PROCEEDINGS OF SCIENCE

Lepton Flavour Universality tests and determination of $|V_{us}|$ using the HFLAV-Tau branching fractions fit

Alberto Lusiani[a,b,*] **on behalf of HFLAV-Tau group**

[a] *INFN, sezione di Pisa, Pisa, Italy*
[b] *Scuola Normale Superiore, Pisa, Italy*

E-mail: alberto.lusiani@sns.it

We present an update of the lepton flavour universality (LFU) tests and of the determination of the CKM matrix element $|V_{us}|$ using the latest HFLAV-Tau global fit of τ branching fractions. The fit accounts for correlations and dependencies on common external inputs. We investigate the impact of using next-to-leading-order QED corrections in the LFU tests.

The European Physical Society Conference on High Energy Physics (EPS-HEP2025)
7-11 July 2025
Marseille, France

*Speaker

https://pos.sissa.it/

PoS(EPS-HEP2025)325

1. HFLAV fit of τ branching fractions, τ mass and τ lifetime

The HFLAV 2023 report [1] describes three fits performed on τ measurements, using the HFLAV procedures to account for the dependencies of the measurements on external inputs like other world averages, or luminosity measurements that are used for different branching fraction measuremnents by the same experimental collaboration.

A set of 171 measurements of τ branching fractions and ratios of branching fractions m_i and an estimate of one nuisance parameter $n_r \pm \sigma_{n_r}$ are fit to obtain 137 fit parameters q_k and 1 nuisance fit parameter p_r. The fit parameters and the nuisance fit parameter are subjected to 91 constraint equations. The minimized χ^2 is

$$\chi^2 = \sum_{ijkl} (m_i - M_{ik} q_k) \left(V^{-1}\right)_{ij} \left(m_j - M_{jl} q_l\right) + \sum_r \frac{(n_r - p_r)^2}{\sigma^2_{n_r}} , \tag{1}$$

where M_{ik} is the model matrix applied to fit parameters to predict measurements and V_{ij} is the measurements' covariance. The available measurements of the τ mass and lifetime are also fit to obtain their averages.

2. Tests of lepton flavour universality

In order to test lepton flavour universality (LFU), we use the expression for the partial width of a heavier lepton $\mathcal{L}$ decay to a lighter lepton ℓ

$$\Gamma[\mathcal{L} \to \nu_{\mathcal{L}} \ell \bar{\nu}_\ell(\gamma)] = \frac{G_{\mathcal{L}} G_\ell m^5_{\mathcal{L}}}{192\pi^3} f^{\mathcal{L}\ell} \left(1 + \delta R^{\mathcal{L}\ell}_\gamma\right) \left(1 + \delta R^{\mathcal{L}\ell}_W\right) , \tag{2}$$

where $f^{\mathcal{L}\ell}$ is a phase space factor that depends on the leptons' masses, $\delta R^{\mathcal{L}\ell}_\gamma$ and $\delta R^{\mathcal{L}\ell}_W$ correspond to the QED and electroweak (EW) radiative corrections, respectively. In the HFLAV 2023 report, as in previous papers, the leading-order (LO) QED correction in α_{QED} has been used,

$$\delta R^{\mathcal{L}\ell}_\gamma = \frac{\alpha(m_{\mathcal{L}})}{2\pi} \left(\frac{25}{4} - \pi^2\right) . \tag{3}$$

Here, we rely on the QED radiative corrections that have been evaluated for the muon decay [2],

$$\delta R^{\mathcal{L}\ell}_\gamma = H^{\mathcal{L}\ell}_1 \frac{\alpha(m_{\mathcal{L}})}{\pi} + H^{\mathcal{L}\ell}_2 \frac{\alpha^2(m_{\mathcal{L}})}{\pi^3} + H^{\mathcal{L}\ell}_3 \frac{\alpha^3(m_{\mathcal{L}})}{\pi^3} \tag{4}$$

$$H^{\mathcal{L}\ell}_1 = \frac{25}{8} - \frac{\pi^2}{2} - \left(9 + 4\pi^2 + 12 \ln\rho\right) \rho + 16\pi^2 \rho^{3/2} \tag{5}$$

$$H^{\mathcal{L}\ell}_2 = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln 2 + H^{\mathcal{L}\ell}_{2,\text{had}} - \frac{5}{4}\pi^2\sqrt{\rho} , \tag{6}$$

