense to regard the doubly heavy diquark and the nucleus of a diquark–light-quark atom, small in extent relative to overall size of the "atom" determined by the motion of the light (strange) quark. This system closely resembles a heavy–light meson with the same light degrees of freedom for the quantum states of the light quark [19], with the important added element that the core can be excited. For applications

$$\frac{df^{(0)}}{dr} + \frac{\kappa}{r} f^{(0)} = (E - V_v + V_s + m_q) f^{(1)}, \quad \frac{df^{(1)}}{dr} - \frac{\kappa}{r} f^{(1)} = (-E + V_v + V_s + m_q) f^{(0)} \quad \text{where} \quad \kappa = 1 + \sigma \cdot L$$

light quark wavefunction → spin of heavy quark

$$\Psi_{n,\ell,j,J,M}(r,\theta,\varphi) = \sum_{S\in\{-\frac{1}{2},+\frac{1}{2}\}} C^{J,M}_{j,m;\frac{1}{2},S} \begin{pmatrix} i f^0_{n,\ell,j}(r) k^+_{\ell,j,m}\ Y^{\ell}_{m-\frac{1}{2}}(\theta,\varphi) \\ i f^0_{n,\ell,j}(r) k^-_{\ell,j,m}\ Y^{\ell}_{m+\frac{1}{2}}(\theta,\varphi) \\ f^1_{n,\ell,j}(r) k^+_{2j-\ell,j,m} Y^{2j-\ell}_{m-\frac{1}{2}}(\theta,\varphi) \\ f^1_{n,\ell,j}(r) k^-_{2j-\ell,j,m} Y^{2j-\ell}_{m+\frac{1}{2}}(\theta,\varphi) \end{pmatrix} \otimes \xi_S \quad \text{and} \quad k^{\pm}_{\ell,j,m} = \begin{cases} +\sqrt{\frac{\ell \pm m + \frac{1}{2}}{2\ell+1}} & \text{for } j = \ell + \frac{1}{2} \\ \pm\sqrt{\frac{\ell \mp m + \frac{1}{2}}{2\ell+1}} & \text{for } j = \ell - \frac{1}{2} \end{cases}$$

Figure 6: Four component Dirac equations for the Cornell potential. Notation defined in [11].

Another way to see that the assumption of separation of scales fails for heavy-heavy-light baryons (with $Q = c$ or $Q = b$) is to compute the RMS separations in the the Q_iQ_j system and the $QQ = H$ core the light quark $q = s$ system . This comparison using the Dirac equation for the Hq system is shown in Figure 7.

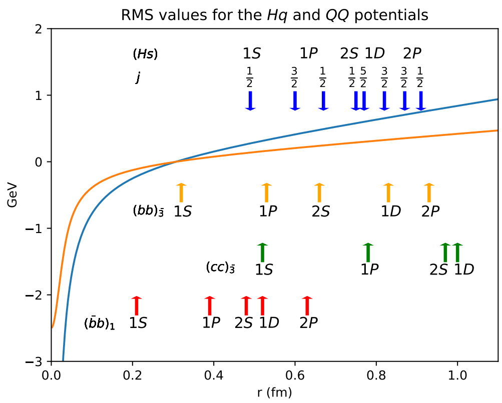


Figure 7: Comparison of the RMS separation between the two heavy quarks QQ to the separation of the heavy diquark system (H) and the strange quark $q = s$ in various states for a doubly heavy baryon systems. For reference the RMS separation between between the b and $\bar{b}$ in a bottonium systems is shown.

4.3 Born-Oppenheimer Effective Theory

One can compute the lowest ($J = 1/2, 3/2, 3/2, 5/2$) P wave states directly in LQCD. Alternatively, one can compute in Lattice QCD ground state energies E(R) for a system with one dynamic light quark in the static potential of two heavy static quarks separated by a distance R. The potentials associated with $O(1/M_Q)$ corrections can also be calculated in this way. Then the total energy and wavefuction for the heavy quark ground system can be obtained by solving the SE for each distinct set of quantum numbers. This allows the complete solution for the lowest doubly heavy S and P states. The Born-Oppenheimer approach for two heavy quarks can be calculated on the lattice [20]. Using this approach and a string potential [21] one can computed the spectrum of doubly heavy baryons see Figure 8.

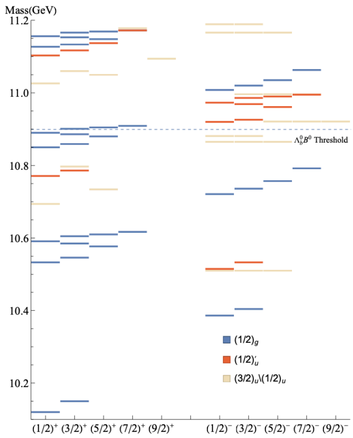


Figure 8: Spectrum of double bottom baryons in terms of j^{η_P} states. Each line represents a state. The spectrum is for the states associated to the $(1/2)_g$ and $(1/2)'_u$ static energies and previous results [22] for the mixed $(3/2)_u/(1/2)_u$ static energies, which do not include hyperfine contributions. The color indicates the static energies that generate each state. From [21].

Both the simple potential model and the Born-Oppenheimer approach suggest that the lowest P wave excitations are below the threshold for Zweig allowed decay and thus should be extremely narrow.

