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Recent measurements in the semi-leptonic D decays at BESIII

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Using 20.3 fb^{-1} and 7.33 fb^{-1} of e^+e^- collision data collected at the center-of-mass energy of 3.773 and 4.128 – 4.226 GeV with the BESIII detector, the non-perturbative nature of QCD in the charm sector is investigated. The decays of $D^+ \to f_0(500)\mu^+\nu_\mu$, $D \to b_1(1235)e^+\nu_e$, $D \to K_1(1270)\mu^+\nu_\mu$ and $D^+ \to K^0_S\pi^0\mu^+\nu_\mu$ are observed for the first time. Based on the amplitude analyses of $D_{(s)} \to hh\ell^+\nu_\ell$ and $hhh\ell^+\nu_\ell$ processes, the scalar($a_0(980)$, $f_0(500)$ and $f_0(980)$), vector($K^*(892)$, $\phi(1020)$, $\rho(770)$), axial-vector($K_1(1270)$, $b_1(1385)$) particles are studied. The measurements of $D \to a^0(980)$, $D \to f_0(500)$, $D \to K^*(892)$, $D \to \rho(770)$, and $D_s \to \phi(1020)$ form factors are presented.

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1. Introduction and formalism

In the standard model (SM), the strong and weak interactions in semileptonic D decays can be well separated, as shown in Fig. 1. The decay amplitude is described as the vector product of the hadronic current and the leptonic current. Using the formalism of $K_{\ell 4}$ decay, the $D_{\ell 4}$ decay can be expressed as [1–6]:

$$\frac{d^5\Gamma}{dm^2 dq^2 d\cos\theta_K d\cos\theta_\ell d\chi} = \frac{G_F^2 |V_{cs}|^2}{(4\pi)^6 m_D^3} \lambda \beta_m \beta_\ell \Big(\mathcal{I}_1 + \mathcal{I}_2 \cos 2\theta_\ell + \mathcal{I}_3 \sin^2\theta_\ell \cos 2\chi + \mathcal{I}_4 \sin 2\theta_\ell \cos\chi + \mathcal{I}_5 \sin\theta_\ell \cos\chi + \mathcal{I}_6 \cos\theta_\ell + \mathcal{I}_7 \sin\theta_\ell \sin\chi + \mathcal{I}_8 \sin 2\theta_\ell \sin\chi + \mathcal{I}_9 \sin^2\theta_\ell \sin 2\chi \Big), \tag{1}$$

using $D^+ \to \bar{K}\pi\ell^+\nu_\ell$ as an example. The prefactor contains Fermi coupling constant G_F, CKM matrix element V_{cs}, and the mass of D (m_D). Other parameters include $\lambda = p_{K\pi} m_D$, with $p_{K\pi}$ the $K\pi$ momentum in the D rest frame, $\beta_m = 2p^*/m$, where p^* is the kaon momentum in the $K\pi$ rest frame and m the $K\pi$ invariant mass, and β_ℓ is defined analogously to β_m but for the $\ell\nu_\ell$ system.

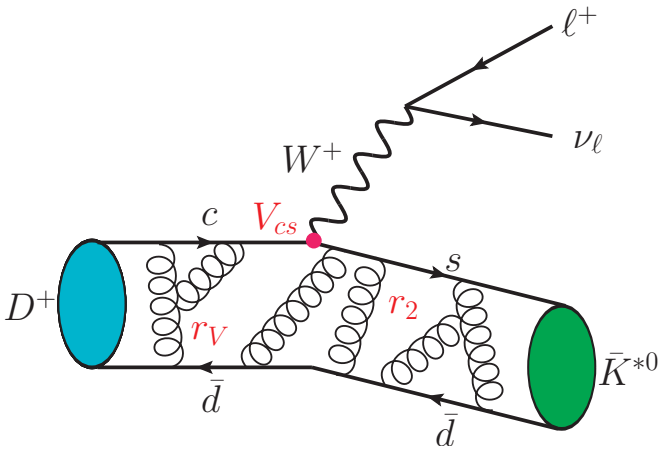

Figure 1: The Feynman diagram of the decay process $D^+ \to \bar{K}^*(892)^0 \ell^+ \nu_\ell$.

