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General Lindblad equation for in-QGP quarkonium evolution

Aoumeur Daddi Hammou,[a,*] **Jean-Paul Blaizot,**[b] **Pol Bernard Gossiaux**[a] **and Thierry Gousset**[a]

[a] *SUBATECH, IMT Atlantique, Nantes Université, CNRS/IN2P3*
Nantes, 44307, France
[b] *Institut de physique théorique, Université Paris-Saclay, CNRS, CEA*
Gif-sur-Yvette 91191, France
E-mail: daddiham@subatech.in2p3.fr, jean-paul.blaizot@ipht.fr, gossiaux@subatech.in2p3.fr, gousset@subatech.in2p3.fr

Accurate modeling and understanding of quarkonium production in AA collisions requires a formalism that preserves the quantum properties of a microscopic $\bar{Q}Q$ systems while treating the interaction of such pairs with the QGP. The open quantum system approach has recently emerged as one of the most fruitful schemes to meet such requirements. However, the quantum master equations (QME) obtained so far in this approach and currently used to make predictions for the upsilon suppression at RHIC and LHC are derived assuming a strict ordering between the QGP temperature T and the energy gaps (ΔE) of the upsilon bound states. This limits their predictive power as the QGP cooling interpolates between the quantum Brownian regime (QBM), $T \gtrsim \Delta E$, and the quantum optical regime (QO), $\Delta E \gtrsim T$. In this paper, we derive and present a more general non-abelian quantum master equation of the Lindblad type, which does not suffer from these limitations and thus allows to faithfully describe the quantum evolution of the pairs during the whole QGP evolution. We also establish the contact with the QME previously obtained in the QBM and QO regimes and provide some concrete perspectives for its efficient solutions.

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*Speaker

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1. Introduction

Over the past decade, the study of quarkonia dynamics in a quark-gluon plasma (QGP) using the open quantum systems (OQS) formalism has enriched our understanding of various physical aspects of this important QGP probe, especially the quantum aspect. However, due to the dynamics of the QGP and its time-dependent temperature, in-QGP quarkonia dynamics cover two regimes of the OQS formalism: the quantum Brownian and optical regimes. Thus far, the quarkonia community has described these two regimes with two different Lindblad equations. Therefore, one must position oneself in a particular regime and use the appropriate equation, which limits the usefulness of the OQS formalism in the study of in-QGP quarkonia.

In order to enhance the success of OQS formalism in describing the out-of-equilibrium evolution of in-QGP quarkonia, it is of paramount importance to be able to encapsulate the physics of the two regimes, in addition to the physics of the transient regime introduced in [1], within a single and universal Lindblad equation. To this end, we identified the Universal Lindblad Equation (ULE) derived by Nathan and Rudner in [2] as a promising candidate for achieving this.

In this proceedings, we apply the aforementioned equation to the quarkonia-QGP system and present a set of coupled singlet-octet ULEs, along with some numerical illustrations.

2. QCD universal Lindblad equation

In this section, the key steps of deriving the QCD universal Lindblad equation (QCD-ULE) are briefly reviewed. Our starting point is the equation for the reduced density matrix of a heavy quark-antiquark pair, written in the interaction picture, [3]

$$\begin{aligned}\frac{d\rho^I(t)}{dt} = -\Bigg(&\int_{t_0}^{t} dt' \int_{\boldsymbol{x}\boldsymbol{x}'} \left[n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\,\rho^I(t')\right] \Delta^{>}(t-t',\boldsymbol{x}-\boldsymbol{x}') \\ &+ \int_{t_0}^{t} dt' \int_{\boldsymbol{x}\boldsymbol{x}'} \left[\rho^I(t')\, n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})\right] \Delta^{<}(t-t',\boldsymbol{x}-\boldsymbol{x}')\Bigg).\end{aligned} \tag{1}$$