where $\rho = \rho^{\mathcal{L}\ell} = m^2_\ell / m^2_{\mathcal{L}}$, $H^{\mu e}_{2,\text{had}} = -0.042 \pm 0.002$, $H^{\mu e}_3 = -15.3 \pm 2.3$, and the modified minimal subtraction scheme has been used. Since $H^{\mathcal{L}\ell}_2$ and $H^{\mathcal{L}\ell}_3$ have not been evaluated for the τ, we will omit them in the following in order to get partial next-to-leading-order (NLO) QED corrections for the τ leptonic decays. The code `alphaQEDr23.f` [3] has been used to compute $\alpha(m_\tau)$. Table 1 reports the LFU-tests' coupling ratios, which have negligible differences when changing from LO to

Table 1: Charged weak current coupling ratios with LO and NLO QED radiative corrections. The expressions $\mathcal{B}_{\mathcal{L}\ell}$ correspond to the τ leptonic branching fractions.

	LO $\delta R_\gamma^{\mathcal{L}\ell}$	NLO $\delta R_\gamma^{\mathcal{L}\ell}$
$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}}$	1.0016 ± 0.0014	1.0016 ± 0.0014
$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau\mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau\mu}(1+\delta R_\gamma^{\tau\mu})(1+\delta R_W^{\tau\mu})}}$	1.0018 ± 0.0014	1.0017 ± 0.0014
$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau\mu}}{\mathcal{B}_{\tau e}} \frac{f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}{f^{\tau\mu}(1+\delta R_\gamma^{\tau\mu})(1+\delta R_W^{\tau\mu})}}$	1.0002 ± 0.0011	1.0001 ± 0.0011

NLO QED radiative corrections. We estimate that the coupling ratios uncertainty has contributions from the present uncertainty on the τ mass in the phase space factors (1.3 ppm), from the uncertainty on $\alpha(m_\tau)$ (0.3 ppm), and from the omission of $H_2^{\mathcal{L}\ell}$ and $H_3^{\mathcal{L}\ell}$ (up to 2.4 ppm, assuming that the two terms may be up to 20 times larger for the τ with respect to the muon).

3. $|V_{us}|$ from τ branching fraction measurements

$|V_{us}|$ is computed using the inclusive τ branching fraction to strange hadronic final states as [4, 5]

$$|V_{us}|_{\tau\text{-OPE-1}} = \sqrt{R_s \Big/ \left[\frac{R_{ud}}{|V_{ud}|^2} - \delta R_{\tau\text{-}SU(3)\text{-break}}\right]} = 0.2184 \pm 0.0021\,, \tag{7}$$

where $R_s = \mathcal{B}_s/\mathcal{B}_e^{\text{uni}} = 0.1632 \pm 0.0027$, $\mathcal{B}_s = 2.908 \pm 0.048$ is the inclusive τ branching fraction to strange hadronic final states, $\mathcal{B}_e^{\text{uni}}$ is the universality-improved electronic τ branching fraction [1], $R_{ud} = R_{\text{had}}^{\text{uni}} - R_s = 3.470 \pm 0.008$, $R_{\text{had}}^{\text{uni}}$ is the ratio between the inclusive hadronic τ decay branching fraction and $\mathcal{B}_e^{\text{uni}}$, $|V_{ud}| = 0.97384 \pm 0.00026$ is taken from a recent average of measurements of superallowed nuclear β decays, neutron decay measurements [6], and $\delta R_{\tau\text{-}SU(3)\text{-break}}$ accounts for the $SU(3)$-breaking effects [7–9]. We mention also two additional $|V_{us}|$ determinations, which could not be updated because of their complexity:

$$|V_{us}|_{\tau\text{-OPE-2}} = 0.2219(22) \quad [10, 11] \tag{8}$$

$$|V_{us}|_{\tau\text{-latt-disp}} = 0.2240(18) \quad [11, 12] \tag{9}$$

An additional evaluation of $|V_{us}|$ uses the lattice QCD calculation of the inclusive τ hadronic decay rate [13],

$$|V_{us}|_{\tau\text{-latt-incl}} = \sqrt{\left(\frac{|V_{us}|^2}{R_s}\right)_{\text{latt-incl}} \cdot R_s} = 0.2189 \pm 0.0019\,, \tag{10}$$

where $(R_s/|V_{us}|^2)_{\text{latt-incl}} = 3.407 \pm 0.022$ [13]. All calculations of the τ inclusive decay rate to hadronic strange final states are not presently including the long-distance isospin-breaking corrections [13].

