PROCEEDINGS OF SCIENCE

Unitarizing Scattering Amplitudes of massive spin-2 Particles in Compact Extra-Dimensions

Dipan Sengupta*

Sydney Consortium for Particle Physics and Cosmology, School of Physics ,
The University of New South Wales, Sydney NSW 2052, Australia

E-mail: dipan.sengupta@unsw.edu.au

We summarize the development of unitarizing scattering amplitudes of massive spin-2 particles in theories of compact extra-dimensions. We show, from a variety of perspectives, the behaviour of scattering amplitudes of massive spin-2 particles at high energies and extract the effective field theory scale in both flat and Anti-de-Sitter background spaces.

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*Speaker

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1. Introduction

Scattering amplitudes are ubiquitous in quantum field theories. It allows us to systematically understand the validity of quantum field theories at every order in perturbation theory. It also allows us to understand the energy scale at which a given theory should be superseded by a larger theory consistent with all the symmetries of the low-energy theory. This is true for all the fundamental forces of nature, including electroweak theory and quantum chromodynamics, as well as gravity.

In recent years, there has been a renewed surge of interest in theories of compactified higher-dimensional gravitational theories. First introduced as a way to unify elctromagnetism and gravity within a 5 dimensional fundamental theory by Kaluza and Klein (KK) [1, 2], compactified theories appear naturally in the broader context of string theory, a candidate for a quantum theory of gravity. Low energy constructions of compactified theories emerged in the late 1990's as an intermediate solution to the hierarchy problem originating from a variety of string compactifications. These low-energy constructions include compactifications over a flat background such as the Arkani-Hamed-Dimopoulous-Dvali (ADD) model[3], as well as compactifications with a bulk Anti-de-Sitter (AdS) geometry, such as the Randall-Sundrum (RS) model[4, 5]. Phenomenological consequences of these models, including solutions to the hierarchy problem, flavor puzzle, electroweak phase transition and constructive approaches such as the clockwork mechanism have been studied in great detail[6, 7].

Recently, phenomenological dark matter models with massive spin-2 particles (known as KK resonances, obtained by compactification of an extra-dimension) as a mediator between the dark and the visible sector have also received a lot of attention. In this case, it was estimated in several works that the matrix elements of these KK spin-2 particles grew as $O\left(\frac{s^3}{M_{KK}^6\Lambda_\pi^2}\right)$ (where s is the squared centre of mass energy, Λ_π the scale of the effective 4D theory, and M_{KK} is the massive spin-2 KK particle), in contradiction with expectations based on unitarity arguments which predict that in compact extra-dimensional theories the scattering amplitude should grow no faster than $O\left(\frac{s}{\Lambda_\pi^2}\right)$. The predictions of the phenomenological dark matter papers with a massive spin-2 KK particle are analogous to the theories of massive gravity as the Fierz-Pauli theory, where scattering amplitudes of matter with massive spin-2 particles grow as $O\left(\frac{s^3}{m_g^6 M_{Pl}^2}\right)$, where m_g is the mass of the spin-2 particle and M_{Pl} the observed 4D Planck mass [8, 9]. This begs a larger question, how do we reconcile the apparent contradictory result of the difference in the high energy scaling behaviour of scattering amplitudes of massive spin-2 particles in massive gravity and KK theories.

The rest of this document briefly illustrates the reconciliation for the $2 \to 2$ elastic scattering of massive spin-2 KK particles in a stabilized AdS background. Further details can be found in the references [10–15] for the pure gravitational sector calculation, in [16, 17] for an exposition of the reconciliation in a general 't-hooft-Feynman gauge using the Goldstone Boson equivalence theorem, and in [17, 18] for scattering of masssive spin-2 KK particles with matter, and finally in [19] a correct estimate of dark matter abundance in KK portal models.