In this equation, $n^a_{\boldsymbol{x}}$ is the color charge density of the heavy quarks,

$$n^a_{\boldsymbol{x}}(x) \equiv \delta\left(\boldsymbol{x}-\boldsymbol{x}_Q\right) t^a_Q - \delta\left(\boldsymbol{x}-\boldsymbol{x}_{\bar{Q}}\right) t^{a*}_{\bar{Q}}, \tag{2}$$

where the matrices t^a_Q and $t^{a*}_{\bar{Q}}$ are the color SU(N_c) algebras in the fundamental representation and its complex conjugate, and $\boldsymbol{x}_Q$ and $\boldsymbol{x}_{\bar{Q}}$ denote the position operators for the heavy quark and antiquark, respectively. The QGP correlators are defined as:

$$\begin{aligned} g_s^2 \left\langle A_0^a(t,\boldsymbol{x})\, A_0^b(t',\boldsymbol{x}')\right\rangle_0 &= \delta^{ab}\Delta^{>}(t-t',\boldsymbol{x}-\boldsymbol{x}'), \\ \Delta^{<}(t-t',\boldsymbol{x}-\boldsymbol{x}') &= \Delta^{>}(t'-t,\boldsymbol{x}'-\boldsymbol{x}), \end{aligned} \tag{3}$$

$\langle\cdots\rangle_0$ denotes the time average with respect to the QGP equilibrium density matrix [3].

Thus far, the resulting master equation obtained by putting Eq. (1) into a Lindblad form has been limited to describing either the quantum Brownian or optical regime, depending on the employed approximation, but not both. In order to overcome this limitation and derive a general Lindblad equation we adopt the scheme established by Nathan and Rudner in [2]. The main approximation

used is the Markovian approximation, which assumes that the medium relaxation time scale is much shorter than that of the system.

The first step of the derivation is based on implementing a mathematical trick at the level of the QGP thermal correlator $\Delta^{>}\left(t-t',\boldsymbol{x}-\boldsymbol{x}'\right)$. This trick consists in writing the QGP correlator as a convolution of the so called "jump correlators" $g\left(t,\boldsymbol{x}\right)$ according to [2]

$$\Delta^{>}\left(t-t',\boldsymbol{x}-\boldsymbol{x}'\right)=\int_{-\infty}^{+\infty}\mathrm{d}v\int_{\boldsymbol{y}}g\left(t-v,\boldsymbol{x}-\boldsymbol{y}\right)\,g\left(v-t',\boldsymbol{y}-\boldsymbol{x}'\right). \tag{4}$$

Moving to the Fourier space, the jump correlator can be defined as follows:[1]

$$g\left(t\right)=\frac{1}{2\pi}\int\mathrm{d}q_0\,e^{-iq_0t}\sqrt{\Delta^{>}\left(q_0\right)}, \tag{5}$$

where its complex conjugate satisfies $g^{*}\left(t\right)=g\left(-t\right)$. It is worth noting that the decay timescales of the QGP thermal correlator and of the jump correlator are of a comparable magnitude, i.e., $g\left(t-v,\boldsymbol{x}-\boldsymbol{y}\right)$ dies off as soon as $|t-v|>\tau_E$.

Upon substituting Eq. (4) into Eq. (1) and integrating the master equation from t_0 to t, we can exploit the fact that both $\Delta^{>}(t)$ and $g(t)$ are a strongly peaked function of t and exchange the time integration domains, thereby removing the time convolution present in Eq. (4). Consequently, this facilitates the construction of terms like $L^{\dagger}L\rho$ and $L\rho L^{\dagger}$, as required by the Lindblad form. For further details on these derivation steps, see [1].

Therefore, by following these steps, the application of the ULE to the quarkonia-QGP system yields the following Lindblad equation:

$$\frac{d\rho\left(t\right)}{dt}=-i\left[H_{Q\bar{Q}}+\Lambda\left(t\right),\rho\left(t\right)\right]+\int_{\boldsymbol{y}}\left(L\left(t,\boldsymbol{y}\right)\rho\left(t\right)L^{\dagger}\left(t,\boldsymbol{y}\right)-\frac{1}{2}\left\{L^{\dagger}\left(t,\boldsymbol{y}\right)L\left(t,\boldsymbol{y}\right),\rho\left(t\right)\right\}\right), \tag{6}$$

where $\rho(t)$ is the density matrix of the reduced system, i.e. the in-QGP quarkonia, and $H_{Q\bar{Q}}$ is system Hamiltonian. The Lindblad operator is given by

$$L\left(t,\boldsymbol{y}\right)=\int_{-\infty}^{+\infty}\mathrm{d}v\int_{\boldsymbol{x}}g\left(t-v,\boldsymbol{y}-\boldsymbol{x}\right)\,U\left(v,t\right)\,n_{\boldsymbol{x}}^{a}\,U^{\dagger}\left(v,t\right), \tag{7}$$

