PROCEEDINGS OF SCIENCE

# Impact of high-energy resummation and GPD evolution on exclusive heavy-quarkonium photoproduction at NLO

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In these proceedings, we introduce and discuss the infamous scale-stability problem of exclusive heavy-quarkonium photoproduction off protons in Collinear Factorisation (CF) at next-to-leading order (NLO) in the strong coupling $\alpha_s$. We confirm this issue and propose a resolution to the problem by resumming leading-logarithmic QCD corrections via High-Energy Factorisation (HEF) in the Doubly-Leading-Logarithmic Approximation (DLA), matched to CF. Our NLO CF $\oplus$ DLA HEF predictions show good agreement with HERA and LHC data and exhibit significantly reduced scale dependence compared to Born-level results. We assess the relative impact of uncertainties arising from different sources, including PDF choice, intrinsic PDF uncertainties and variations of the factorisation and renormalisation scales, and compare them with the corresponding experimental uncertainties. The results bear implications for the interpretation of experimental measurements at past, current, and future facilities, including HERA, the LHC, and the EIC.

*The European Physical Society Conference on High Energy Physics (EPS-HEP2025)*
*7-11 July 2025*
*Marseille, France*

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## 1. Introduction

The long-standing problem of poor scale stability in exclusive heavy-quarkonium photoproduction in the perturbative QCD (pQCD) Collinear Factorisation (CF) framework has posed a significant challenge in reliably interpreting experimental measurements and comparing theoretical predictions with data. The issue was first identified over 20 years ago in [1], where the perturbatively calculable coefficient functions at next-to-leading order (NLO) in the strong coupling $\alpha_s$ for the photoproduction process off protons, $\gamma p \to Vp$, with $V = J/\psi, \Upsilon, \psi', \ldots$ denoting a heavy quarkonium, were first presented. Since then, several dedicated efforts to resolve this issue have appeared in the literature. These include an approach to judiciously fix the factorisation scale [4], another based on the explicit resummation of high-energy logarithms in the leading-logarithmic approximation (LLA) [5, 6], and a further one advocating a crucial so-called '$Q_0$' subtraction [7]. Such efforts have the important upshot of mitigating the scale dependence of pQCD computations within the CF framework, thereby enabling a wide range of meaningful phenomenological applications - such as probes of gluon saturation and information on gluon parton distribution functions (PDFs) - which become reliable only once the theoretical uncertainties are under control.

In these proceedings, we present a solution that integrates CF and High-Energy Factorisation (HEF), resumming an important class of high-energy logarithms in a manner consistent with the NLO DGLAP evolution of Generalised Parton Distributions (GPDs). We further discuss how this approach complements and extends previous efforts.

## 2. Collinear factorisation

The amplitude $A$ for exclusive quarkonium photoproduction at NLO within CF can be expressed as a convolution of the hard-scattering coefficient functions $C_i$ and the GPDs $F_i$, corresponding to the quark ($i = q$) and gluon ($i = g$) channels, together with a Distribution Amplitude (DA) $\phi^V_{Q\bar{Q}}$ describing the hadronisation of the open heavy-quark pair $Q\bar{Q}$ into the quarkonium $V$. In the non-relativistic static limit, the DA reduces to a delta function constraining the heavy-quark pair to be produced with zero relative velocity and it follows that

$$A = -\varepsilon^{\gamma}_{\lambda} \cdot \varepsilon^{*V}_{\lambda'} \sum_{i=q,g} \int_{-1}^{1} dX\, C_i(X, \xi; \mu_F, \mu_R)\, F_i(X, \xi; t, \mu_F), \tag{1}$$

where $\varepsilon^{\gamma}_{\lambda}$ ($\varepsilon^{*V}_{\lambda'}$) is the polarisation vector for the incoming photon (outgoing quarkonium) and $X$ and $\xi$ denote parton momentum fractions. The factorisation and renormalisation scales are denoted by $\mu_F$ and $\mu_R$, respectively. In the Bjorken limit at leading twist, the partons participating in the $t$-channel colour-singlet exchange are collinear along the light-cone $P^+$ direction[1] , carrying momentum fractions $X + \xi$ and $X - \xi$, see Fig. 1 for the set up.[2]

The coefficient functions to NLO in $\alpha_s$ for the photoproduction case considered here were derived in [1].[3] It is enlightening to study these functions in the limit of large $\gamma p$ centre-of-mass

[1]We adopt the light-cone basis, where a generic four-vector $v^\mu$ has components $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and $\mathbf{v}_\perp = (v^1, v^2)$, and work in the center-of-mass frame of the $\gamma p$ system, with the $z$-axis chosen along the proton direction.

