PROCEEDINGS OF SCIENCE

# Combination and checks of highly correlated measurements of the muon precession frequency in magnetic field for the final FNAL measurement of the muon magnetic anomaly

**Alberto Lusiani[a,b,*] on behalf of the Muon $g-2$ Collaboration**

[a] *INFN, sezione di Pisa, Pisa, Italy*
[b] *Scuola Normale Superiore, Pisa, Italy*

*E-mail:* alberto.lusiani@sns.it

We describe the procedures developed to assess the consistency of and combine the multiple measurements of the muon spin-precession frequency $\omega_a$ performed by the Muon $g-2$ collaboration for its 2025 determination of the muon magnetic anomaly. Building on the methods introduced in the 2023 Muon $g-2$ analysis, we evaluate the correlations among multiple $\omega_a$ measurements on the same data by using bootstrap techniques. This approach enables robust consistency checks and a statistically sound combination of the independent analyses.

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## 1. Introduction

The 2025 muon magnetic anomaly measurement by the Muon $g-2$ collaboration [1] relies on measurements of the muon spin precession frequency $\omega_a$ in a magnetic field, performed in four datasets collected from 2020 to 2023 and labelled NoRF, xRF, xyRF5 and xyRF6, according to the usage of radiofrequency on the electric quadrupoles to suppress horizontal and vertical muon beam oscillations. The $\omega_a$ frequencies have been fitted on each of the four datasets by five analysis groups, using a total of 20 different fits, employing a variety of different methods. Statistical consistency checks have been performed to validate all analyses, and an optimal combination of the measurements has been used to provide the $\omega_a$ measurements for the magnetic anomaly determination.

## 2. Consistency checks

While two $\omega_a$ measurements on different datasets are expected to be consistent within their statistical uncertainties and the stability of the magnetic field in the storage ring, two $\omega_a$ measurements on the same dataset may be naively expected to be consistent within their quoted systematic uncertainties. Actually, when measurement methods and procedures differ [1], the uncertainty on the difference between two $\omega_a$ measurements $m_2 - m_1$ (residual) has a statistical component, which is typically dominant and is determined by the statistical correlation $\rho_{12}$ between the two measurements:

$$\sigma^2(m_2 - m_1) = \sigma_1^2 - 2\rho_{12}\sigma_1\sigma_2 + \sigma_2^2 \,. \tag{1}$$

In the 2023 muon magnetic anomaly measurement [2], the correlation between any two $\omega_a$ measurements has been estimated according to a model that summed "decorrelation components" related to how the two different measurements differ regarding a set of analysis features [3], like the method of reconstruction of the muon-decay positrons' energy in the calorimeters, or the muon-decay events weighting as a function of the just-mentioned reconstructed energy. Dedicated studies on fits differing only on a single feature have estimated the related "decorrelations", using toy Monte Carlo or bootstrap [4] techniques.

The $\omega_a$ measurements are obtained by fitting time-binned histograms, where the bin content, as summarized in Table 1, is either based on muon-decay positron counting, or on the total energy of the same detected positrons (Q or "charge" method). Positrons are either counted when their energy exceeds a threshold (T or "threshold" method), or with weights determined by their expected energy-dependent asymmetry (the amplitude of the modulation of the detected decays over time due to the evolving alignment between the muon spin and momentum, therefore this last method is named A or "asymmetry" method).

In the 2025 measurement, the correlation between any two measurements relying on muon-decay counting (T, A) has been estimated with the bootstrap technique on the full datasets NoRF, xRF, xyRF5 and xyRF6. The covariance $V(m_1, m_2)$ between two $\omega_a$ measurements is estimated as

$$V(m_1, m_2) = \frac{\sum_b (m_{1,b} - \overline{m}_1)(m_{2,b} - \overline{m}_2)}{N_b - 1} \,, \tag{2}$$

**Table 1:** $\omega_a$ measurement methods.

| | description | bin content | weighting |
|---|---|---|---|
| T | threshold | event counts | above-threshold energy |
| A | asymmetry-weighted | event counts | energy-dependent asymmetry |
| Q | charge | integrated energy | |

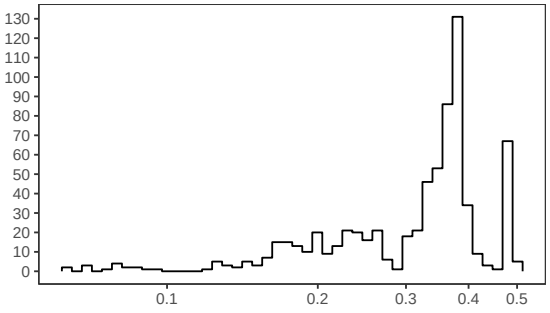


**Figure 1:** Ratio between the uncertainty on the residual between any two $\omega_a$ measurements and the quadrature sum of their individual uncertainties. The rightmost peak corresponds to residuals between Q and A/T measurements; the second-rightmost peak corresponds to residuals between A and T measurements. Smaller residual uncertainties correspond to analyses that use the same A, T, or Q method but differ in other features.

