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Quark pairing in sQGP

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We obtain a quark pairing gap in sQGP by solving the coupled Dyson-Schwinger equations for quark propagator and quark gluon vertex in the Nambu-Gorkov basis which is widely applied to study the color superconductivity. We acquire a quark pairing gap at small chemical potential which is related to the dimension two gluon condensate and hence, its generation mechanism differs from the conventional color superconducting phase located only at large chemical potential. The gap persists up to $2-3\,T_c$ and vanishes at higher temperature. Such a quark pairing together with a second order phase transition characterizes a partial-deconfined new phase in sQGP and distinct from the phase with quasi quarks and gluons.

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1. Introduction

There are many exotic properties for QCD matter in the region near above the chiral phase transition temperature $T = T_c$. Many observations both from experimental data and theoretical computations indicate that the QCD matter in the temperature region $T \in [T_c, 3T_c]$ is not simply a phase consist of chiral symmetric deconfined quarks and gluons.

Here we first introduce the framework of functional QCD approach, especially a computationally minimal scheme [1]. Following this, we compute the coupled Dyson-Schwinger equations (DSEs) of quark propagator and quark gluon vertex [2], and we find that despite of a vanishing chiral condensate as $\langle\bar{\Psi}\Psi\rangle = 0$, a finite quark pairing gap of $\langle\bar{\Psi}_C\Psi\rangle$ emerges at $T \sim T_c - 3T_c$ after extending the Dyson-Schwinger equation into Nambu-Gorkov basis [3, 4]. In particular, the pairing is closely related to the non Abelian effect in quark gluon vertex which has not been included in the previous studies [5–7] of the color superconducting phase located at low temperature and high chemical potential. The quark pairing is genered due to the glue dynamics since it is related to the dimensinoal two gluon condensate. This new mechansim extends the color superconducting phase into small chemical potential and high temperature region.

Moreover, at chiral limit, the quark pairing becomes vanishing above the temperature $T \sim 3T_c$ which defines a new phase transition and indicates that the sQGP can be characterized as a completely different phase compared to the wQGP phase. This novel finding may shed light on the phenomena in heavy ion collisions and deepen the understanding of QCD.

2. Minimal scheme in functional QCD approach

To study the QCD matter at finite temperature and density, the key element is the DSE or gap equation for the quark propagator. The full quark propagator at finite density and temperature provides access to many observables that are related to the chiral and deconfinement phase transition. The quark DSE is given by

$$\begin{aligned}\mathbf{S}(\vec{p},\tilde{\omega}_m)^{-1} &\equiv i\tilde{\omega}_m)\gamma_0 C(\vec{p},\tilde{\omega}_m) + i\vec{p}\cdot\vec{\gamma}A(\vec{p},\tilde{\omega}_m) + B(\vec{p},\tilde{\omega}_m) = \mathbf{S}_0^{-1} + \mathbf{\Sigma}(\vec{p},\tilde{\omega}_m)\,, \\ \mathbf{\Sigma}(\vec{p},\tilde{\omega}_m) &= T\sum_{n=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\, g^2\mathbf{D}_{\mu\nu}(\vec{p}-\vec{q},\Omega_{mn};T,\mu)\times\mathbf{\Gamma}_\mu^{a,0}\,\mathbf{S}(\vec{q},\tilde{\omega}_l)\,\mathbf{\Gamma}_\nu^a(\vec{q},\tilde{\omega}_n,\vec{p},\tilde{\omega}_m)\,,\end{aligned} \tag{1}$$

where $\mathbf{D}_{\mu\nu}(k) = (\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\frac{Z(k^2)}{k^2}$ is the full gluon propagator with gluon momenta $k = q - p$, and $\mathbf{\Gamma}_\nu$ is the full quark-gluon vertex. The four momentum is defined as:$p = (\vec{p}, \tilde{\omega}_m)$ with Matsubara frequency $\tilde{\omega}_m = ((2m+1)\pi T, \mu)$. The diagrammatic depiction of Eq. (1) is provided in Fig.(1).

For the gluon propagator, it is relatively separate from the computation of quark propagator. One can apply the vacuum data of gluon propagator from lattice QCD, and simply compute the one loop difference between vacuum and finite temperature and chemical potential. Here we apply the parameterisation in Ref. [8] of the 2+1-flavour gluon propagator in the vacuum, which combines the data from quantitative functional QCD and from lattice QCD.

