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NNLOCAL: completely local subtractions

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The computation of higher-order corrections to cross sections relevant at LHC involves the evaluation of phase-space integrals that exhibit soft and collinear divergences. The subtraction of these divergences is a key ingredient to obtain fully-differential predictions for physical observables. We discuss a subtraction method to handle these divergences based on the construction of universal local counterterms. The integration of the counterterms is carried out analytically, giving a strong control on the numerical stability of our predictions. We implement our method in a numerical program, that we dub NNLOCAL, and validate it by computing the fully-differential NNLO cross-section for Higgs boson production in gluon-gluon fusion.

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1. Introduction

Our knowledge of elementary particles and their interaction is encoded in the Standard Model (SM) of particle physics. In the last decades, the SM has been intensively tested at collider experiments, like the Large Hadron Collider (LHC) at CERN, and the agreement among theoretical predictions and experimental data is remarkable. One of the most important confirmations of the SM was the discovery of the Higgs boson at LHC, in July 2012 [1, 2]. Despite its successes, we know that the SM is not the end of the story. Some observed phenomena, like matter-antimatter asymmetry and the small neutrino masses, cannot be described within the SM framework. In addition to that, physics beyond the SM can manifest itself indirectly as a small deviation between SM predictions and experimental measurements. For all these reasons, precision tests of the SM become crucial.

At LHC, the effect of the strong force is predominant: for this reason it is necessary to rely on accurate Quantum Chromodynamics (QCD) calculations to make meaningful comparison with data. Theoretical predictions for observables at colliders are obtained using *perturbation theory*: cross sections are expanded in a power series in the strong coupling constant α_s. One of the key aspects for increasing precision is to include higher-order corrections in the perturbative expansion. This requires to take into account Feynman diagrams with additional real and virtual emissions with respect to the Born process. Both real and virtual diagrams suffer from infrared (IR) divergences, that are reached in the limits in which the partons are soft and/or collinear to each other. The sum of real and virtual contributions is finite, and therefore all the IR divergences cancel in the final result. However, the phase space integrations are typically not feasable in an analytical way, especially when considering differential distributions. For this reason, it is necessary to reorganize the various contributions of the computation such that the IR cancellation takes place explicitly, and a numerical integration can be carried out without additional complications. One possibility for doing so is offered by *local subtraction methods.* The key idea of local subtraction is to define some counterterms that have the same singular behaviour as the matrix element in the IR limits. The subtraction of these counterterms makes the phase space integration finite. However, to preserve the correct final result, they have to be added back integrated over the phase space of the additional radiation. The poles in the dimensional regulator ϵ of the integrated counterterms will cancel with the explicit poles of virtual amplitudes, leading to a finite expression suitable for numerical integration.

At Next-to-Leading Order (NLO), there are well established subtraction techniques [3, 4], that have been implemented in several public codes. On the other side, the formulation of local subtraction methods at Next-to-Next-to-Leading order (NNLO) is still a very active field of research, and several approaches have been proposed [5–10]. Here, we focus on an extension of the CoLoRFulNNLO subtraction method [11–17] to hadronic collisions. In the CoLoRFulNNLO method, the counterterms are defined based on the universal structure of QCD amplitudes in IR limits, that are extended over the whole phase space using suitable momentum mappings. The counterterms are then integrated analytically over the radiation phase space. We implement our method in a parton-level Monte Carlo code, that we dub `NNLOCAL` [18], and as a proof-of-concept we compute the NNLO corrections to the cross section for Higgs boson production in gluon-gluon fusion in the Higgs effective field theory (HEFT) approximation.

This proceeding is organised as follows: in Sec. 2, we give a brief review of the CoLoRFulNNLO subtraction formalism. Sec. 3 is devoted to the analytic integration of the counterterms, while in Sec. 4 we show some numerical results obtained with the `NNLOCAL` code. Finally, we draw our conclusions in Sec. 5.

2. The subtraction method

We consider a hadron-hadron collision leading to the production of a colorless system X. At partonic level, the NNLO correction to the cross section for this process is

$$\sigma_{ab}^{\mathrm{NNLO}} = \int_{X+2} d\sigma_{ab}^{\mathrm{RR}} J_{X+2} + \int_{X+1} \left(d\sigma_{ab}^{\mathrm{RV}} + d\sigma_{ab}^{C_1} \right) J_{X+1} + \int_{X} \left(d\sigma_{ab}^{\mathrm{VV}} + d\sigma_{ab}^{C_2} \right) J_X \,, \tag{1}$$