The angular coefficients can be expanded in terms of four form factors $\mathcal{F}_{1,2,3,4}$ as

$$\begin{aligned}
\mathcal{I}_1 =& \frac{\beta_\ell}{4}\left[\left(1+\frac{m_\ell^2}{q^2}\right)|\mathcal{F}_1|^2+\frac{3}{2}\left(1+\frac{m_\ell^2}{3q^2}\right)\sin^2\theta_K\left(|\mathcal{F}_2|^2+|\mathcal{F}_3|^2\right)+\frac{2m_\ell^2}{q^2}|\mathcal{F}_4|^2\right],\\
\mathcal{I}_2 =& -\frac{\beta_\ell^2}{4}\left[|\mathcal{F}_1|^2-\frac{1}{2}\sin^2\theta_K\left(|\mathcal{F}_2|^2+|\mathcal{F}_3|^2\right)\right],\\
\mathcal{I}_3 =& -\frac{\beta_\ell^2}{4}\left[\left(|\mathcal{F}_2|^2-|\mathcal{F}_3|^2\right)\right]\sin^2\theta_K,\\
\mathcal{I}_4 =& \frac{\beta_\ell^2}{2}\,\mathrm{Re}\left(\mathcal{F}_1\mathcal{F}_2^*\right)\sin\theta_K,\\
\mathcal{I}_5 =& \beta_\ell\,\mathrm{Re}\left[\mathcal{F}_1\mathcal{F}_3^*+\frac{m_\ell^2}{q^2}\mathcal{F}_4\mathcal{F}_2^*\right]\sin\theta_K,\\
\mathcal{I}_6 =& \beta_\ell\,\mathrm{Re}\left[\mathcal{F}_2\mathcal{F}_3^*\sin^2\theta_K-\frac{m_\ell^2}{q^2}\mathcal{F}_1\mathcal{F}_4^*\right],\\
\mathcal{I}_7 =& \beta_\ell\,\mathrm{Im}\left[\left(\mathcal{F}_1\mathcal{F}_2^*\right)+\frac{m_\ell^2}{q^2}\mathcal{F}_4\mathcal{F}_3^*\right]\sin\theta_K,\\
\mathcal{I}_8 =& \frac{\beta_\ell^2}{2}\,\mathrm{Im}\left(\mathcal{F}_1\mathcal{F}_3^*\right)\sin\theta_K,\\
\mathcal{I}_9 =& -\frac{\beta_\ell^2}{2}\,\mathrm{Im}\left(\mathcal{F}_2\mathcal{F}_3^*\right)\sin^2\theta_K.
\end{aligned} \tag{2}$$

The form factors $\mathcal{F}_{1,2,3,4}$ can be expanded into partial waves to show their explicit dependence on θ_K. If only S- and P-waves are kept, one obtains:

$$\begin{aligned}
\mathcal{F}_1 &= \sum_{l=0}^{\infty}\mathcal{F}_{1,l}\,P_l\left(\cos\theta_K\right)=\mathcal{F}_{10}+\mathcal{F}_{11}\cos\theta_K,\\
\mathcal{F}_2 &= \sum_{l=1}^{\infty}\frac{1}{\sqrt{l(l+1)}}\mathcal{F}_{2,l}\,\frac{d}{d\cos\theta_K}P_l\left(\cos\theta_K\right)=\frac{1}{\sqrt{2}}\mathcal{F}_{21},\\
\mathcal{F}_3 &= \sum_{l=1}^{\infty}\frac{1}{\sqrt{l(l+1)}}\mathcal{F}_{3,l}\,\frac{d}{d\cos\theta_K}P_l\left(\cos\theta_K\right)=\frac{1}{\sqrt{2}}\mathcal{F}_{31},\\
\mathcal{F}_4 &= \sum_{l=0}^{\infty}\mathcal{F}_{4,l}\,P_l\left(\cos\theta_K\right)=\mathcal{F}_{40}+\mathcal{F}_{41}\cos\theta_K.
\end{aligned} \tag{3}$$