[2]It follows that $\xi$ describes the net longitudinal momentum asymmetry between the initial and final states with momenta $p$ and $p'$.

[3]See [2, 3] for the corresponding electroproduction coefficient functions at NLO.



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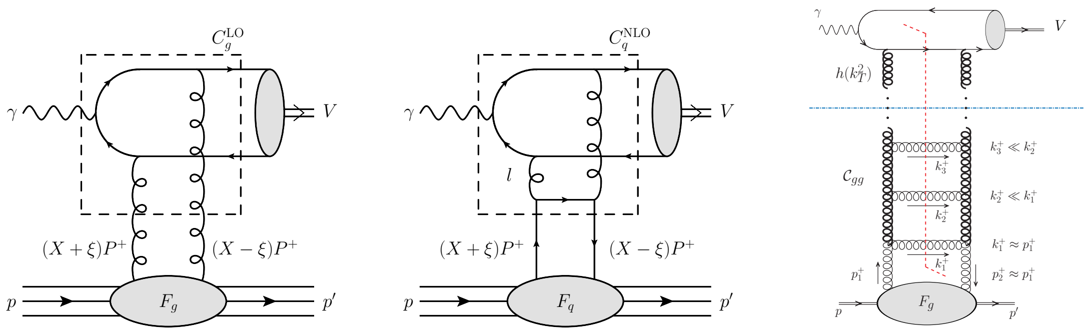


**Figure 1:** Example LO (left) and NLO (middle) contributions to $\gamma p \to V + p$, where $V$ denotes the heavy quarkonium. In these diagrams, the momentum $P \equiv (p + p')/2$ and $l$ is the loop momentum. The momentum fractions of the left and right partons are $x = X + \xi$ and $x' = X - \xi$ respectively. Also shown (right) is a typical Feynman diagram contributing to the imaginary part of the photoproduction amplitude in the $\xi/X \ll 1$ region in the LLA, see text for details.

energies $W_{\gamma p}^2 \gg M_V^2$, or equivalently small $\xi \ll 1$, viz., $\xi \approx M_V^2/(2W_{\gamma p}^2)$, where $M_V$ is the mass of the heavy quarkonium. In this limit, one finds that

$$C^{\mathrm{NLO}}_{\{q,g\}}(X,\xi;\mu_F,\mu_R) = -\frac{i\pi c|X|}{2\xi}\frac{\alpha_s(\mu_R)}{\pi}\ln\left(\frac{m_Q^2}{\mu_F^2}\right)\{2C_F, C_A\}\,, \tag{2}$$

where $c = 4\pi\alpha_s(\mu_R)ee_Q R_V(0)/(m_Q^{3/2}\sqrt{2\pi N_c})$. Here, $e_Q$ is the electric charge of the heavy quark in units of the positron charge $e$ in natural units, $R_V(0)$ is the radial wave function at the origin, and $m_Q = M_V/2$ is the mass of the heavy quark. Upon integration, noting that at small $x$ the gluon and quark GPDs scale like a constant and $1/X$, respectively, the purely imaginary high-energy amplitude becomes directly proportional to $\ln(1/\xi)$, leading to a logarithmic enhancement in the small $\xi \ll |X| \ll 1$ region. This is indicative of the need for resummation of the tower of logarithms $\sim (\alpha_s \ln(1/\xi))^n$. Indeed, in the DGLAP evolution of low $\xi$ GPDs, the probability of emitting a gluon is strongly enhanced by the large value of $\ln(1/\xi)$. As a consequence, the contributions from the GPD and from the coefficient function no longer fully compensate each other when the factorisation scale $\mu_F$ is varied.[4] In [4], it was proposed to effectively resum contributions of the type $\sim (\alpha_s \ln(1/\xi)\ln(m_Q^2/\mu_F^2))^n$ at amplitude level by setting the factorisation scale to $\mu_F = m_Q$ in part of the calculation. This approach removes this double-logarithmic enhancement from the NLO amplitude at small $\xi$ and was shown to greatly reduce the scale dependence. This approach, however, misses terms that may contain a single $\ln(1/\xi)$ that does not depend on $\mu_F$.[5] A method to compute these missing terms at a given order in $\alpha_s$ has been developed and employed to evaluate contributions that arise beyond NLO [5, 6], with the aim of achieving a resummation of all such