For the quark gluon vertex, we apply the dominant two structures, Dirac structure $\mathcal{T}_\mu^{(1)} = \gamma_\mu$ and Pauli structure $\mathcal{T}_\mu^{(4)} = \sigma_{\mu\nu}k_\nu$ as:

$$\Gamma_\mu(q,p) = \mathcal{T}_\mu^{(1)}(q,p)\,\lambda^{(1)}(q,p) + \mathcal{T}_\mu^{(4)}(q,p)\,\lambda^{(4)}(q,p)\,, \tag{2}$$

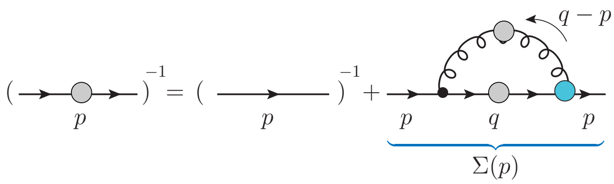

Figure 1: Quark DSE for the quark self energy $\Sigma(p)$. The classical vertex is the one with small black dot. The classical quark propagator is depicted with a straight black line.Full propagators and vertices are depicted with blobs.

where the dressing of the classical tensor structure is constrained by the STIs for the quark-gluon vertex. We shall use

$$\lambda^{(1)}(q,p) = g_s F(k^2)\Sigma_A(q,p)\,, \qquad \Sigma_A(q,p) = \frac{A(p)+A(q)}{2}\,, \tag{3}$$

with the ghost dressing function $F(k^2) = k^2\, G_c(k)$, where $G_c(k)\delta^{ab}$ is the ghost propagator.

For $\lambda^{(4)}$, several studies suggest that it is proportional to differences of the scalar quark dressing function. It also carries the RG-scaling of the quark and anti-quark leg of the quark-gluon vertex. The RG-scaling of any vertex dressing λ^i also has to accommodate the RG-scaling of the gluon momentum as $\propto 1/Z^{1/2}(k^2)$ as in Refs. [9, 10]. Hence, in the vacuum we choose

$$\lambda^{(4)}(q,p) = \frac{g_s}{Z^{1/2}(k)}\,\Delta_B(q,p)\,, \qquad \Delta_B(q,p) = \frac{B(p)-B(q)}{p^2-q^2}\,. \tag{4}$$

This then completes the minimal scheme to describe QCD phase diagram in the Dyson-Schwinger equation approach. Next we will extend this scheme in Nambu-Gorkov basis to study the quark pairing.

3. The Dyson-Schwinger equations in Nambu-Gorkov basis

To study the quark pairing, one may apply the Nambu-Gorkov basis as:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \bar{\Psi} = \begin{pmatrix} \bar{\psi}, \bar{\psi}_C \end{pmatrix}, \tag{5}$$

with $\psi_C = C\psi^*$ the charge-conjugator spinor obtained through the charge conjugation matrix $C = i\gamma^2\gamma^4$. The free quark propagator in the new basis is simply a doubling of degrees of freedom and at finite temperature and chemical potential it can be written as:

$$\mathbf{S}_0^{-1}(p) = \begin{pmatrix} \mathrm{i}\gamma\cdot p + \mathrm{i}\gamma_4\mu, & 0 \\ 0, & \mathrm{i}\gamma\cdot p - \mathrm{i}\gamma_4\mu \end{pmatrix} \tag{6}$$

where m_0 is the current quark mass and $p = (\boldsymbol{p}, \omega_n)$, with $\omega_n = (2n+1)\pi T$ being the quark Matsubara frequency, and μ the quark chemical potential.

One can then denote the self energy of quark propagator and the respective quark propagator as:

$$\mathbf{\Sigma}(p) \equiv \begin{pmatrix} \Sigma_+(p), & \Phi_-(p) \\ \Phi_+(p), & \Sigma_-(p) \end{pmatrix}, \qquad \mathbf{S} = \begin{pmatrix} G_+, & F_- \\ F_+, & G_- \end{pmatrix}, \tag{7}$$

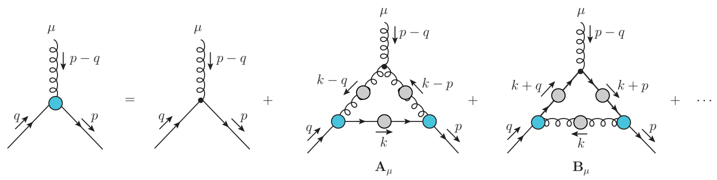

Figure 2: The Dyson Schwinger equations for the quark gluon vertex. The diagram B_μ in quark gluon vertex can also be found in Abelian gauge theory, while the diagram A_μ with the three gluon vertex represents the non Abelian effect.