But for five body decay, the general formalism is hard to extract, and the decay amplitude of $D\to\bar{K}_1e^+\nu$ is constructed as the product of $D\to\bar{K}_1W^*$, $\bar{K}_1$ decay and W^* decay as [7]:

$$A=(V-A)^{\mu\eta}\cdot\Big[\sum_{\lambda_W}\epsilon^*(\lambda_W)_\mu\epsilon(\lambda_W)_\rho\Big]\cdot\Big[\sum_{\lambda_{K_1}}\epsilon^*(\lambda_{K_1})_\eta\epsilon(\lambda_{K_1})_\sigma\Big]\cdot\mathcal{R}_{\bar{K}_1}\cdot J^\sigma\cdot L^\rho=H_\rho L^\rho, \tag{4}$$

where $L_\mu=\bar{u}_\nu\gamma_\mu(1-\gamma_5)v_l$ is the leptonic current of $W^*\to e^+\nu_e$ and the hadronic part H_ρ is given

as

$$H_\rho = \mathcal{R}_{\bar{K}_1} \cdot (V-A)^{\mu\eta} g_{\mu\rho}(g_{\eta\sigma} - \frac{p_\eta^{K_1} p_\sigma^{K_1}}{M_{K_1}^2}) J^\sigma = \mathcal{R}_{\bar{K}_1} \cdot \left(\frac{2iA(q^2)}{m_D - M_{K_1}} \epsilon^{\rho J p^D p^{K_1}} \right.$$
$$\left. -V_1(q^2)(M_{K_1} - m_D) \left[\frac{(J \cdot p^{K_1})}{M_{\bar{K}_1}^2} p_{K_1}^\rho - J^\rho \right] - \frac{V_2(q^2)}{m_D - M_{K_1}} \left[\frac{(J \cdot p^{K_1})(p^{K_1} \cdot q)}{M_{\bar{K}_1}^2} - (J \cdot q) \right] (p_{K_1} + p_D)^\rho \right), \tag{5}$$

where $\epsilon(\lambda_{W/\bar{K}_1})$ is the polarization vector of W^* and $\bar{K}_1$. The vector and axial-vector currents are given as

$$V^\mu = V^{\mu\eta} \epsilon^*(\lambda_{K_1})_\eta = \left(-2M_{K_1} V_0(q^2) \frac{q^\eta q^\mu}{q^2} - (m_D - M_{K_1}) V_1(q^2) \left[g^{\mu\eta} - \frac{q^\eta q^\mu}{q^2} \right] \right.$$
$$\left. + V_2(q^2) \left(\frac{q^\eta}{m_D - M_{K_1}} \right) \left[(p_D + p_{K_1})^\mu - \frac{m_D^2 - M_{K_1}^2}{q^2} q^\mu \right] \right) \epsilon^*(\lambda_{K_1})_\eta, \tag{6}$$
$$A^\mu = A^{\mu\eta} \epsilon^*(\lambda_{K_1})_\eta = \left(-\frac{2iA(q^2)}{m_D - M_{K_1}} \epsilon^{\mu\eta p^D p^{K_1}} \right) \epsilon^*(\lambda_{K_1})_\eta.$$

The current J^μ of decay $\bar{K}_1 \to K^- \pi\pi$ is constructed following Ref. [8]. Potential intermediate states $\bar{K}^*(892) \to K^-\pi$, $\rho(770) \to \pi\pi$, $\omega \to \pi\pi$, and $(\bar{K}\pi)_{S\text{-wave}} \to \bar{K}\pi$ are considered.