2. Gravitational Sector of the Model

Without loss of generality, we parametrize the 5 dimensional metric as, [13, 14], we write the Lagrangian for the stabilized 5D model on a curved AdS background as,

$$S = \int_V d^4x\, dy \left(\mathcal{L}_{\text{EH}} + \mathcal{L}_{\phi\phi} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{GHY}} + \Delta\mathcal{L}\right), \tag{1}$$

where $\mathcal{L}_{\text{EH}}$ is the 5D Einstein-Hilbert term, $\mathcal{L}_{\phi\phi}$ and $\mathcal{L}_{\text{pot}}$ are the kinetic and potential energy terms of the stabilizing bulk scalar field $\hat{\phi}(x, y)$, $\mathcal{L}_{\text{GHY}}$ is the Gibbons-Hawking-York term [20, 21] required for a well-posed variational problem in the gravitational spin-2 KK and the spin-0 GW sector. Finally, $\Delta\mathcal{L}$ is a total derivative term added to the action to eliminate the mixing between the tensor and scalar fields in 5D. The specific forms can be found in [12, 14].

In unitary gauge, the 5D metric for an RS-like background is parametrized as [14]

$$G_{MN} = \begin{pmatrix} w g_{\mu\nu} & 0 \\ 0 & -v^2 \end{pmatrix} \text{ and } G^{MN} = \begin{pmatrix} g^{\mu\nu}/w & 0 \\ 0 & -1/v^2 \end{pmatrix}, \tag{2}$$

in terms of coordinates $x^M = (x^\mu, y)$, where $y \in (-\pi r_c, +\pi r_c]$ parametrizes an orbifolded extra-dimension (where y and $-y$ are identified); for convenience we define $\varphi \equiv y/r_c$, and use y or φ interchangeably as the coordinate of the fifth dimension. Here

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu}(x_\mu, y), \tag{3}$$

where $\eta_{\mu\nu}$ is the usual 4D "mostly-minus" metric and $\hat{h}_{\mu\nu}(x, y)$ parametrizes metric fluctuations. The quantities w and v are defined as,

$$\begin{aligned} w &= \exp\left[-2\left(A(y) + \frac{e^{2A(y)}}{2\sqrt{6}}\kappa\, \hat{r}(x, y)\right)\right] \\ v &= \left(1 + \frac{e^{2A(y)}}{\sqrt{6}}\kappa\, \hat{r}(x, y)\right), \end{aligned} \tag{4}$$

where the function $A(y)$ specifies the background AdS geometry[1] and $\hat{r}(x, y)$ parametrizes scalar metric fluctuations around this background. We normalize the Lagrangian such that the 5D gravitational coupling κ is related to the 5D fundamental Planck mass M_5 according to $\kappa^2 = 4/M_5^3$.

The GW stabilization mechanism [22, 23] relies on the existence of a non-trivial background scalar field configuration $\phi_0(y)$ such that the size of the extra dimension is stabilized by a competition between the kinetic (gradient) and potential energies of this field, which is "pinned" to different values at the two ends (branes) of the orbifold. We expand the bulk scalar $\hat{\phi}(x, y)$ as

$$\hat{\phi}(x, y) \equiv \frac{1}{\kappa}\phi \equiv \frac{1}{\kappa}\left[\phi_0(y) + \hat{f}(x, y)\right], \tag{5}$$

where scalar fluctuations are encoded in $\hat{f}(x, y)$.[2]

[1] For RS1 [4] models, in which the extra dimension is unstabilized, $A(y) = k|y|$ as sourced by brane and bulk cosmological constants.with k being the curvature of the AdS background.

[2] The factors of κ (in units of energy$^{-3/2}$) included in Eq. (5) are defined so that ϕ_0 and $\hat{f}$ are dimensionless in natural units.

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For any arbitrary well behaved and bounded potential that stabilizes the AdS bulk geometry, the quadratic fluctuation terms of the full Lagrangian in Eq. (1), once compactified with proper boundary conditions, generates a pair of Sturm-Liouville (SL) equation that can be solved to yield the KK spectrum of the gravitational (spin-2) sector and the scalar (spin-0) sector. A detailed description of the procedure to bring the KK mode Lagrangian to a quadratic form can be found in [13, 14].