with $\Phi_-(p_4, \vec{p}) = \gamma_4 \Phi_+^\dagger(-p_4, \vec{p})\gamma_4$ and

$$G_\pm = \left\{[G_{\pm,0}]^{-1} + \Sigma_\pm - \Phi_\mp([G_{\mp,0}]^{-1} + \Sigma_\mp)^{-1}\Phi_\pm\right\}^{-1}, \tag{8}$$

$$F_\pm = -([G_{\mp,0}]^{-1} + \Sigma_\mp)^{-1}\Phi_\pm G_\pm. \tag{9}$$

with the bare quark-gluon vertex:

$$\mathbf{\Gamma}_\mu^{a,0} = \begin{pmatrix} \frac{\lambda^a}{2}\gamma_\mu, & 0 \\ 0, & -\frac{(\lambda^a)^t}{2}\gamma_\mu \end{pmatrix}, \tag{10}$$

The most important upgrade here is a full consideration of the quark gluon vertex Γ_ν^a. The dressed vertex is in principle a matrix as:

$$\mathbf{\Gamma}_\nu^a = \begin{pmatrix} \frac{\lambda^a}{2}\Gamma_\nu, & \gamma_2(\Xi_\nu)^T\gamma_2\frac{(\lambda^a)^T}{2} \\ \frac{\lambda^a}{2}\Xi_\nu, & \frac{(\lambda^a)^T}{2}\gamma_2(\Gamma_\nu)^T\gamma_2 \end{pmatrix}. \tag{11}$$

Instead of applying some parametrization to the vertex, here we solve the Dyson-Schwinger equation for the vertex especially with the non Abelian diagram A_μ in Fig. (2) which is found to be dominant [8]. The Abelian contribution B_μ is $1/N_c^2$ suppressed in comparison to A_μ due to the color matrices. The DSE for the vertex Γ_μ with the non-Abelian diagram is expressed as

$$\Gamma_\mu^a(q,p) = g_s P_{\mu\nu}(p-q)\,\Gamma_\nu^{a,0} + \mathbf{A}_\mu^a(q,p), \tag{12}$$

with the non-Abelian contributions given by

$$\mathbf{A}_\mu^a(q,p) = \frac{Z_1 N_c}{2} P_{\mu\nu}(p-q) \times = \int_k \Gamma_{\nu\alpha\beta}^{abc,0}\, G_A^{b,b'}(k-q)\, G_A^{c,c'}(k-p)\, \Gamma_\alpha^{b'}(k,p)\, G_q(k)\, \Gamma_\beta^{c'}(q,k).$$

In the above formulas, $N_c = 3$ for $SU(3)$, and $\Gamma_{\nu\alpha\beta}^{abc,0}$ denotes the classical three-gluon vertex,

$$\Gamma_{\nu\alpha\beta}^{abc,0} = f^{abc} g_s\left[(2k-p-q)_\nu g_{\alpha\beta} + (2q-p-k)_\alpha g_{\nu\beta} + (2p-q-k)_\beta g_{\alpha\nu}\right]. \tag{13}$$

With such a setup for the quark gap equation in Eq. (11) and the equation for quark gluon vertex in Eq. (12) together with the input of gluon propagator, the equations are closed and can be solved numerically. However, we would like to introduce some further approximations to have a more direct and simple conclusion without losing the reliability.

4. Solving the equation in the infrared

We focus on the pseusocalar gap at chiral limit and neglect the momentum dependence of the mass gap as $\Phi^\dagger(p) = \Phi(p) = \Delta\gamma_5\mathcal{M}$ with $\mathcal{M}$ the color and flavor matrices. Here we choose the color flavor locking channel as $\mathcal{M} = \frac{\lambda_f^2}{2}\bigotimes\frac{\lambda^2}{2} + \frac{\lambda_f^5}{2}\bigotimes\frac{\lambda^5}{2} + \frac{\lambda_f^7}{2}\bigotimes\frac{\lambda^7}{2}$ and equalize the three flavor quarks. Consequently, the trace of the color matrices of the off diagonal part is similar to the diagonal part with and additional factor $1/2$. Note that the choices of the channel and the flavor will not change our main conclusion as they simply change a common factor in the off diagonal part of the self energy which do not have impact on the existence of the pairing gap as will elaborate below. For the quark gluon vertex, we apply the similar approximation. We neglect the momentum dependence of the coefficients $\lambda_i(p,q)$ with their momentum chosen to be at the infrared limit with $\vec{q} = \vec{p} = 0$ and the zeroth Matsubara frequency only with $q_4 = p_4 = \pi T + i\mu$. The treatment of the momentum dependence here is similar to the NJL model for the dynamical quark mass generation. Since the dynamical generated mass function is finite in the infrared and goes to zero in ultraviolet, such an approximation is effective if one considers the nonperturbative properties in the infrared instead of the perturbative running behavior.