We compute $|V_{us}|/|V_{ud}|$ from the ratio of branching fractions $\mathcal{B}_{10/9} = \mathcal{B}(\tau^- \to K^-\nu_\tau)/\mathcal{B}(\tau^- \to \pi^-\nu_\tau) = 0.0644 \pm 0.0009$ using the equation [14]:

$$\frac{\mathcal{B}(\tau^- \to K^-\nu_\tau)}{\mathcal{B}(\tau^- \to \pi^-\nu_\tau)} = \frac{f_{K\pm}^2\,|V_{us}|^2}{f_{\pi\pm}^2\,|V_{ud}|^2}\frac{\left(m_\tau^2 - m_K^2\right)^2}{\left(m_\tau^2 - m_\pi^2\right)^2}(1+\delta_{\tau K/\tau\pi})\,, \tag{11}$$

where $\delta_{\tau K/\tau\pi} = (0.10 \pm 0.80)\%$ [15] is the radiative correction for the ratio of partial widths $\Gamma(\tau \to K\nu_\tau[\gamma])/\Gamma(\tau \to \pi\nu_\tau[\gamma])$. Using the ratio of decay constants $f_{K\pm}/f_{\pi\pm} = 1.1934 \pm 0.0019$ from the 2023 web update of the FLAG 2021 lattice QCD averages with N_f = 2+1+1 [16–22], we obtain $|V_{us}|/|V_{ud}| = 0.2289 \pm 0.0019$. By using the above-mentioned $|V_{ud}|$ value, we compute $|V_{us}|_{\tau K/\pi} = 0.2229 \pm 0.0019$.

$|V_{us}|$ is calculated using the branching fraction $\mathcal{B}(\tau^- \to K^-\nu_\tau)$ as

$$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2\,|V_{us}|^2\,\tau_\tau m_\tau^3\left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{\text{EW}}^{\tau h}(1+\delta_{\tau K})\,. \tag{12}$$

We use $f_{K\pm} = 155.7 \pm 0.3$ MeV from the 2023 web update of the FLAG 2021 lattice QCD averages with N_f = 2+1+1 [16–18, 20, 21, 23]. The universal short-distance electroweak correction for τ hadronic decays is $S_{\text{EW}}^{\tau h} = S_{\text{EW}}^{R\tau h} \cdot S_{\text{EW}}^{\text{sub,lep}} = 1.01910 \pm 0.00030$, where the radiative correction for the τ spectral functions is $S_{\text{EW}}^{R\tau h} = 1.02350 \pm 0.00030$ [24–26] and the sub-leading universal short-distance correction for the τ leptonic decays is $S_{EW}^{\text{sub,lep}} = 0.9957$ [24]. The long-distance radiative correction for $\mathcal{B}(\tau^- \to K^-\nu_\tau)$ is $\delta_{\tau K} = (-0.15 \pm 0.57)\%$ [15]. The physical constants G_F and $\hbar$ are taken from CODATA 2018 [27]. We obtain $|V_{us}|_{\tau K} = 0.2224 \pm 0.0017$.

We list in the following $|V_{us}|$ as predicted by the unitarity of the CKM matrix, $(|V_{us}|_{\text{uni}})^2 = 1 - |V_{ud}|^2 - |V_{ub}|^2$ (where the $|V_{ub}|$ is negligible because of its small size, $|V_{ub}| = 0.00382 \pm 0.00020$ [28]), $|V_{us}|$ evaluated with measurements of kaons' properties [6], the above $|V_{us}|$ calculations using τ measurements, the average of the τ exclusive evaluations ($|V_{us}|_{\tau K/\pi}$ & $|V_{us}|_{\tau K}$), the average of all τ evaluations, and all discrepancies with respect to $|V_{us}|_{\text{uni}}$:

