PROCEEDINGS OF SCIENCE

# Towards HH at NNLO QCD: the $n_h^2$ contribution

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The virtual corrections for $gg \to HH$ at next-to-leading order (NLO) in QCD have been efficiently approximated using a Taylor expansion in the limit of a forward kinematics. The same method has been recently applied to the calculation of a subset of the next-to-NLO corrections, which are desirable given the significant impact, at NLO, of the uncertainty due to the choice of the top mass renormalization scheme. In these proceedings, we report on the progress in the calculation of another contribution at NNLO, given by diagrams in which the two Higgs bosons couple to different top quark loops. For this contribution a naive Taylor expansion cannot be used, and we discuss an approach based on asymptotic expansions in different kinematic limits.

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## 1. Introduction

The production of two Higgs bosons via gluon fusion is a key probe of the Higgs trilinear self-coupling, and measuring this process at the LHC is one of the main goals of the forthcoming High-Luminosity phase. The inclusion of QCD corrections beyond the leading-order contribution [1, 2] in the theoretical prediction for $gg \to HH$ is very important, first and foremost because these corrections are known to be large.

The calculation of the next-to-leading-order (NLO) QCD corrections requires the evaluation of multi-scale two-loop box diagrams with massive lines. These challenging contributions have been computed after long efforts, using either numerical methods [3–5] or symbolic approximations based on expansions [6–12]. The inclusion of NNLO QCD corrections is desirable in order to reduce the residual theoretical uncertainty associated to the choice of renormalization scheme and scale for the top-quark mass [13–16].

Given the extreme complexity in the evaluation of the three-loop box diagrams in the virtual NNLO corrections, the only approaches to estimate the latter developed in the literature are based on expansions. Furthermore, the nature of the mass scheme uncertainty forces one to retain some dependence on the top-quark mass in the calculation [17–20].

In these proceedings, we are interested in an expansion in the forward kinematics, which has been applied on a subset of the virtual NNLO corrections in Refs. [21, 22], showing encouraging results. This method has been developed at NLO QCD in Ref. [12] and it is based on a simple Taylor expansion. Not all topologies at NNLO, though, can be evaluated with this convenient approach: diagrams that admit $t$-channel cuts through massless lines, like the ones shown in Fig. 1, require an asymptotic expansion, which is more involved. In these diagrams, the two Higgs bosons always couple to two independent heavy-fermion loops, and we denote them as the $n_h^2$ contribution. In these proceedings, we discuss the progress in the application of forward expansions to the $n_h^2$ contribution. We present the results of Ref. [23] for the reducible diagrams in Sect. 2 and we elaborate on a strategy to compute the irreducible diagrams in Sect. 3, before concluding in Sect. 4.

## 2. Reducible $n_h^2$ contribution

In Ref. [23] we computed the contribution from diagrams like the one on the left of Fig. 1. Such reducible diagrams can be decomposed as a product of one- and two-loop subamplitudes with different complexities. The most complicated part in the calculation are two-loop three-point subamplitudes with one Higgs boson, one on-shell gluon and one off-shell gluon, with the latter connecting to a $t$-channel (or $u$-channel) gluon propagator in the full three-loop diagrams.

In the following, we focus on the calculation of the $gg^*H$ subamplitude. We decompose the latter in terms of Lorentz structures

$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = V_0 \left( F_a\, g^{\alpha\beta}(q_s \cdot q_2) + F_b\, q_s^\alpha q_2^\beta + F_c\, q_2^\alpha q_s^\beta + F_d\, q_s^\alpha q_s^\beta + F_e\, q_2^\alpha q_2^\beta \right), \quad (1)$$

where $q_s$ and $q_2$ are the momenta of the off-shell and on-shell gluon, respectively, and $V_0$ is a normalization factor. The form factors $F_i$ are scalar functions of the Higgs mass, $m_H$, the top-quark mass, $m_t$ and the virtuality of the off-shell gluon, $q_s^2$. We perform an integration-by-parts (IBP) reduction, using `KIRA` [24], retaining full dependence on these three scales and arriving at

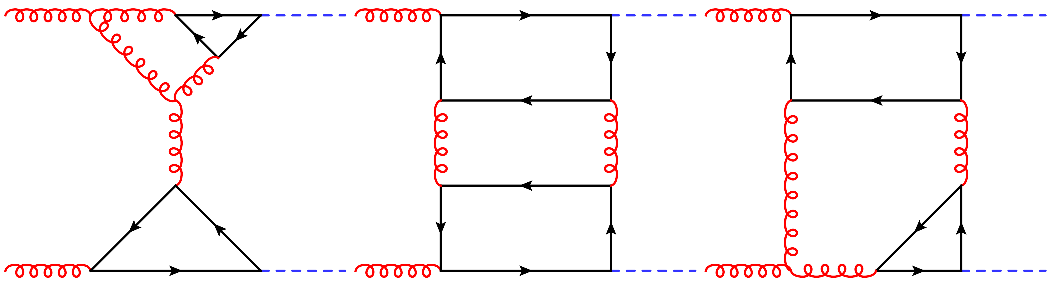

**Figure 1:** Representative Feynman diagrams for the three-loop virtual corrections to $gg \to HH$. The diagrams discussed in this contribution feature the external Higgs bosons coupling to two distinct top-quark loops, represented with solid black lines.