2. BESIII data, detector and analysis method

The BESIII detector [9] records symmetric e^+e^- collisions provided by the BEPCII storage ring [10] in the center-of-mass energy ($\sqrt{s}$) range from 1.84 to 4.95 GeV, with a peak luminosity of 1.1×10^{33} cm^{-2}s^{-1} achieved at $\sqrt{s} = 3.773$ GeV. BESIII has collected large data samples in these energy regions [11–13]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The magnetic field was 0.9 T in 2012, which affects 11% of the total J/ψ data. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the dE/dx resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution in the plastic scintillator TOF barrel region is 68 ps, while that in the end cap region was 110 ps. The end cap TOF system was upgraded in 2015 using multigap resistive plate chamber technology, providing a time resolution of 60 ps [14].

During the new data taking period, we recorded an integrated luminosity of 20.3 fb^{-1} at the $\psi(3770)$, yielding samples of $\bar{D}^0 \sim 1.69 \times 10^7$ and $D^- \sim 1.10 \times 10^7$. In addition, data collected for $e^+e^- \to D_s^\pm D_s^{*\mp}$ at $\sqrt{s}$ = 4.128-4.226 GeV provide about $D_s^- \sim 0.8 \times 10^6$ events, and $e^+e^- \to D_s^{*+} D_s^{*-}$ at $\sqrt{s}$ = 4.237-4.669 GeV contribute a further $D_s^- 0.12 \times 10^6$ events. Figure 2 shows the BESIII data.

Due to the e^+e^- annihilation data near threshold, the double-tag method is employed, benefiting from a clean environment and low background ratio. Undetectable neutrinos are extracted as

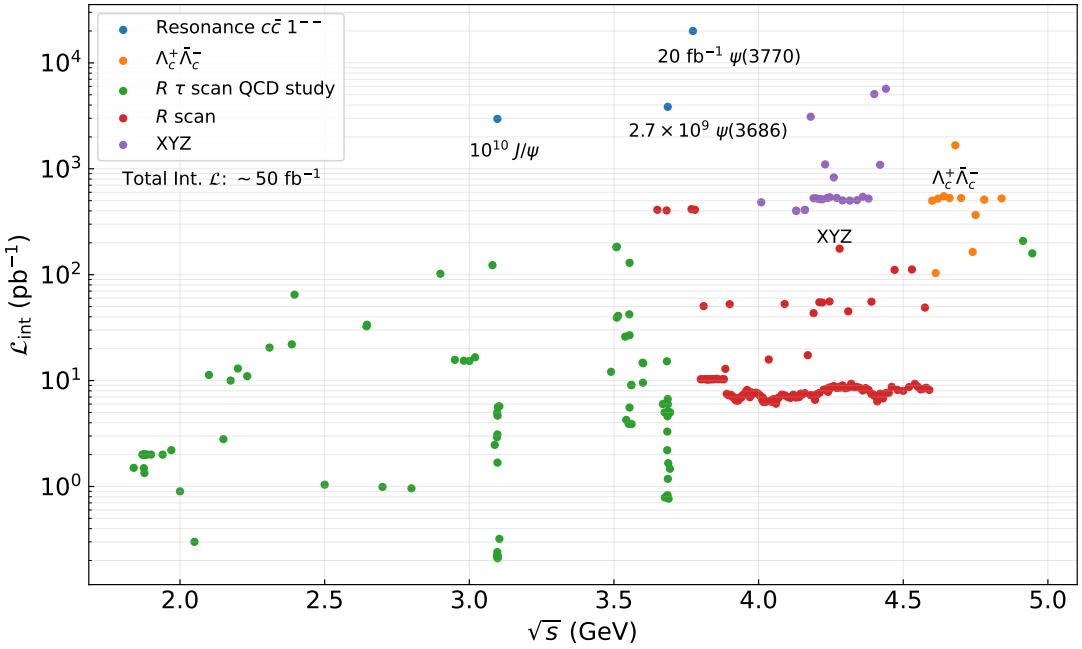

Figure 2: The BESIII data.

$U_{\rm miss} = E_{\rm miss} - |\vec{p}_{\rm miss}|$ and $M^2_{\rm miss} = E^2_{\rm miss} - |\vec{p}_{\rm miss}|^2$ to determine (semi-)leptonic signals, the branching fraction is calculated as

$$\mathcal{B} = \frac{N_{\rm DT}}{N_{\rm ST}\epsilon_{\rm DT}/\epsilon_{\rm ST}}, \tag{7}$$

and systematic uncertainties on the ST mostly cancel out.