For the tensor fluctuations, which give the masses and the wavefunctions of the spin-2 KK modes, the tensor field $\hat{h}_{\mu\nu}(x,y)$ can be decomposed into a tower of 4D KK states $\hat{h}^{(n)}_{\mu\nu}(x)$ with $\varphi \equiv y/r_c$,

$$\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{r_c}} \sum_{n=0}^{+\infty} \hat{h}^{(n)}_{\mu\nu}(x)\, \psi_n(\varphi)\ , \tag{6}$$

r_c being the radius of the compact extra dimension and $\psi_n(\varphi)$ the 5D wavefunction of the n^{th} mode that satisfies the SL differential equation:

$$\partial_\varphi[e^{-4A}\, \partial_\varphi \psi_n] = -\mu_n^2 e^{-2A} \psi_n. \tag{7}$$

These wavefunctions satisfy the Neumann boundary conditions where $(\partial_\varphi \psi_n) = 0$ at $\varphi \in \{0, \pi\}$. The dimensionless eigenvalues $\mu_n = m_n r_c$ give the masses m_n of the n^{th} spin-2 KK mode. The wavefunctions follow an orthonormality and complteness relation according to,

$$\int_{-\pi}^{+\pi} d\varphi \quad e^{-2A}\, \psi_m\, \psi_n = \delta_{m,n},\ \text{orthonormality}\ , \tag{8}$$

$$\delta(\varphi_2 - \varphi_1) = e^{-2A} \sum_{j=0}^{+\infty} \psi_j(\varphi_1)\, \psi_j(\varphi_2)\ ,\ \ \text{completeness.} \tag{9}$$

The difference between an unstabilized and stabilized model in the above is encoded in the new background geometry with a modified warp factor $A(y)$.

For the spin-0 sector, in which the metric fluctuation and the bulk scalar mix proportional to vacuum expectation value (VEV) of the background scalar field $\phi_0(\varphi)$, the KK decomposition of the 5D scalar field $\hat{r}(x,y)$ into a tower of spin-0 KK modes proceeds by introducing extra-dimensional wavefunctions $\gamma_i(\varphi)$ and a tower of 4D scalar fields $\hat{r}^{(i)}(x)$ parameterized as follows:

$$\hat{r}(x,y) = \frac{1}{\sqrt{r_c}} \sum_{i=0}^{+\infty} \hat{r}^{(i)}(x)\, \gamma_i(\varphi)\ . \tag{10}$$

The eigenmode equation that brings the 5D scalar spin-0 Lagrangian to canonical form is given by [13, 14, 24, 25],

$$\partial_\varphi\left[\frac{e^{2A}}{(\phi_0')^2}(\partial_\varphi \gamma_i)\right] - \frac{e^{2A}}{6}\gamma_i = -\mu^2_{(i)}\, \frac{e^{4A}}{(\phi_0')^2}\, \gamma_i \left\{1 + \frac{2\,\delta(\varphi)}{\left[2\ddot{V}_1 r_c - \frac{\phi_0''}{\phi_0'}\right]} + \frac{2\,\delta(\varphi-\pi)}{\left[2\ddot{V}_2 r_c + \frac{\phi_0''}{\phi_0'}\right]}\right\}, \tag{11}$$

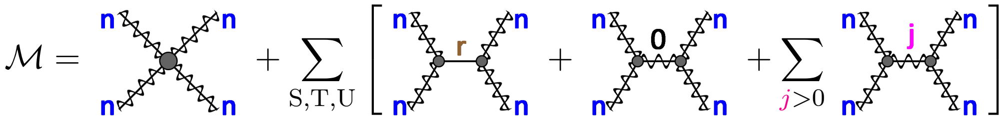

Figure 1: Elastic scattering of spin-2 KK particles of mode number n, as a sum of 4 gauge invariant diagrams at leading order

3. Scattering Amplitudes and energy growth

We want to assess the high energy behaviour of the scattering amplitude corresponding to the Feynman diagrams in Fig. In order to carry out scattering amplitude calculations we require to define the polarizations of external spin-2 KK gravitons.