where $d\sigma_{ab}^{\mathrm{RR}}$, $d\sigma_{ab}^{\mathrm{RV}}$ and $d\sigma_{ab}^{\mathrm{VV}}$ are the double real, real-virtual and double virtual cross sections, while $d\sigma_{ab}^{\mathrm{C_1}}$ and $d\sigma_{ab}^{\mathrm{C_2}}$ are the collinear remnants. The function J_X is a generic IR safe observable. In Eq. (1), all the integrals are IR divergent, but their sum is finite. In order to regularize the IR singularities, we introduce several counterterms. Focusing on the double real contribution, it is regularized as

$$\sigma_{ab}^{\mathrm{NNLO,RR}} = \int_{X+2} \left[d\sigma_{ab}^{\mathrm{RR}} J_{X+2} - d\sigma_{ab}^{\mathrm{RR,A_1}} J_{X+1} - d\sigma_{ab}^{\mathrm{RR,A_2}} J_X + d\sigma_{ab}^{\mathrm{RR,A_{12}}} J_X \right] , \tag{2}$$

that, by construction, it is finite in $d = 4$ dimensions and can be integrated numerically. The counterterms introduced have the following interpretation:

- $d\sigma_{ab}^{\mathrm{RR,A_1}}$ regularizes the singly-unresolved limits.
- $d\sigma_{ab}^{\mathrm{RR,A_2}}$ regularizes the doubly-unresolved limits.
- $d\sigma_{ab}^{\mathrm{RR,A_{12}}}$ cancels the singularity coming from the singly-unresolved limits of $d\sigma_{ab}^{\mathrm{RR,A_2}}$, and from the doubly-unresolved limits of $d\sigma_{ab}^{\mathrm{RR,A_1}}$.

The precise definitions of the counterterms are quite involved and we will not report them here. For the sake of this proceeding, it is necessary just to highlight some of their features. To cancel the IR singularities, the counterterms are defined such that they coincide with QCD matrix elements in the IR limits. Their extension over the whole phase space is done through suitable momentum mappings, that are defined such that the Born and the radiation phase spaces factorize. In addition to that, their definition is such that all the overlaps between singly and doubly unresolved limits are treated in a process-independent way. To preserve the result, each counterterm needs to be added back in its integrated version. In this proceeding we focus on the integration of the counterterm $d\sigma_{ab}^{\mathrm{RR,A_2}}$ over the double-emission phase space:

$$\int_2 d\sigma_{ab}^{\mathrm{RR,A_2}} \,. \tag{3}$$

3. Integration of the A_2 counterterm

In this section we will discuss the computational strategy employed for the integration of the $d\sigma_{ab}^{\mathrm{RR,A_2}}$ counterterm. It is defined to regularize the doubly-unresolved limits of matrix elements with two extra emissions, and its explicit definition is based on the double and triple collinear splitting kernels, and double soft eikonal factors [19]. The analytical integration is performed by sorting all the terms appearing in the counterterm into *integral topologies*, and after that we perform an Integration-By-Part (IBP) reduction to a basis of *master integrals* (MIs), using reverse unitarity [20] to construct IBP identities among phase space integrals. To cross-check our results, we compute the MIs using two different strategies: direct integration and the differential equations method [21, 22].

To illustrate our computational strategy, we consider an explicit example. The counterterm that regularizes the configurations in which the emitted partons r and s are collinear to the initial-state parton a is

$$\begin{aligned} C_{ars}^{IFF(0)}(\{p\}_{X+2}; p_a, p_b) &= (8\pi\alpha_s\mu^{2\epsilon})^2 \frac{1}{x_{a,rs}} \frac{1}{s_{ars}^2} \\ &\times \langle \mathcal{M}^{(0)}_{(ars)b,X}(\hat{p}_a, \hat{p}_b, \{\hat{p}\}_X) | \hat{P}^{(0)}_{(ars)rs} | \mathcal{M}^{(0)}_{(ars)b,X}(\hat{p}_a, \hat{p}_b, \{\hat{p}\}_X) \rangle \mathcal{F}(x_{a,rs}, \xi_{a,rs}\xi_{b,rs}) \end{aligned} \tag{4}$$

where $\hat{P}^{(0)}_{(ars)rs}$ is the triple collinear splitting kernel [19] and $s_{ij} = (p_i + p_j)^2$, where $i, j \in \{a, r, s\}$, and $|\mathcal{M}^{(0)}_{(ars)b,X}(\hat{p}_a, \hat{p}_b, \{\hat{p}\}_X)\rangle$ is the Born matrix element in color-spin space, evaluated on the set of mapped momenta $\{\hat{p}\}$. We point out that the mapped momenta depend on two convolution variables ξ_a and ξ_b. The variables $x_{a,rs}$ and $\xi_{a,rs}, \xi_{b,rs}$ are defined as