3. Recent results

With the world's largest $D\bar{D}$ samples near threshold, which contain the new $D\bar{D}$ data taken during 2022-2024, we obtain a coherent picture of charm semileptonic transitions. First observations of $D^+ \to f_0(500)\mu^+\nu_\mu$ [21], $D \to b_1(1235)\ell^+\nu_\ell$ [26], $D \to K_1(1270)\mu^+\nu_\mu$ [25] and $D^+ \to K^0_S\pi^0\mu^+\nu_\mu$ [18] expand the accessible hadronic spectrum and enable amplitude level studies of scalar, vector and axial-vector dynamics. High statistics measurements of $D \to \bar{K}^*(892)\ell^+\nu_\ell$ [15–18], $D^0 \to \rho(770)e^+\nu_e$ [22] and $D_s^+ \to \phi(1020)\mu^+\nu_\mu$ [20] yield precise branching fractions and helicity form factor ratios r_V and r_2, and determine $f_+(0)\,|V_{cq}|$ with improved accuracy. No deviation from lepton flavor universality is seen within current uncertainties. An angular moment, five dimensional test of CP symmetry in $D^+ \to K^0_S\pi^0\ell^+\nu_\ell$ finds no significant CP violation and sets mode dependent limits using $\langle A_i\rangle$ and $\langle S_i\rangle$ observables, see Fig. 4. Partial wave analyses confirm sizable S-P interference in $K\pi$ and $\pi\pi$ systems, consistent with the $D_{\ell 4}$ formalism, and the $D \to K_1(\to K\pi\pi)\ell\nu$ framework allows a first constraint on the axial channel with controlled substructure inputs. Exploiting the form factor ratios r_V and r_A together with decay rate information, we place a data driven constraint on the $K_1(1270)$-$K_1(1400)$ mixing angle θ_{K_1}, see Fig. 3. These results are listed in Table 1

These results sharpen inputs for lattice and dispersive fits, reduce hadronic uncertainties in $|V_{cs}|$ and $|V_{cd}|$ extractions, and provide new benchmarks for light meson dynamics in the nonperturbative regime. With the ongoing 20.3 fb^{-1} at $\psi(3770)$ and 7.33 fb^{-1} at 4.128-4.226 GeV, we will extend to fully differential determinations in (q^2, m, Ω), perform z-expansion, model independent form factor fits, refine amplitude analyses for axial-vector modes, and probe CP-odd observables linked to $\mathcal{I}_{7,8,9}$. The BESIII threshold program will thus continue to deliver precision charm semileptonic physics and stringent tests of the Standard Model.

Table 1: Summary of BESIII recent semileptonic decays of D and D_s mesons: measured branching fractions and form-factor parameters. Vector modes report (r_V, r_2); scalar modes report $f_+(0)|V_{cq}|$; axial-vector modes report (r_V, r_A). Unless stated otherwise, quoted $\mathcal{B}$ values include the relevant subdecay branching fractions.