A helicity-λ spin-2 KK graviton carries five polarizations $\varepsilon^{\mu\nu}_{\lambda(k)}$. These can be grouped into two transverse, and three longitudinal polarizations, which can be split into two helicity-1 modes and one helicity-0 mode, defined respectively as [12],

$$\lambda = \pm 2, \;\; \varepsilon^{\mu\nu}_{\pm 2} = \varepsilon^{\mu}_{\pm 1}\varepsilon^{\nu}_{\pm 1}, \tag{12}$$

$$\lambda = \pm 1, \;\; \varepsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}\left[\varepsilon^{\mu}_{\pm 1}\varepsilon^{\nu}_{0} + \varepsilon^{\mu}_{0}\varepsilon^{\nu}_{\pm 1}\right], \tag{13}$$

$$\lambda = 0, \quad \varepsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}\left[\varepsilon^{\mu}_{+1}\varepsilon^{\nu}_{-1} + \varepsilon^{\mu}_{-1}\varepsilon^{\nu}_{+1} + 2\varepsilon^{\mu}_{0}\varepsilon^{\nu}_{0}\right], \tag{14}$$

where $\varepsilon^{\mu}_{\pm 1}$ are the usual polarization vectors for the photon and the helicity-0 polarization is defined by,

$$\varepsilon^{\mu}_{0}(k_2) = \frac{E_{k_2}}{M_{KK}}\left(\sqrt{1 - \frac{M^2_{KK}}{E^2_{k_2}}},\ \hat{k}\right). \tag{15}$$

Choosing the centre-of-momentum frame for the incoming particles with four-vectors p_1 and p_2, and outgoing four-vectors k_1 and k_2, the outgoing energies E_{k_1} and E_{k_2} can be expressed in terms of the Mandelstam variable s and the mass of the graviton M_G as,

$$E_{k_1} = \frac{s - M^2_{KK}}{2\sqrt{s}}, \qquad E_{k_2} = \frac{s + M^2_{KK}}{2\sqrt{s}}. \tag{16}$$

In order to extract the high energy behaviour of the amplitude, we consider the scattering of two longitudinal KK gravitons,

$$h^{(n)}h^{(n)} \to h^{(n)}h^{(n)}. \tag{17}$$

and expand the matrix element as a power series is $\sqrt{s}$,

$$\mathcal{M}_{\lambda\bar{\lambda}}(s,\theta) = \sum_{\sigma\in\mathbb{Z}} \widetilde{\mathcal{M}}^{(\sigma)}_{\lambda\bar{\lambda}}(\theta)s^{\sigma/2}. \tag{18}$$

We observe that while the scattering amplitudes of individual diagrams grow as fast as $O(s^5)$, the sum of all diagrams at every order from $O(s^5)$ to $O(s^2)$, once all KK modes are taken into account in the intermediate propagators in the s,t and u channel diagrams [10–13, 16, 18]. This is ensured by a set of sum-rules that relates various 3 and 4 point coupling structures depicted in Fig. 1, and provides an exact cancelation that can be evaluated analytically. In a stabilized Randall-Sundrum model, this ensures that at high energies that scattering amplitudes of spin-2 KK modes grow as $\mathcal{M} \simeq \frac{s}{\Lambda_\pi^2}$, where $\Lambda_\pi = M_{pl}e^{-kr_c\pi}$, is the energy scale untill which the RS model is valid perturbatively.

Analogous to the case of W^+W^- scattering, the precise pattern of cancelation at every power of $O(s)$ such that the final amplitude grows like $O(s)$ is due to the unitary gauge choice. In a't-Hooft-Feynman gauge one can show that the high energy growth grows no faster than $O(s)$ from the outset [16–18], ensured by a goldstone equivalence theorem analogous to W^+W^- scattering.

4. Conclusion

We have analyzed the high energy behaviour of scattering amplitudes of massive spin-2 Kaluza-Klein gravitons in extra-dimensional models. We have shown that while individual diagrams grow as fast $O(s^5)$ in unitary gauge, delicate cancellations driven by higher dimensional diffeomorphism and Ward Identities ensure that the final amplitude grow no master than $O(s)$, with an effective 4D field theory scale determined only by the curvature and the radius of compactification. Further development include studies of scattering amplitudes for brane localized curvature [26], as well as multi-brane models such as in [27].

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