and the Lamb-shift term $\Lambda\left(t\right)$ is defined by

$$\begin{aligned}\Lambda\left(t\right)=&\frac{1}{2i}\int_{-\infty}^{+\infty}\mathrm{d}v\int_{-\infty}^{+\infty}\mathrm{d}v'\,\mathrm{sign}\left(v-v'\right)\int_{\boldsymbol{y}\boldsymbol{x}\boldsymbol{x}'}g\left(v-t,\boldsymbol{x}-\boldsymbol{y}\right)g\left(t-v',\boldsymbol{y}-\boldsymbol{x}'\right)\\&\times U\left(v,t\right)\,n_{\boldsymbol{x}}^{a}\,U^{\dagger}\left(v,t\right)\,U\left(v',t\right)\,n_{\boldsymbol{x}'}^{a}\,U^{\dagger}\left(v',t\right),\end{aligned} \tag{8}$$

with $U\left(v,t\right)=e^{-iH_Q\left(t-v\right)}$ being the evolution operator.

The universality and regime independence of Eq. (6) can be shown through the demonstration that this equation can be approximated by the quantum Brownian or quantum optical master equations upon the implementation of the appropriate approximation for each regime, see [1] for further details.

Upon tracing out the center-of-mass and projecting on the color and energy bases, following closely [4], we obtain the following set of coupled singlet-octet ULEs

[1] Notice that the coupling constant g_s is implicitly included in the definition of the jump correlator function.

$$\frac{d\tilde{\rho}\,(t)}{dt} = -\,i\left[H_Q + \tilde{\Lambda}, \tilde{\rho}\,(t)\right] + \int_z \sum_{n=0}^{3} \gamma_n \Bigg(\tilde{L}_n\,(z)\,\tilde{\rho}\,(t)\,\tilde{L}_n^{\dagger}\,(z) - \frac{1}{2}\left\{\tilde{L}_n^{\dagger}\,(z)\,\tilde{L}_n\,(z)\,, \tilde{\rho}\,(t)\right\}\Bigg), \tag{9}$$

with

$$\tilde{\rho}\,(t) = \begin{pmatrix} \tilde{\rho}_{\mathrm{s}}\,(t) & 0 \\ 0 & \tilde{\rho}_o\,(t) \end{pmatrix}, \qquad \tilde{H}_Q = \begin{pmatrix} \tilde{H}_s & 0 \\ 0 & \tilde{H}_o \end{pmatrix}, \qquad \tilde{\Lambda}\,(t) = \begin{pmatrix} \tilde{\Lambda}_s & 0 \\ 0 & \tilde{\Lambda}_o \end{pmatrix}, \tag{10}$$

and

$$\gamma_0 = \gamma_1 = \begin{pmatrix} N_c^2 - 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \gamma_2 = \gamma_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{11}$$

the tildes refer to the operators after tracing out the center-of-mass.

The Lindblad operators in this equation can be expressed as

$$\tilde{L}_0 = \int_{\boldsymbol{k}} \sum_n |n\rangle\langle n|\tilde{L}_0|\boldsymbol{k}\rangle\langle\boldsymbol{k}| \equiv \int_{\boldsymbol{k}} \sum_n \tilde{L}_0\,(n,\boldsymbol{k})\,|n\rangle\langle\boldsymbol{k}|, \tag{12}$$

$$\tilde{L}_1 = \int_{\boldsymbol{k}} \sum_n |\boldsymbol{k}\rangle\langle\boldsymbol{k}|\tilde{L}_1|n\rangle\langle n| \equiv \int_{\boldsymbol{k}} \sum_n \tilde{L}_1\,(\boldsymbol{k},n)\,|\boldsymbol{k}\rangle\langle n|, \tag{13}$$

$$\tilde{L}_2 = \int_{\boldsymbol{k},\boldsymbol{k}'} |\boldsymbol{k}\rangle\langle\boldsymbol{k}|\tilde{L}_2|\boldsymbol{k}'\rangle\langle\boldsymbol{k}'| \equiv \int_{\boldsymbol{k},\boldsymbol{k}'} \tilde{L}_2\,(\boldsymbol{k},\boldsymbol{k}')\,|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|, \tag{14}$$