$$\begin{aligned}
&|V_{us}|_{\text{uni}} &&= 0.2272 \pm 0.0011 && 0.0\sigma && \left[\sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2}\quad \text{(CKM unitarity)}\right], && (13)\\
&|V_{us}|_{K\ell 3} &&= 0.2233 \pm 0.0005 && -3.2\sigma && [\mathcal{B}_{K\ell 3}]\,, && (14)\\
&|V_{us}|_{K\ell 2} &&= 0.2250 \pm 0.0005 && -1.7\sigma && [\mathcal{B}_{K\ell 2}]\,, && (15)\\
&|V_{us}|_{\tau\text{-OPE-1}} &&= 0.2184 \pm 0.0021 && -3.6\sigma && [\mathcal{B}(\tau^- \to X_s^-\nu_\tau)]\,, && (16)\\
&|V_{us}|_{\tau\text{-latt-incl}} &&= 0.2189 \pm 0.0019 && -3.7\sigma && [\mathcal{B}(\tau^- \to X_s^-\nu_\tau)]\,, && (17)\\
&|V_{us}|_{\tau K/\pi} &&= 0.2229 \pm 0.0019 && -2.0\sigma && [\mathcal{B}(\tau^- \to K^-\nu_\tau)/\mathcal{B}(\tau^- \to \pi^-\nu_\tau)]\,, && (18)\\
&|V_{us}|_{\tau K} &&= 0.2224 \pm 0.0017 && -2.3\sigma && [\mathcal{B}(\tau^- \to K^-\nu_\tau)]\,, && (19)\\
&|V_{us}|_{\tau\ \text{excl}} &&= 0.2225 \pm 0.0017 && -2.3\sigma && [\text{tau exclusive average}]\,, && (20)\\
&|V_{us}|_{\tau} &&= 0.2208 \pm 0.0014 && -3.6\sigma && [\text{tau average}]\,. && (21)
\end{aligned}$$

All correlations arising from using the τ branching fractions fit results and common external inputs are accounted for. The systematic uncertainties of $|V_{us}|_{\tau\text{-OPE-1}}$ and $|V_{us}|_{\tau\text{-latt-incl}}$ have been assumed to be uncorrelated. The correlation between $f_{K\pm}$ and $f_{K\pm}/f_{\pi\pm}$ is conservatively set to 100%. We assume that $\delta_{\tau\pi}$ and $\delta_{\tau K}$ are uncorrelated [15] and that $\delta_{\tau K/\tau\pi} \simeq \delta_{\tau K} - \delta_{\tau\pi}$.

PoS(EPS-HEP2025)325

References

[1] S. Banerjee, E. Ben-Haim, F. Bernlochner, E. Bertholet, M. Bona, A. Bozek et al., *Averages of b-hadron, c-hadron, and τ-lepton properties as of 2023*, 2024.

[2] Particle Data Group collaboration, J. Erler and A. Freitas, *Electroweak Model and Constraints on New Physics*, *Prog. Theor. Exp. Phys.* **2022** (2022) 083C01.

[3] F. Jegerlehner, *$\alpha_{qed}(s)$ for precision physics at the FCC-ee/ILC*, in *Theory for the FCC-ee* (A. Blondel, J. Gluza, S. Jadach, P. Janot and T. Riemann, eds.), vol. CERN-2020-003 of *CERN Yellow Reports: Monographs*, p. 9. CERN, Geneva, 2020. DOI.

[4] E. Gámiz, J. Prades, M. Jamin, F. Schwab and A. Pich, *Determination of m_s and $|V_{us}|$ from hadronic tau decays*, *JHEP* **01** (2003) 060, [hep-ph/0212230].

[5] E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, *$|V_{us}|$ and m_s from hadronic tau decays*, *Phys. Rev. Lett.* **94** (2005) 011803, [hep-ph/0408044].

[6] V. Cirigliano, A. Crivellin, M. Hoferichter and M. Moulson, *Scrutinizing CKM unitarity with a new measurement of the Kμ3/Kμ2 branching fraction*, *Phys. Lett. B* **838** (2023) 137748, [2208.11707].

[7] E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, *$|V_{us}|$ and m_s from hadronic tau decays*, *Nucl. Phys. Proc. Suppl.* **169** (2007) 85, [hep-ph/0612154].

[8] E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, *Theoretical progress on the V_{us} determination from tau decays*, *PoS* **KAON** (2008) 008, [0709.0282].

[9] K. Maltman, *A Critical look at V_{us} determinations from hadronic τ decay data*, *Nucl. Phys. Proc. Suppl.* **218** (2011) 146, [1011.6391].

[10] K. Maltman, R. J. Hudspith, R. Lewis, C. E. Wolfe and J. Zanotti, *A resolution of the inclusive flavor-breaking sum rule τ V_{us} puzzle*, 1510.06954.