Channel	$E_{\rm cm}$ (GeV)	$\mathcal{L}$ (fb^{-1})	Process	$\mathcal{B}$(%)	form factor parameters
Vector modes (K^*, ρ, ϕ) — report (r_V, r_2)					
$D^0 \to \bar{K}^0\pi^- e^+\nu_e$ [15]	3.773	7.93	$D^0\bar{D}^0$	$1.444 \pm 0.022_{\rm stat} \pm 0.024_{\rm syst}$	...
$D^0 \to K^*(892)^- e^+\nu_e$ [15]	3.773	7.93	$D^0\bar{D}^0$	$2.039 \pm 0.032_{\rm stat} \pm 0.034_{\rm syst}$	$r_V = 1.48 \pm 0.05_{\rm stat} \pm 0.02_{\rm syst}$ $r_2 = 0.70 \pm 0.04_{\rm stat} \pm 0.02_{\rm syst}$
$D^0 \to \bar{K}^0\pi^- \mu^+\nu_\mu$ [16]	3.773	7.93	$D^0\bar{D}^0$	$1.373 \pm 0.020_{\rm stat} \pm 0.023_{\rm syst}$	...
$D^0 \to K^*(892)^- \mu^+\nu_\mu$ [16]	3.773	7.93	$D^0\bar{D}^0$	$1.948 \pm 0.033_{\rm stat} \pm 0.036_{\rm syst}$	$r_V = 1.37 \pm 0.09_{\rm stat} \pm 0.03_{\rm syst}$ $r_2 = 0.76 \pm 0.06_{\rm stat} \pm 0.02_{\rm syst}$
$D^0 \to K^-\pi^0 \mu^+\nu_\mu$ [17]	3.773	7.93	$D^0\bar{D}^0$	$0.773 \pm 0.014_{\rm stat} \pm 0.012_{\rm syst}$	...
$D^0 \to K^*(892)^- \mu^+\nu_\mu$ [17]	3.773	7.93	$D^0\bar{D}^0$	$2.073 \pm 0.039_{\rm stat} \pm 0.032_{\rm syst}$	$r_V = 1.46 \pm 0.11_{\rm stat} \pm 0.04_{\rm syst}$ $r_2 = 0.71 \pm 0.08_{\rm stat} \pm 0.03_{\rm syst}$
$D^+ \to K^0_S\pi^0 e^+\nu_e$ [18]	3.773	20.3	D^+D^-	$0.943 \pm 0.012_{\rm stat} \pm 0.010_{\rm syst}$	...
$D^+ \to K^0_S\pi^0 \mu^+\nu_\mu$ [18]	3.773	20.3	D^+D^-	$0.896 \pm 0.017_{\rm stat} \pm 0.008_{\rm syst}$	...
$D^+ \to \bar{K}^*(892)^0 e^+\nu_e$ [18]	3.773	20.3	D^+D^-	$5.29 \pm 0.07_{\rm stat} \pm 0.06_{\rm syst}$	$r_V = 1.42 \pm 0.03_{\rm stat} \pm 0.02_{\rm syst}$
$D^+ \to \bar{K}^*(892)^0 \mu^+\nu_\mu$ [18]	3.773	20.3	D^+D^-	$4.99 \pm 0.10_{\rm stat} \pm 0.05_{\rm syst}$	$r_2 = 0.71 \pm 0.08_{\rm stat} \pm 0.03_{\rm syst}$
$D^0 \to \rho^-(770) e^+\nu_e$ [19]	3.773	7.93	$D^0\bar{D}^0$	$1.439 \pm 0.033_{\rm stat} \pm 0.027_{\rm syst}$	$r_V = 1.548 \pm 0.079_{\rm stat} \pm 0.041_{\rm syst}$ $r_2 = 0.823 \pm 0.056_{\rm stat} \pm 0.026_{\rm syst}$
$D_s^+ \to \phi(1020) \mu^+\nu_\mu$ [20]	4.128-4.226	7.33	$D_s^+D_s^{*-}$	$2.25 \pm 0.09_{\rm stat} \pm 0.07_{\rm syst}$	$r_V = 1.58 \pm 0.17_{\rm stat} \pm 0.2_{\rm syst}$ $r_2 = 0.71 \pm 0.14_{\rm stat} \pm 0.02_{\rm syst}$
Scalar modes (f_0, a_0) — report $f_+(0)\|V_{cq}\|$, the subdecay BFs are not divided					
$D^+ \to f_0(500) \mu^+\nu_\mu$ [21]	3.773	2.93	D^+D^-	$5.29 \pm 0.07_{\rm stat} \pm 0.06_{\rm syst}$	$f_+(0)\|V_{cq}\| = 0.143 \pm 0.014_{\rm stat} \pm 0.011_{\rm syst}$
$D_s^+ \to f_0(980) e^+\nu_e$ [22]	4.128-4.226	7.33	$D_s^+D_s^{*-}$	$1.72 \pm 0.13_{\rm stat} \pm 0.10_{\rm syst}$	$f_+(0)\|V_{cq}\| = 0.504 \pm 0.017_{\rm stat} \pm 0.035_{\rm syst}$
$D^0 \to a_0(980)^- e^+\nu_e$ [23]	3.773	7.93	$D^0\bar{D}^0$	$1.72 \pm 0.13_{\rm stat} \pm 0.10_{\rm syst}$	$f_+(0)\|V_{cq}\| = 0.504 \pm 0.017_{\rm stat} \pm 0.035_{\rm syst}$
Axial-vector modes (K_1, b_1) — report (r_V, r_A), for b_1 channel the subdecay BFs are not divided					
$D^+ \to \bar{K}_1(1270)^0 e^+\nu_e$ [24]	3.773	20.3	D^+D^-	$(2.27 \pm 0.11_{\rm stat} \pm 0.07_{\rm syst} \pm 0.07_{\rm input}) \times 10^{-3}$	$r_V = (-4.3 \pm 1.0_{\rm stat} \pm 2.5_{\rm syst}) \times 10^{-2}$
$D^0 \to K_1(1270)^- e^+\nu_e$ [24]	3.773	20.3	$D^0\bar{D}^0$	$(1.02 \pm 0.06_{\rm stat} \pm 0.06_{\rm syst} \pm 0.03_{\rm input}) \times 10^{-3}$	$r_A = (-11.2 \pm 1.0_{\rm stat} \pm 0.9_{\rm syst}) \times 10^{-2}$
$D^+ \to \bar{K}_1(1270)^0 \mu^+\nu_\mu$ [25]	3.773	20.3	D^+D^-	$(2.36 \pm 0.20^{+0.18}_{\rm stat-0.27_{syst}} \pm 0.48_{\rm input}) \times 10^{-3}$	...
$D^0 \to K_1(1270)^- \mu^+\nu_\mu$ [25]	3.773	20.3	$D^0\bar{D}^0$	$(0.78 \pm 0.11^{+0.05}_{\rm stat-0.09_{syst}} \pm 0.15_{\rm input}) \times 10^{-3}$	...
$D^0 \to b_1(1235)^- e^+\nu_e$ [26]	3.773	20.3	$D^0\bar{D}^0$	$(0.72 \pm 0.18^{+0.06_{\rm syst}}_{\rm stat-0.08_{syst}}) \times 10^{-4}$	...