$$\tilde{L}_3 = \int_{\boldsymbol{k},\boldsymbol{k}'} |\boldsymbol{k}\rangle\langle\boldsymbol{k}|\tilde{L}_3|\boldsymbol{k}'\rangle\langle\boldsymbol{k}'| \equiv \int_{\boldsymbol{k},\boldsymbol{k}'} \tilde{L}_3\,(\boldsymbol{k},\boldsymbol{k}')\,|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|, \tag{15}$$

where the expressions of the matrix elements $\tilde{L}_0\,(n,\boldsymbol{k})$, $\tilde{L}_1\,(\boldsymbol{k},n)$, and $\tilde{L}_{2,3}\,(\boldsymbol{k},\boldsymbol{k}')$ are given by

$$\tilde{L}_0\,(n,\boldsymbol{k}) = -i\sqrt{\frac{1}{2N_c}} \int_{\boldsymbol{q}} e^{i\boldsymbol{q}\cdot\boldsymbol{z}}\, g\left(\frac{k^2}{M} - E_n, \boldsymbol{q}\right) \langle n|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|\boldsymbol{k}\rangle \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{16}$$

$$\tilde{L}_1\,(n,\boldsymbol{k}) = -i\sqrt{\frac{1}{2N_c}} \int_{\boldsymbol{q}} e^{i\boldsymbol{q}\cdot\boldsymbol{z}}\, g\left(E_n - \frac{k^2}{M}, \boldsymbol{q}\right) \langle\boldsymbol{k}|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|n\rangle \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{17}$$

$$\tilde{L}_2\,(\boldsymbol{k},\boldsymbol{k}') = -i\sqrt{\frac{N_c^2 - 4}{4N_c}} \int_{\boldsymbol{q}} e^{i\boldsymbol{q}\cdot\boldsymbol{z}}\, g\left(\frac{k'^2}{M} - \frac{k^2}{M}, \boldsymbol{q}\right) \langle\boldsymbol{k}|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|\boldsymbol{k}'\rangle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{18}$$

$$\tilde{L}_3\,(\boldsymbol{k},\boldsymbol{k}') = \sqrt{\frac{N_c}{4}} \int_{\boldsymbol{q}} e^{i\boldsymbol{q}\cdot\boldsymbol{z}}\, g\left(\frac{k'^2}{M} - \frac{k^2}{M}, \boldsymbol{q}\right) \langle\boldsymbol{k}|C_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|\boldsymbol{k}'\rangle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{19}$$

with

$$S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}} \equiv 2\sin\left(\frac{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}{2}\right), \qquad C_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}} \equiv 2\cos\left(\frac{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}{2}\right), \tag{20}$$

where $\hat{\boldsymbol{s}}$ is the relative coordinate operator between the heavy quark pair.

Similarly, the singlet and octet Lamb-shift terms can be written as

$$\begin{aligned}\tilde{\Lambda}_s &= \sum_{n,n'} |n\rangle\langle n|\tilde{\Lambda}_s|n'\rangle\langle n'| \equiv \sum_{n,n'} \tilde{\Lambda}_s\,(n,n')\,|n\rangle\langle n'|,\\ \tilde{\Lambda}_o &= \int_{\boldsymbol{k}\boldsymbol{k}'} |\boldsymbol{k}\rangle\langle\boldsymbol{k}|\tilde{\Lambda}_o|\boldsymbol{k}'\rangle\langle\boldsymbol{k}'| \equiv \int_{\boldsymbol{k}\boldsymbol{k}'} \tilde{\Lambda}_o\,(\boldsymbol{k},\boldsymbol{k}')\,|\boldsymbol{k}\rangle\langle\boldsymbol{k}'|,\end{aligned} \tag{21}$$

with

$$\tilde{\Lambda}_s\,(n,n') = -C_F \int_{\boldsymbol{q},\boldsymbol{k}} \mathrm{P}\int_{q_0} \frac{g\,(q_0+E_n-E_{n'},\boldsymbol{q})\,g\,(q_0,\boldsymbol{q})}{q_0-E_{n'}+\frac{k^2}{M}} \langle n\left|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}\,|\boldsymbol{k}\rangle\langle\boldsymbol{k}|\,S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}'}\right|n'\rangle, \tag{22}$$