[11] K. Maltman et al., *Current Status of inclusive hadronic τ determinations of $|V_{us}|$*, *SciPost Phys. Proc.* **1** (2019) 006.

[12] RBC, UKQCD collaboration, P. Boyle, R. J. Hudspith, T. Izubuchi, A. Jüttner, C. Lehner, R. Lewis et al., *Novel $|V_{us}|$ Determination Using Inclusive Strange τ Decay and Lattice Hadronic Vacuum Polarization Functions*, *Phys. Rev. Lett.* **121** (2018) 202003, [1803.07228].

[13] Extended Twisted Mass collaboration, C. Alexandrou et al., *Inclusive Hadronic Decay Rate of the τ Lepton from Lattice QCD: The $\bar{u}s$ Flavor Channel and the Cabibbo Angle*, *Phys. Rev. Lett.* **132** (2024) 261901, [2403.05404].

[14] BaBar collaboration, S. Banerjee, *Lepton Universality, $|V_{us}|$ and search for second class current in tau decays*, in *34th International Conference on High Energy Physics*, 11, 2008. 0811.1429.

[15] M. A. Arroyo-Ureña, G. Hernández-Tomé, G. López-Castro, P. Roig and I. Rosell, *Radiative corrections to $\tau \to \pi(K)\nu\tau[\gamma]$: A reliable new physics test*, *Phys. Rev. D* **104** (2021) L091502, [2107.04603].

[16] Y. Aoki et al., *FLAG Review 2021*, 2111.09849.

[17] S. Aoki et al., "Flag review 2023 web update." http://flag.unibe.ch/2021/Media?action=AttachFile&do=get&target=FLAG_2023_webupdate.pdf, March, 2023.

[18] Extended Twisted Mass collaboration, C. Alexandrou et al., *Ratio of kaon and pion leptonic decay constants with Nf=2+1+1 Wilson-clover twisted-mass fermions*, *Phys. Rev. D* **104** (2021) 074520, [2104.06747].

[19] N. Miller et al., *F_K/F_π from Möbius Domain-Wall fermions solved on gradient-flowed HISQ ensembles*, *Phys. Rev. D* **102** (2020) 034507, [2005.04795].

[20] HPQCD collaboration, R. J. Dowdall, C. T. H. Davies, G. P. Lepage and C. McNeile, *V_{us} from π and K decay constants in full lattice QCD with physical u, d, s and c quarks*, *Phys. Rev. D* **88** (2013) 074504, [1303.1670].

[21] ETM collaboration, N. Carrasco et al., *Leptonic decay constants f_K, f_D, and f_{D_s} with $N_f = 2 + 1 + 1$ twisted-mass lattice QCD*, *Phys. Rev.* **D91** (2015) 054507, [1411.7908].

[22] Fermilab Lattice and MILC collaborations, A. Bazavov et al., *B- and D-meson leptonic decay constants from four-flavor lattice QCD*, *Phys. Rev. D* **98** (2018) 074512, [1712.09262].

[23] Fermilab Lattice and MILC collaborations, A. Bazavov et al., *Charmed and light pseudoscalar meson decay constants from four-flavor lattice QCD with physical light quarks*, *Phys. Rev. D* **90** (2014) 074509, [1407.3772].

[24] M. Davier, S. Eidelman, A. Hocker and Z. Zhang, *Confronting spectral functions from e+ e- annihilation and tau decays: Consequences for the muon magnetic moment*, *Eur. Phys. J. C* **27** (2003) 497–521, [hep-ph/0208177].

[25] A. Pich, *Precision Tau Physics*, *Prog. Part. Nucl. Phys.* **75** (2014) 41, [1310.7922].

[26] M. Davier, A. Hoecker, A. M. Lutz, B. Malaescu and Z. Zhang, *Tensions in $e^+e^- \to \pi^+\pi^-(\gamma)$ measurements: the new landscape of data-driven hadronic vacuum polarization predictions for the muon $g-2$*, 2312.02053.

[27] E. Tiesinga, P. J. Mohr, D. B. Newell and B. N. Taylor, *CODATA recommended values of the fundamental physical constants: 2018*, *Rev. Mod. Phys.* **93** (2021) 025010.

[28] Particle Data Group collaboration, R. L. Workman et al., *Review of Particle Physics*, *PTEP* **2022** (2022) 083C01.