$$\begin{aligned} x_{a,rs} &= 1 - \frac{s_{ar} + s_{br}}{s_{ab}} - \frac{s_{as} + s_{bs}}{s_{ab}}, \\ \xi_{a,rs} &= \sqrt{\frac{s_{ab} - s_{b(rs)}}{s_{ab} - s_{a(rs)}} \frac{s_{ab} - s_{(rs)(ab)} + s_{rs}}{s_{ab}}}, \\ \xi_{b,rs} &= \sqrt{\frac{s_{ab} - s_{a(rs)}}{s_{ab} - s_{b(rs)}} \frac{s_{ab} - s_{(rs)(ab)} + s_{rs}}{s_{ab}}}, \end{aligned} \tag{5}$$

where we introduced the notations $s_{c(jk)} = s_{cj} + s_{ck}$ and $s_{(cd)(jk)} = s_{c(jk)} + s_{d(jk)}$. The function $\mathcal{F}(x, y)$ is defined as

$$\mathcal{F}(x, y) = \left(\frac{x}{y}\right)^2, \tag{6}$$

and its role is to cancel unphysical singularities associated to the factor $x_{a,rs}$.

The counterterm of Eq. (4) has to be integrated over the double radiation phase space, whose integration measure is [23]

$$d\Phi_{II,FF} = \frac{d^d p_r}{(2\pi)^{d-1}} \frac{d^d p_s}{(2\pi)^{d-1}} \delta_+(c_1)\delta_+(c_2)\delta_+(c_3)\delta_+(c_4), \tag{7}$$

where the phase space constraints c_1 and c_2 are the on-shellness of the momenta p_r and p_s, while the constraints c_3 and c_4 come from the phase space factorization. Explicitly, the constraints are

given by

$$
\begin{aligned}
&c_1 = p_r^2, \quad c_2 = p_s^2\,, \\
&c_3 = (p_a + p_b - p_r - p_s)^2 - \xi_a \xi_b s_{ab}\,, \\
&c_4 = \xi_a (s_{ab} - s_{ar} - s_{as}) + \xi_b (s_{ab} - s_{br} - s_{bs})\,.
\end{aligned} \tag{8}
$$

To derive IBP relations among phase space integrals, the delta functions are treated as cut propagators:

$$
\delta_+(c_i) \to \left(\frac{1}{c_i}\right)_c\,, \tag{9}
$$

where in IBP relations, the cut propagators satisfy the condition

$$
\left(\frac{1}{c_i}\right)_c^{\nu} = 0\,, \quad \text{for } \nu \leq 0\,. \tag{10}
$$

We find that the integrated triple collinear counterterm is a linear combination of integrals belonging to 17 different integral topologies. For brevity, we will report only some of them, and the complete definition of the families will be given in a future publication

$$
\begin{aligned}
F_1(n_1,n_2,n_3) &= \int d\Phi_{II,FF} \frac{1}{s_{as}^{n_1}(s_{ar}+s_{as})^{n_2}(s_{ab}-s_{ar}-s_{br})^{n_3}}\,, \\
F_2(n_1,n_2,n_3) &= \int d\Phi_{II,FF} \frac{1}{s_{ar}^{n_1}(s_{ar}+s_{as})^{n_2}(s_{ar}+s_{br})^{n_3}}\,, \\
F_3(n_1,n_2,n_3) &= \int d\Phi_{II,FF} \frac{1}{s_{ar}^{n_1}(s_{ab}-s_{ar}-s_{br})^{n_2}(s_{ar}+s_{as}-s_{ab})^{n_3}}\,, \\
F_4(n_1,n_2,n_3) &= \int d\Phi_{II,FF} \frac{1}{s_{ar}^{n_1}(s_{ab}-s_{ar}-s_{br})^{n_2}(s_{ar}+s_{as}+s_{br}+s_{bs})^{n_3}}\,, \\
&\vdots \\
F_{17}(n_1,n_2,n_3) &= \int d\Phi_{II,FF} \frac{1}{s_{as}^{n_1}(s_{ar}+s_{as}-s_{rs})^{n_2}(s_{ar}+s_{br})^{n_3}}\,.
\end{aligned} \tag{11}
$$

After an IBP reduction we find that the integrated counterterm $\int_2 d\sigma_{ab}^{\mathrm{RR,A_2}}$ is a linear combination of 42 master integrals. For each integral topology, we set up a system of differential equations in the convolution variables ξ_a, ξ_b, that is solved by finding a suitable transformation to a canonical basis [24]. After the phase space integration, the last step to be performed is the integration over the convolution variables. This is done by constructing a suitable distributional expansion.