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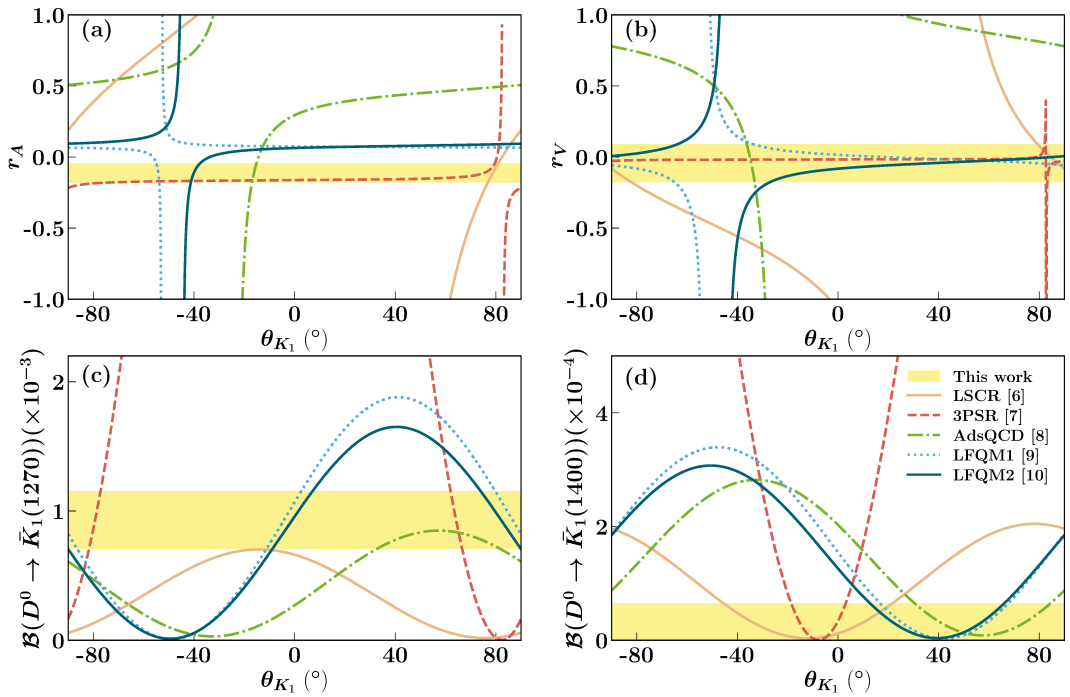