For the diagonal part of the quark gluon vertex, the Pauli term is proportional to the mass function and thus vanishing in the chiral symmetric phase. The Dirac term is constrained via the Slavnov-Taylor identities (STIs). This implies that the diagonal part can be simply considered as the bare vertex as:

$$\Gamma_\nu = F(k^2)\gamma_\nu, \tag{14}$$

For the off-diagonal part Ξ_ν, we take the Pauli term which is supposed to be the dominant structure in analogy to the diagonal part. This then completes the discussion of the vertex structures and we have for the off-diagonal part of the quark gluon vertex as:

$$\Xi_\mu(p,q) = \lambda_4\sigma_{\mu\nu}(p-q)^\nu. \tag{15}$$

With the above setup, the DSE of the quark-gluon vertex in Eq. (12) can be greatly simplified. After tracing out all the color and Dirac matrices, one has for the off diagonal part of quark gluon vertex:

$$\lambda_4 = Z\lambda_4^2\Delta(1-\overline{\cos^2\langle k,q\rangle}) = \frac{1}{2}Z\lambda_4^2\Delta, \quad Z = \frac{2g^2}{3}\int dk\frac{4\vec{k}^2}{k^2}G^2(\bar{k}^2), \tag{16}$$

with $\bar{k} = (2n\pi T, \vec{k})$. Here we've introduced the angular average between the transfer momentum k in the vertex and the loop momentum q in the vertex DSE. Note that there is also a term proportional to Δ in the self energy part in Eq. (16), however, it only has a quadratic contribution of Δ in the solution of λ_4 and thus can be neglected from the equation.

One then obtain the off diagonal part of quark gluon vertex as:

$$\Xi_\nu = \sigma_{\nu\mu}(p_\mu - q_\mu)\frac{1}{Z\Delta}, \tag{17}$$

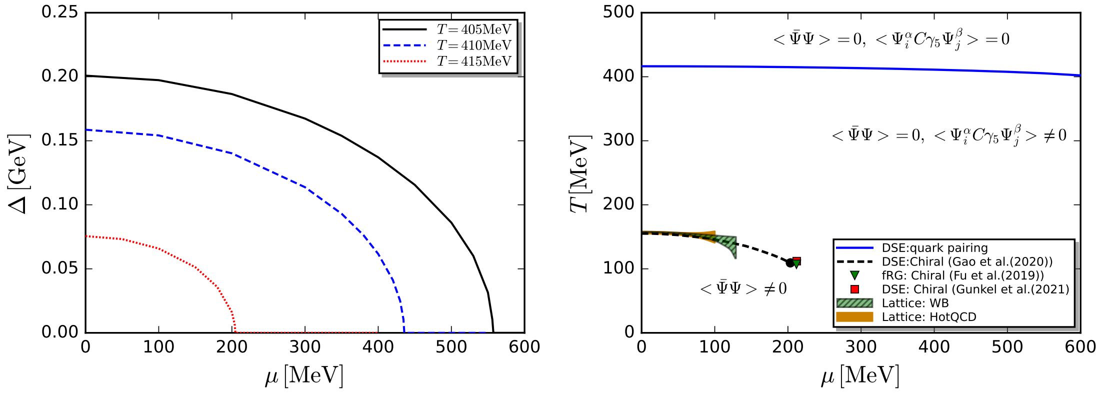

Figure 3: The obtained quark chemical potential μ dependence of Δ at several temperatures (*left panel*), and the respective PT line as the solid line (*right panel*). For comparison, the dashed line in the phase diagram is the chiral PT line from Ref. [9] together with the critical end point from other functional QCD calculations [12, 13], and the colored bands are the lattice QCD results from Ref. [14] (WB) and Ref. [15] (HotQCD).

and putting Eq. (17) in the gap equation, one directly has for the non-diagonal part:

$$\Delta = -\delta_m \Delta + \frac{K}{Z\Delta}, \tag{18}$$
$$K = 2g^2 \int dk \frac{k_4(k_4 - p_4) + \vec{k}^2}{k_4^2 + \vec{k}^2} G(\bar{k}^2), \delta_m = 4g^2 \int dk \frac{k_4 k_4^* - \vec{k}^2}{(k_4^2 + \vec{k}^2)^2} F(\bar{k}^2) G(\bar{k}^2).$$

This then gives us a simple expression for the quark pairing gap, and one has: $\Delta = \sqrt{\frac{K}{Z(1+\delta_m)}}$. Note that Z is positive and δ_m is similar to a renormalization constant and typically of order $O(10^{-1})$. Therefore, if $K > 0$, one can obtain a finite solution for Δ, else if $K < 0$, Eq. (16) and Eq. (18) only allow the trivial solution as $\lambda_4 = \Delta = 0$.