4. The NNLOCAL code

The full subtraction formalism, including all the analytical expressions for the integrated counterterms, has been implemented in the parton level Monte Carlo code NNLOCAL [18], that is publicly available at https://github.com/nnlocal/nnlocal. To test our code, we compute the inclusive NNLO cross section for Higgs boson production in gluon fusion in the Higgs effective field theory (HEFT) approximation. We find perfect agreement with the code n3loxs [25]. Since NNLOCAL is fully differential in all the particle momenta, we also test it for the computation of

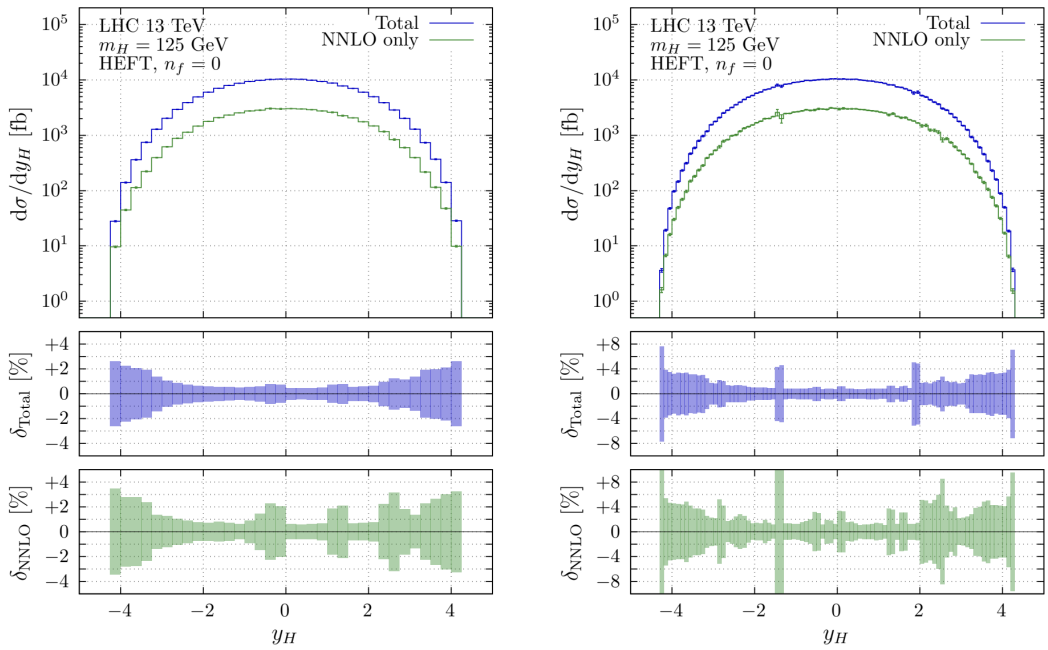

Figure 1: The rapidity distribution of the Higgs boson at NNLO in the HEFT with $n_f = 0$ light quarks. The distribution on the left has a bin width $\Delta y = 0.25$, while the one on the right has a bin width of $\Delta y = 0.1$. The lower panels show the relative error for the total distribution and the NNLO contribution. The error bands shown are the estimated Monte Carlo uncertanties.

differential observables. In Fig. 1 we present the rapidity distribution of a Higgs boson with mass $m_H = 125$ GeV at the 13 TeV at LHC in the HEFT approximation, considering $n_f = 0$ light quarks. The runtime was about 1 hour on a MacBook Pro laptop with M2 processor and 8 CPU cores. In the left plot, with a bin width $\Delta y = 0.25$, we can see a good numerical convergence and stability. To test our code on more demanding conditions, we also plot the same distribution with a bin width of $\Delta y = 0.1$. We still see an overall good convergence, but because of the fine binning, some spikes arise due to the phenomenon of *misbinning*. This problem can be solved by considering *robust estimators* for the bin values.

5. Conclusions and outlook

The computation of higher order corrections for collider observables is extremely important for precision tests of the Standard Model. The CoLoRFulNNLO method is designed to handle fully differential NNLO calculations numerically, by subtracting the IR divergences using suitable local counterterms. We perform an analytic integration of the counterterms needed for color singlet production in hadronic collisions at NNLO. The subtraction method is implemented in the public code `NNLOCAl`.

Our proof-of-concept code is the first building block for the applications of the CoLoRFulNNLO method to hadronic collisions. Its future extension and refinement will make it a useful program to perform NNLO QCD computations for a variety of processes at LHC.

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