[4]The mean number of gluons in the logarithmic interval $\Delta \ln \mu_F$ is given by $\langle n\rangle \simeq \frac{\alpha_s N_c}{\pi}\ln(1/\xi)\Delta\ln(\mu_F^2)$. For $\xi \sim 10^{-3} - 10^{-4}$ and with the conventional scale variation from $\mu_F/2$ to $2\mu_F$, this yields $\langle n\rangle \gg 1$. In contrast, the NLO coefficient function accounts for the virtual correction of at most one gluon.

[5]An analogous scale-fixing prescription was proposed for the $P_T$-integrated cross sections of inclusive hadro- [8] and photoproduction [9] of heavy quarkonia.

terms. The first few terms are given by

$$A \sim 1 + z \ln\left(\frac{m_Q^2}{\mu_F^2}\right) + z^2\left(\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{m_Q^2}{\mu_F^2}\right)\right) + \ldots, \qquad z^n \sim \alpha_s^n \ln^n(1/\xi). \tag{3}$$

As shown in Ref. [6], resummation of such high-energy terms up to the sixth order leads to a markedly improved and scale-stable result. In our approach, we resum higher-order QCD corrections using High-Energy Factorisation (HEF) in the Doubly-Logarithmic Approximation (DLA) matched to CF. This approach is consistent with the NLO DGLAP evolution of GPDs and provides for a uniformly accurate description of the exclusive quarkonium photoproduction observable over a wide range in $W_{\gamma p}$. We now turn to discuss our procedure in more detail.

## 3. High-energy resummation and matching to CF

At higher orders in $\alpha_s$, the CF coefficient function $C_i(X, \xi; \mu_F, \mu_R)$, expressed in terms of the variable $\rho = \xi/X$, receives corrections of the form $\propto (\alpha_s^n \ln^{n-1}(1/|\rho|))/|\rho|$. These contributions become significant in the region $|\rho| \ll 1$, where their resummation defines the Leading-Logarithmic Approximation (LLA). This resummation can be achieved through the HEF framework developed for inclusive processes, and can be extended to exclusive reactions by invoking the optical theorem.

In the LLA, the dominant contribution to the imaginary part arises from Feynman diagrams such as Fig. 1, right where the thick helical lines represent Reggeised gluons in the $t$-channel which are strongly ordered in $k^+$. The lower part of the cut diagram corresponds to the usual BFKL ladder, integrated over the transverse momenta $\mathbf{k}_\perp^2$ of all emitted gluons. This computation yields a universal HEF resummation factor $C_{gi}(\rho, \mathbf{k}_\perp^2; \mu_F, \mu_R)$ at LLA, while the upper part gives rise to a process-dependent coefficient function, $h(\mathbf{k}_\perp^2)$. Together, they define the resummed HEF coefficient function,

$$C_i^{(\mathrm{HEF})}(\rho; \mu_F, \mu_R) = -\frac{i}{|\rho|}\int_0^\infty d\mathbf{k}_\perp^2 C_{gi}(|\rho|, \mathbf{k}_\perp^2; \mu_F, \mu_R) h(\mathbf{k}_\perp^2), \tag{4}$$

which replaces $C_i$ in Eqn. (1).

In our approach, at this stage to maintain consistency with GPD evolution, only the doubly-logarithmic subset of LLA corrections, the terms $\propto (\alpha_s^n(\mu_R)\ln^{n-1}(1/\rho)\ln^{n-1}(\mu_F^2/\mathbf{k}_\perp^2))/|\rho|$, is retained in $C_{gi}$. This is because $C_{gi}$ in the LLA generates $\mu_F$-dependent terms which will not be compensated by the fixed-order evolution of GPDs. Doing so provides the resummed coefficient function in HEF to DLA accuracy. Full details are given in [10]. The two approximations to the CF coefficient function so far discussed, the NLO in $\alpha_s$ in CF and the DLA in HEF, can now be suitably combined using a subtractive-matching prescription,

$$C_i^{(\mathrm{match.})}(X, \xi) = C_i^{\mathrm{LO}}(X, \xi) + C_i^{\mathrm{NLO}}(X, \xi) + \left[\check{C}_i^{(\mathrm{HEF})}(\xi/|X|) - C_i^{(\mathrm{NLO,asy.})}(X, \xi)\right]\theta(|X| - \xi). \tag{5}$$