$$\begin{aligned}\tilde{\Lambda}_o\,(\boldsymbol{k},\boldsymbol{k}') = &-\frac{1}{4}\int_{\boldsymbol{q}\boldsymbol{p}} \mathrm{P}\int_{q_0} \frac{g\left(q_0-\frac{k'^2}{M}+\frac{k^2}{M},\boldsymbol{q}\right)g\,(q_0,\boldsymbol{q})}{q_0-\frac{k'^2}{M}+\frac{p^2}{M}}\\ &\times\left(\frac{N_c^2-4}{N_c}\langle\boldsymbol{k}|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}'}|\boldsymbol{k}'\rangle + N_c\langle\boldsymbol{k}|C_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|C_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}'}|\boldsymbol{k}'\rangle\right)\\ &-\frac{1}{2N_c}\sum_m\int_{\boldsymbol{q}} \mathrm{P}\int_{q_0} \frac{g\left(q_0-\frac{k'^2}{M}+\frac{k^2}{M},\boldsymbol{q}\right)g\,(q_0,\boldsymbol{q})}{q_0-\frac{k'^2}{M}+E_m}\langle\boldsymbol{k}|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}}|m\rangle\langle m|S_{\boldsymbol{q}\cdot\hat{\boldsymbol{s}}'}|\boldsymbol{k}'\rangle,\end{aligned} \tag{23}$$

where the symbol “P" denotes the Cauchy principal value.

The operators $\tilde{L}_0$ and $\tilde{L}_1$ describe the transitions between the singlet states of energy E_n and the octet states of energy k^2/M, i.e., they describe the dissociation and regeneration. Similarly, the operators $\tilde{L}_{2,3}$ describe the transitions among octet states, as reflected by the argument of the jump correlator functions.

3. Illustrations

In this section we shed some light on the singlet-octet ULEs through computing some physical observables of particular interest to quarkonia phenomenology. In particular, the dissociation of in-QGP quarkonia represents a pivotal component of their dynamic behavior. Consequently, it is important to provide an accurate description of this phenomenon in order to ensure a comprehensive understanding of this probe. The focus in this section will be on the singlet states decay through singlet-to-octet transitions.

For simplicity, we consider a one dimensional dynamics and use Pöschl-Teller potential in the singlet sector. In addition, the jump correlator function $g(q_0,\boldsymbol{q})$ is computed in the Hard Thermal Loop (HTL) limit.

Figure 1 presents the decay rates of the singlet states that result from singlet-to-octet transitions. This rates are given by [2]

[2] In this section, the coupling constant g_s is extracted from the jump correlator function definition and is written explicitly.

$$\langle m|\Gamma_s|n\rangle = \frac{4\pi\alpha_S C_F}{\hbar c}\sum_k \int_z \int_{q'} e^{-iq'z} g\left(E_m^{(s)} - E_k^{(o)}, q'\right)\langle k|S_{q's}|m\rangle^\star \times \int_q e^{iqz} g\left(E_n^{(s)} - E_k^{(o)}, q\right)\langle k|S_{qs}|n\rangle, \tag{24}$$

where the rates are computed considering the cases of transitions with and without energy gap.

The usual trends are observed, i.e., increase with both T and n. The equivalent quantities in the "no energy-gap" approximation are also displayed. In the case $m_D = T$, one observes a a noticeable reduction resulting from the energy gaps, especially for low T and the deepest bound states. In the case $m_D = 2T$, the coupling with the medium is stronger and the dissociation is more efficient. Consequently, in this case, the energy gap plays a negligible role.