where the index $b$ denotes the $b$'th bootstrap sample of a set of $N_b = 200$, and $\overline{m}_k$ denotes the average of $m_{k,b}$ over the bootstrap samples. The correlations between the "charge" or Q $\omega_a$ measurements and themselves or the T or A measurements have been estimated using $\omega_a$ measurements in $N_{\text{sd}} = 34$ sub-datasets. The correlation $\rho_{12}$ between two $\omega_a$ measurements has been numerically computed to satisfy

$$\frac{\chi^2}{\text{n.d.o.f.}} = \frac{1}{N_{\text{sd}} - 1} \sum_i \frac{\left[(m_{2,i} - m_{1,i}) - \left\langle m_{2,i} - m_{1,i} \right\rangle\right]^2}{\sigma_{1,i}^2 - 2\rho_{12}\sigma_{1,i}\sigma_{2,i} + \sigma_{2,i}^2} = 1 \,, \tag{3}$$

where $\left\langle m_{2,i} - m_{1,i} \right\rangle$ is the weighted average of the residual over the sub-datasets, which is subtracted in order to compute $\rho_{12}$ using measurements with analysis-group-dependent blinding offsets. Figure 1 reports the ratio between the uncertainty on the residual between any two $\omega_a$ measurements, evaluated using the above-mentioned estimated correlations, and the sum in quadrature of the respective $\omega_a$ measurements uncertainties (which would be the residual uncertainty if the two measurements were uncorrelated).

The consistency of the $\omega_a$ measurements with analysis-group-dependent blinding offsets has

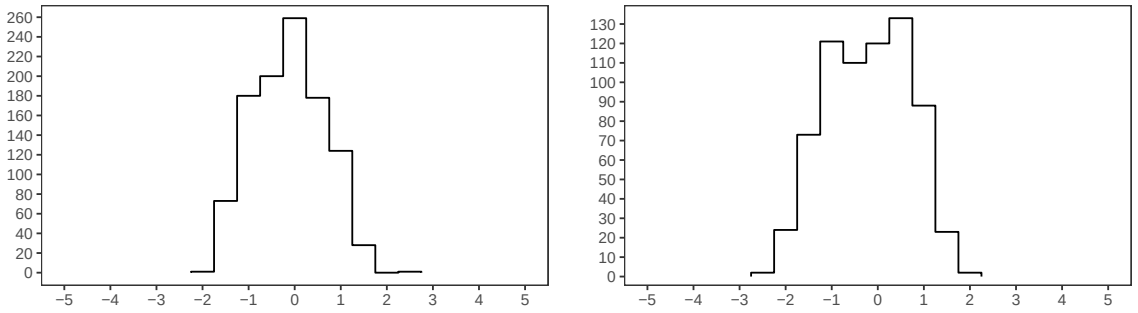

**Figure 2:** Residuals' pulls of "blind" $\omega_a$ measurements "double differences" (left) and of $\omega_a$ measurements (right). Both distributions are reasonably Normal.

been checked on the distribution of the pulls

$$p(a1, a2, d1, d2) = \frac{(m_{d2,a2} - m_{d2,a1}) - (m_{d1,a2} - m_{d1,a1})}{\sqrt{\sigma^2(m_{d2,a2} - m_{d2,a1}) + \sigma^2(m_{d1,a2} - m_{d1,a1})}}\,, \tag{4}$$

where $a1$, $a2$ denote two different analyses and $d1$, $d2$ denote two different datasets. The "double difference" removes the unknown blinding offsets. The analysis-group-dependent blinding offsets are removed when all analyses have been finalized and the above pulls are normally distributed as expected. After removing the analysis-group-dependent blinding offsets, the pulls

$$p(a1, a2) = \frac{m_{a2} - m_{a1}}{\sigma(m_{a2} - m_{a1})} \tag{5}$$

are used to perform a more statistically stringent consistency check. Figure 2 shows the "blind" "double differences" pulls and the final pulls on the $\omega_a$ measurements.

## 3. Per-dataset $\omega_a$ averages

Following the same approach as the 2023 analysis [2], we average, for each of the four datasets collected from 2020 to 2023, a total of 7 $\omega_a$ measurements performed with the most precise A method. In order to obtain a robust average with conservative uncertainties, all $\omega_a$ measurements are equally weighted, and both the statistical and systematic uncertainties are assumed to be 100% correlated. According to the correlations estimated in the previous section, this conservative procedure produces a statistical uncertainty that is about 1.5% larger than the optimal one.

## 4. Acknowledgments

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