5. Summary

- The nature of the $B_s(1P) j^P = \frac{1}{2}^+$ states is still an open question. Comparing the decays of the $D_s(1P)$ states and the $B_s(1P)$ states with give valuable information into the microscopic nature of the $j^P = \frac{1}{2}^+$ 1P states.
- The naive analog between the $Q\bar{q}$ mesons and the QQq baryons fails for the ccq, bbq, bcq systems. This is because the low-lying excitations of the QQ core are of the same order as that of the excitations of the light quark ($\bar{q}$). Expect a complicated spectrum of low P wave QQq states. Some of the lowest 1P (ccs, cbs, bbs) states may be stable to Zweig allowed decays.

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References

[1] Particle Data Group, S. Navas et al., *Review of Particle Physics*, *Phys. Rev. D*, **110(3)** (2024) 030001 and 2025 update.

[2] E. Eichten and C. Quigg, *Heavy-quark symmetry implies stable heavy tetraquark mesons* $Q_iQ_j\bar{q}_k\bar{q}_l$, *Phys. Rev. Lett.*, **119(20)** (2017) 202002 [1707.09575].

[3] CMS Collaboration, A. Hayrapetyan et al., *New Structures in the J/ψJ/ψ Mass Spectrum in Proton-Proton Collisions at s=13 TeV*, *Phys. Rev. Lett.* **132(11)** (2024) 111901 [2306.07164].

[4] W. Bardeen, E. Eichten and C. T. Hill, *Chiral multiplets of heavy - light mesons*, *Phys. Rev. D*, **68** (2003) 054024 [hep-ph/0305049] .

[5] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, *Dynamically generated 0+ heavy mesons in a heavy chiral unitary approach*, Phys. Lett. B**641** (2006) 278 [hep-ph/0603072] .

[6] Feng-Kun Guo, Peng-Nian Shen, and Huan-Ching Chiang, *Dynamically generated 1+ heavy mesons*, *Phys. Lett. B* **647** (2007) 133 [hep-ph/0610008].

[7] F.-K. Guo, C. Hanhart, and U.-G. Meissner, *Interactions between heavy mesons and Goldstone bosons from chiral dynamics*, *Eur. Phys. J. A* **40** (2009) 171 [0901.1597].

[8] Zi-Le Zhang, Zhan-Wei Liu, Si-Qiang Luo, Ping Chen, and Zhi-Hui Guo, *Masses and radiative decay widths of Ds0*(2317) and Ds1'(2460) and their bottom analogs*, *Phys. Rev. D* **110(9)** (2024) 094037 [2409.05337].

[9] Meng Han, Wei Li, Su-Yan Pei, Ting-Ting Liu, and Guo-Li Wang, *Decay properties of* $D^*_{s0}(2317)$ *as a conventional* $c\bar{s}$ *meson*, *Chin. Phys. C*, **47(7)** (2023) 073102 [2305.04250].

[10] LHCb Collaboration, R. Aaij et al., *Observation of the doubly charmed baryon* Ξ^{++}_{cc}, *Phys. Rev. Lett.***119(11)** (2017) 112001 [2504.05063].

[11] M. Di Pierro and E.Eichten, *Excited Heavy - Light Systems and Hadronic Transitions*, *Phys. Rev. D* **64** (2001) 114004 [hep-ph/0104208].

[12] A. M. Green, J. Koponen, C. Michael, C. McNeile, and G. Thompson, *Excited B mesons from the lattice*, *Phys. Rev. D* **69** (2004) 094505 [hep-lat/0312007].

[13] R. J. Hudspith and D. Mohler, *Exotic tetraquark states with two $\bar{b}$ quarks and $J^P = 0^+$ and 1^+ B_s states in a nonperturbatively tuned lattice NRQCD setup*, *Phys. Rev. D* **107(11)** (2023) 114510 [2303.17295].

[14] LHCb Collaboration, R. Aaij et al., *Observation of five new narrow Ω_c^0 states decaying to $\Xi_c^+ K^-$*, *Phys. Rev. Lett.* **118(18)** (2017) 182001 [1703.04639].

[15] LHCb Collabiration, R. Aaij et al., *Observation of New Ω_c^0 States Decaying to the* Ξ*c+K- Final State*, *Phys. Rev. Lett.* bf 131(13) (2023) 131902 [2302.04733].

[16] LHCb Collaboration, R. Aaij et al., *First Determination of the Spin-Parity of* Ξ*c(3055)+,0 Baryons*, *Phys. Rev. Lett.* **134(8)** (2025) 081901 [2409.05440].

[17] A. Nakamura and T. Saito, *QCD color interactions between two quarks*, *Phys. Lett. B* **621** (2005) 171 [hep-lat/0512043].

[18] T. Takahashi, H. Suganuma, Y. Nemoto, and H. Matsufuru, *Detailed analysis of the three quark potential in SU(3) lattice QCD*, *Phys. Rev. D* **65** (2002) 114509 [hep-lat/0204011].

[19] M. Savage and M. Wise, *Spectrum of baryons with two heavy quarks*, *Phys. Lett.* **B248** (1990) 177.

[20] J. Najjar and G. Bali, *Static-static-light baryonic potentials*, PoS LAT2009 (2009) 089 [0910.2824].

[21] J. Soto and J. T. Castellà, *Effective QCD string and doubly heavy baryons*, *Phys. Rev. D* **104** (2021) 074027 [2108.00496].

[22] J. Soto and J. T. Castellà, *Effective field theory for double heavy baryons at strong coupling*. *Phys. Rev. D* **102(1)** (2020) 014013, Erratum *Phys.Rev.D* **104** (2021) 059901 [2005.00551].