a decomposition of the two-loop form factors in terms of 46 master integrals (MI). In order to have a reliable approximation for any value of $q_s^2$, we carry out two independent expansions in the following kinematic limits

$$m_H^2 \ll q_s^2, m_t^2 \qquad\qquad q_s^2 \ll m_H^2, m_t^2. \tag{2}$$

Both expansions are asymptotic expansions. Following the strategy of the expansion by regions, each expansion requires the calculation of a "hard" and a "soft" region, which can be identified using `asy.m` [25].

- In the small-$m_H$ expansion (denoted as $m_H^{\text{EXP}} \to 0$), the expansion in the hard region is implemented as a Taylor expansion in $m_H^2$; a subsequent IBP reduction leads to single-scale MIs that are a subset of the topologies of Ref. [12] and can be evaluated semi-analytically using the "expand-and-match" method [26]. The expansion in the soft region is carried out in the large-mass-expansion [27, 28].

- In the small-$q_s^2$ expansion (denoted as $q_s^2 \to 0$), the expansion in the hard region is again a Taylor expansion, with MIs well known from the $gg \to H$ amplitude at NLO in QCD. The expansion in the soft region is obtained by direct integration of the relevant integrals using Mellin-Barnes techniques, with the help of the packages `HarmonicSums` [29–40], `Sigma` [41, 42] and `EvaluateMultiSums` [43, 44].

The results for one of the form factors are shown in Fig. 2. One can see that the combination of the small-$q_s^2$ expansion and the small-$m_H$ expansion accurately reproduces the results of an exact numerical evaluation using `AMFLow` [45] for almost any value of $q_s^2$. The region $-q_s^2 \simeq 10^4$ GeV is covered by a different version of the small-$m_H$ expansion, denoted as $m_H \to 0$ in Fig. 2, in which only the master integrals are expanded while their coefficients are exact. The results for the two-loop $gg^*H$ subamplitude can then be combined with other one-loop subamplitudes, such that we obtain results for the reducible contribution for the two form factors for $gg \to HH$ that are reliable and accurate over the complete phase space.

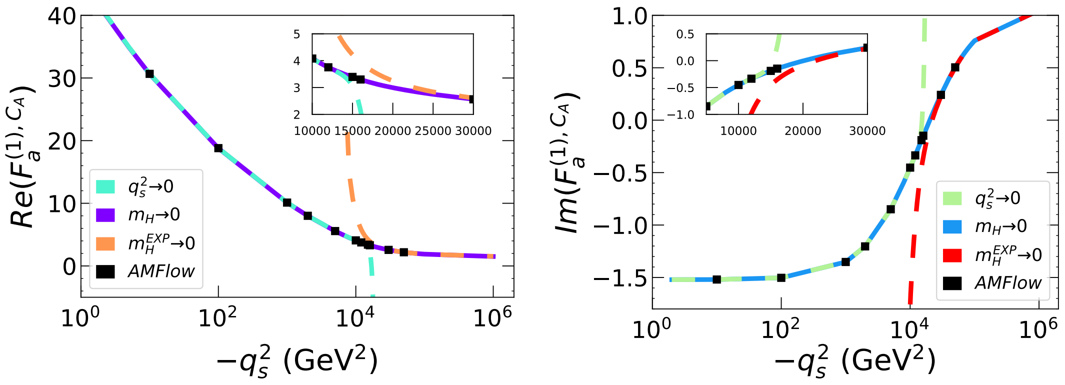


**Figure 2:** Results for one of the form factors contributing to the $gg^*H$ subamplitude. Different expansions are plotted against the exact evaluation using `AMFLow`. The inset shows the region in the gluon virtuality where the $m_H \to 0$ expansion is needed.

## 3. Irreducible $n_h^2$ contribution

Diagrams like the ones in the middle and right of Fig. 1 are genuinely three-loop, and we evaluate them in the forward limit, defined as

$$m_H^2, t \ll s, m_t^2. \tag{3}$$

Also this limit implies an asymptotic expansion, with varying number and nature of the regions depending on the topologies considered. The contributions from the hard region can always be computed with a Taylor expansion following Refs. [21, 22]. On the other hand, we observe that the treatment of the non-hard regions in momentum-space representation is rather complicated, and we found that the evaluation of the relevant integrals using Mellin-Barnes techniques showed promising results for planar topologies.

## 4. Conclusion

We have discussed how the $n_h^2$ contribution represents a new challenging class of diagrams that appears in the calculation of the virtual NNLO QCD corrections to $gg \to HH$. We have shown that expansion methods offer a promising way to obtain accurate results for this contribution. Contrary to what observed so far in the application of forward expansions, these new topologies require asymptotic expansions, which makes the calculation more complicated.

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