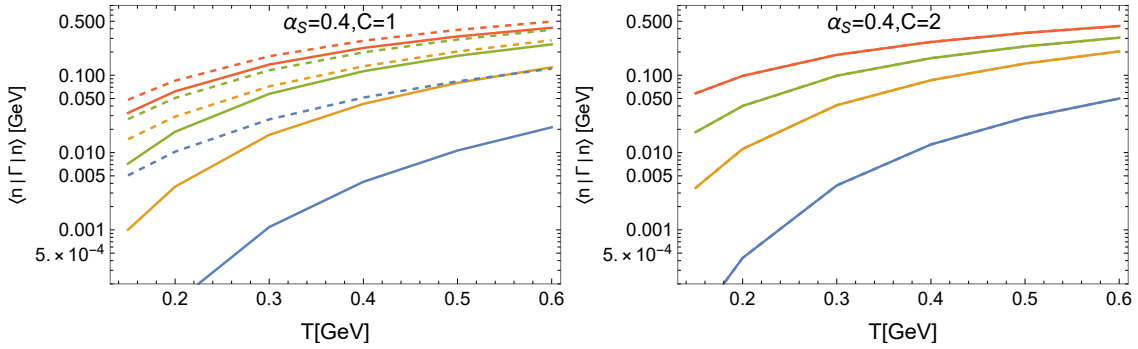

Figure 1: Left: Plain curves illustrate Γ of eq (24) for $m = n$, and $m_D = T$. We consider the quarkonium states $n = 1$ (blue), $n = 2$ (orange), $n = 3$ (green) , and $n = 4$ (red). Dashed curves represent the same quantities with no energy gap taken into account. Right: same for $m_D = 2T$.

In order to gain further insight into the melting of in-QGP quarkonia bound states, we compute the spectral normalized density defined as follows

$$\rho_s(E) := \frac{1}{\pi}\Im\left(\mathrm{tr}\frac{1}{\hat{H}_s^{\mathrm{eff}} - E\hat{I}}\right). \tag{25}$$

where $\hat{H}_s^{\mathrm{eff}}$ is the effective Hamiltonian given by

$$\hat{H}_s^{\mathrm{eff}} := \hat{H}_s - \frac{i}{2}\hat{\Gamma}_s, \tag{26}$$

with $\hat{H}_s$ being the singlet component of system Hamiltonian and $\hat{\Gamma}_s$ is defined as follows:

$$\hat{\Gamma}_s := (N_c^2 - 1)\int_z \hat{L}_1^\dagger(z)\,\hat{L}_1(z) \tag{27}$$

This quantity is illustrated on fig. 2, showing a continuous transition from a “multiple peaks" to a broad structure passing from low to high temperature. Notice that the $\psi(2S)$ peak is already "melted" for T a low as 0.15 GeV while the $\psi(1S)$ peak survives for temperatures as high as 0.6 GeV. It is worth to note that for the HTL description of the QGP, the off-diagonal elements of $\hat{\Gamma}_s$ in

Eq. (24) are found to play a very little role in the peak broadening and for the spectral function in general.

It is evident that the evaluation of the genuine spectral density is a considerably more complex process, as it should take into account all couplings between the singlet and the octet sectors.

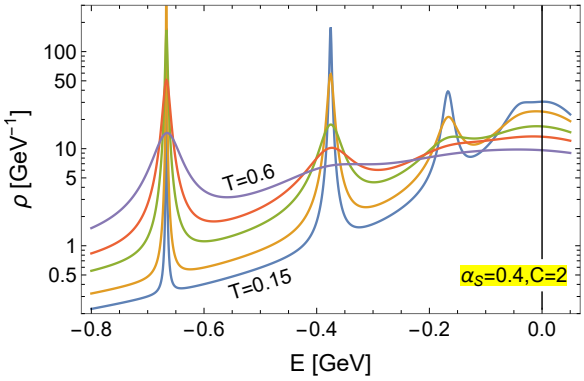

Figure 2: Spectral density according to eq. (25) for $T = 0.15$, 0.2, 0.3, 0.4 and 0.6 GeV.

4. Conclusions

The objective of this study was to establish a unified quantum description of the in-QGP quarkonia dynamics by deriving a general Lindblad equation. The latter is designed to be regime-independent and capable of capturing the physics of the quantum, Brownian, and optical regimes. To this end, the Universal Lindblad Equation (ULE), as derived by Nathan and Rudner in [2], was applied to the QGP-quarkonia system. Doing so, we succeeded to obtain a set of coupled singlet and octet ULEs. A presentation of preliminary numerical results was also made, with a focus on the singlet states decay and associated observables.

The Lindblad form of our derived singlet-octet ULEs guarantees the preservation of positivity, a crucial prerequisite for attaining reliable physical outcomes. Furthermore, the Lindblad form facilitates the implementation of certain numerical efficient methods, including the quantum trajectory method. We plan to employ this method to solve numerically the singlet-octet ULEs thereby enabling the exploration of observables that are of particular relevance to quarkonia phenomenology. Furthermore, the aforementioned equations facilitate the investigation of the transient regime between the quantum Brownian and optical regimes, a regime that has hitherto remained unexplored.

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