The $\theta(|X| - \xi)$ ensures that the resummation is active only in the DGLAP region, $|X| > \xi$, of GPDs, see Eqn. (2). The hat symbol on $\hat{C}_i^{(\mathrm{HEF})}(\xi/|X|)$ indicates that the leading-order $\alpha_s^0$ term has been subtracted from it. For $\rho \ll 1$, that is $\xi \ll |X| < 1$, one has $C_i^{(\mathrm{NLO})}(X, \xi) \to C_i^{(\mathrm{NLO,asy.})}(X, \xi)$. Likewise, when $\rho \to 1$, that is $\xi \simeq |X| < 1$, it follows that $\hat{C}_i^{(\mathrm{HEF})}(\xi/|X|) \to C_i^{(\mathrm{NLO,asy.})}(X, \xi)$, since $\ln^n(1/|\rho|) \to 0$. Here, the matched coefficient function coincides with the fixed-order CF result.

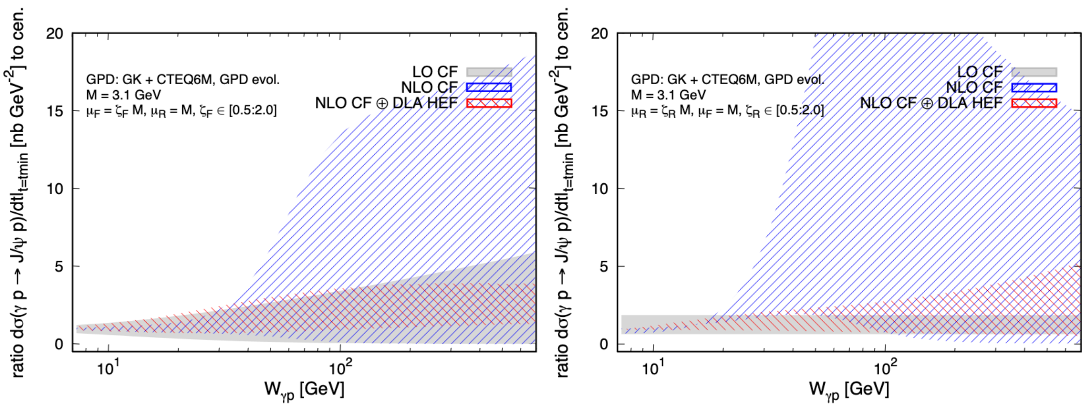


**Figure 2:** Normalised $t$-differential cross section for $J/\psi$ production, showing LO CF, NLO CF, and NLO CF $\oplus$ DLA HEF $\mu_F$ (left) and $\mu_R$ (right) scale uncertainties.

## 4. Results

We use the Goloskokov-Kroll (GK) model of GPDs [11] in terms of double distributions which naturally encode the well-known properties of GPDs, including polynomiality, Lorentz invariance and correct forward limits [12, 13]. In this way, we generate GPDs from input PDFs at an input scale of $\mu_0 = 2$ GeV and perform full leading-logarithmic GPD evolution [14] in the `PARTONS` framework [15] interfaced to `APFEL++` [16].

In Fig. 2, we show the relative $\mu_F$ (left panel) and $\mu_R$ (right panel) scale dependence of the LO CF, NLO CF and the NLO CF $\oplus$ DLA HEF results for $J/\psi$ production as a function of $W_{\gamma p}$. We show the ratio of differential cross sections, $d\sigma/dt|_{t_{\min}}$, with $t_{\min} \approx 0$ in the $W_{\gamma p}$ region considered, obtained by varying the scales by the conventional factor of two. For the $\mu_F$ scale variation, the NLO CF prediction (blue band) highlights the large, erratic scale dependence of the fixed-order CF computation, whereas our resummed prediction (red band) exhibits a smoother, monotonically increasing energy dependence, similar to the LO prediction (grey band), but with significantly reduced sensitivity to scale variations. For the $\mu_R$ variation, we observe that the uncertainty of the NLO CF $\oplus$ DLA HEF prediction gradually increases at higher energies and eventually exceeds that of the LO CF result. This behaviour arises from the $\mu_R$ dependence of the hard-pomeron contribution at DLA, which is expected to be reduced in an extension of our approach to NLLA.