One may further expand K in Eq.(19) as:

$$K = 2\langle g^2 A_\mu^2 \rangle - 2\langle g^2 \frac{k_4 p_4}{k^2} G(\bar{k}^2) \rangle, \tag{19}$$

The first term in the r.h.s is the dimension-2 gluon condensate without the color matrices. The term contains a quadratic divergence that needs to be subtracted, and our calculation shows that it has weak temperature dependence on the temperature domain we considered here. The second term is typically proportional to temperature with a power law scale. Therefore, there exists a second order phase transition at temperature $T_\Delta > T_c$. Above T_Δ, one has $\Delta = 0$, and below T_Δ, one has:

$$\Delta^2 \propto \langle g^2 A_\mu^2 \rangle [1 - (T/T_\Delta)^\alpha], \quad \alpha \approx 2.16. \tag{20}$$

One can further proceed the computation at finite chemical potential. At finite chemical potential, the pairing gap Δ is in general complex similar to the mass function. Here we simply compute the average of Δ with both the first positive and negative Matsubara frequency of p, q as $p_4^\pm = q_4^\pm = \pm\pi T + i\mu$. This allows one to define the phase transition taking place at where the average of Δ vanishes. We depict the μ dependence of the pairing at several temperatures and finally, the respective phase diagram together with the chiral phase transition from the previous studies in Fig. (3). In general, the quark pairing exists above T_c up to $T \approx 2-3T_c$, which coincides in fact with the previous conjectures of a possible new phase existing at this area [11, 16, 17]. The discontinuous derivative of the quark pairing at T_Δ further characterizes this new phase that is distinct from the weakly coupled quark gluon plasma with quasi quarks and gluons.

5. Discussions and outlooks

We firstly emphasize here that this finite gap is induced purely by the non-Abelian effect of the interaction vertex as it generates the $\frac{1}{\Delta}$−type interaction. The interaction with $\frac{1}{\Delta}$ naturally induces a finite gap in the gap equation as in Eq. (18). Note that this specific form makes the gap robust against the choices of the coupling strength, the channels and the flavors of the pairing. However, it needs to mention that since the pairing is related to the gluon condensate, a strong coupling strength is still required.

Following similar procedures, one also sees that the Abelian diagram B_μ in the quark gluon vertex contributes the terms which lead to the relation $\lambda_i \propto \frac{g^2}{2N_c}\Delta$ instead of $1/\Delta$, and consequently, without the non-Abelian vertex, the DSEs without the quadratic correction of the pairing in general only allow the trivial solution and the gap is vanishing. Therefore, The conventional color superconducting phase which exists with only the Abelian contribution or even without the off-diagonal part of quark gluon vertex is distinct from the quark pairing pattern found here.

It is interesting to note that the quark pairing phase has a large overlap with the chiral spin symmetric phase. The bosonic quark pairing and the proposed fermionic field with the string-like interaction in chiral spin symmetric phase are two distinct collective modes, which are reminiscent of the duality between the pairing and vortex dynamics. A further investigation on their relation and generating mechanism may reveal some new features of QCD matter. Moreover, the possible heavy ion collision experiments signals The quark paring indicates the color superconducting property and thus, it is interesting to investigate further in this region on the transport properties.

Besides, there is another question left, that is, does the quark pairing gap still exist in the chiral symmetry breaking phase. The coupled equations of gap equation and quark gluon vertex do not seem to prevent the gap appearing in the chiral symmetry breaking phase. However, if inserting the gap into the diagonal part of the gap equation, it will not have the impact on the quark mass function as discussed in the setup of the quark gluon vertex, instead, it only contributes to wave function renormalization in the Dirac structure $\not{p}$ of the gluon propagator. Therefore, it shares some features and functionings with the A_0^a condensate which roughly speaking shifts the temporal component of $\not{p}$. The existence of the quark pairing in chiral symmetry breaking phase and its relation with A_0^a condensate requires a complete computation in the chiral symmetry breaking phase with the Nambu-Gorkov basis, which will be investigated in the near future.

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