Figure 3: Comparisons of (a) r_A, (b) r_V, (c) $\mathcal{B}(D^0 \to K(1270)^- e^+ \nu_e)$, and (d) $\mathcal{B}(D^0 \to K(1400)^- e^+ \nu_e)$ measured in this work and predicted by various theoretical approaches as a function of θ_{K_1}. The yellow bands show $\pm 5\sigma$ limit for (a), (b), (c), and the upper limit at 90% CL for (d).

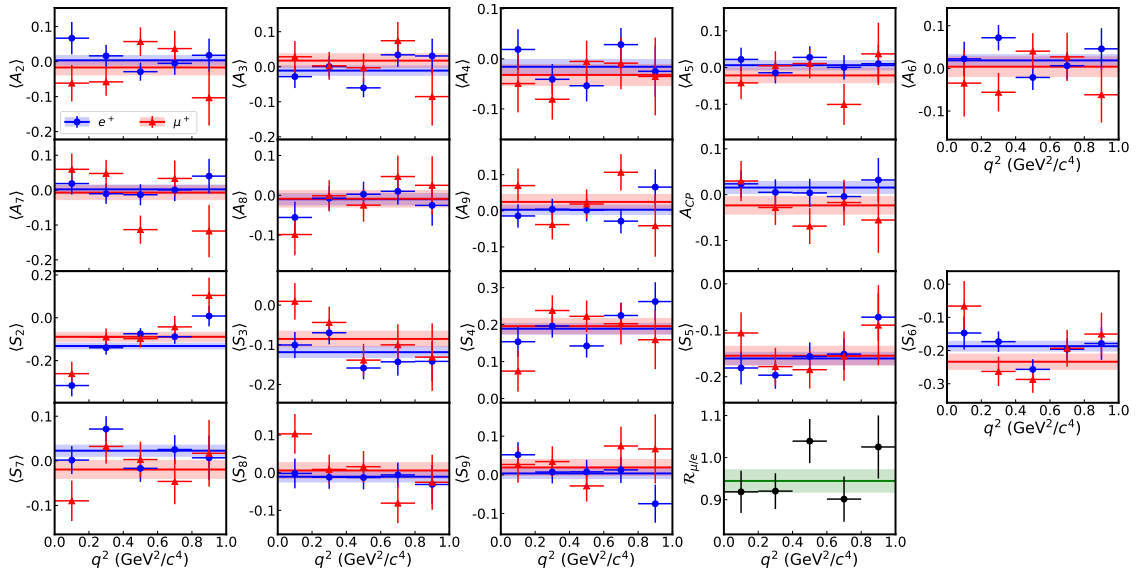

Figure 4: Measured observables $\langle A_i \rangle$, $\langle S_i \rangle$, $\mathcal{R}_{\mu/e}$ and the CP asymmetry for $D^+ \to \bar{K}^{*0} \ell^+ \nu_\ell$ in different q^2 regions with $0.8 < m_{\bar{K}^{*0}} < 1.0$ GeV$^2/c^4$. The e^+ (μ^+) results are shown in blue (red). The horizontal lines and shaded bands represent the mean value and 1 standard deviation across full q^2 range.

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