In Fig. 3, we present the $t$-differential cross section as a function of $W_{\gamma p}$ for $V = J/\psi$ (left) and $V = \Upsilon$ (right) production, compared to precise $t$-differential data from H1 [17] and ZEUS [18], respectively. The GPDs are constructed using input CT18NLO PDFs [19], with the light-red shaded band indicating the 9-point scale variation and the hatched bands showing the individual $\mu_F$ and $\mu_R$ scale variations. Central cross-section predictions based on GPDs constructed from a variety of input PDFs assess the spread in description arising from modern PDF sets. Also shown for the $V = J/\psi$ case is the propagated PDF uncertainty in the cross section through the dark-red shaded band. This is obtained by performing a PDF uncertainty analysis of the Hessian eigenvector members of CT18NLO. We have checked that the broadening of this uncertainty band for larger $W_{\gamma p}$ in the 'upper' direction is due to the behaviour of the gluon PDF of member 57 in this region.

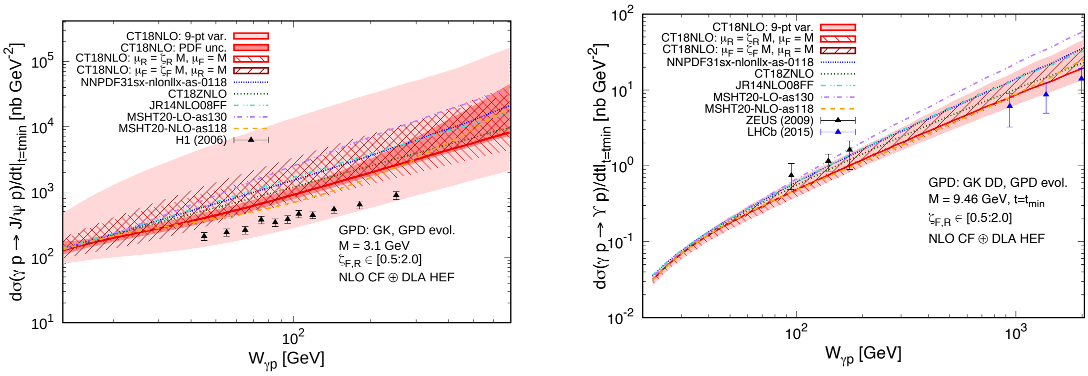


**Figure 3:** $t$-differential cross-section predictions as a function of $W_{\gamma p}$ compared to experimental measurements for $J/\psi$ (left) and $\Upsilon$ (right) production. The light-red band shows the 9-point scale variation, the dark-red band the propagated PDF uncertainty in the cross section, while the hatched bands show the separate $\mu_F$ and $\mu_R$ scale variations. Also shown are central cross-section predictions based on a variety of PDF sets.

For $J/\psi$ production, the data lie within the 9-point scale variation band, with central predictions slightly above the measurements. The spread in the various central predictions is comparable to the size of the PDF uncertainty band of CT18NLO for larger $W_{\gamma p}$. In the case of $\Upsilon$, which probes a larger scale than $J/\psi$, the 9-point scale variation band is narrower and our predictions show good agreement with precise data from ZEUS [18], as well as with large $W_{\gamma p}$ photoproduction data extracted from ultraperipheral collisions at the LHC [20]. Extension of our approach to NLLA, consideration of relativistic-$v^2$ corrections and more flexible GPD modelling is left for future work.

## Acknowledgements

I am supported by the Marie Skłodowska-Curie Action ("AutomOnium"), funded by the European Union under grant agreement No. 101204057. I also acknowledge support from the Agence Nationale de la Recherche (ANR) via the grant ANR-20-CE31-0015 ("PrecisOnium"), as well as the IDEX Paris-Saclay "Investissements d'Avenir" (ANR-11-IDEX-0003-01) through the GLUODYNAMICS project funded by the "P2IO LabEx (ANR-10-LABX-0038)", the French CNRS via the IN2P3 projects "GLUE@NLO" and "QCDFactorisation@NLO", and the COPIN-IN2P3 project #12-147 "kT factorisation and quarkonium production in the LHC era".

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