PROCEEDINGS OF SCIENCE

Renormalization-group improved prediction of $H \to gg$ decay rate

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We investigate the renormalization-group scale and scheme dependence of the $H \to gg$ decay rate at the order N^{4}LO within the renormalization group summed perturbation theory, which incorporates the resummation of all RG-accessible logarithms up to the leading and four subsequent sub-leading logarithmic contributions of the perturbative expansion. In addition, we analyze the higher-order behaviour of the $H \to gg$ decay width using the asymptotic Padé approximant method. A complementary assessment of the higher-order structure is carried out using the asymptotic Padé-Borel approximant method. The predictions obtained from the Padé-Borel approximant framework are found to be consistent with those derived from the asymptotic Padé approximant analysis.

The European Physical Society Conference on High Energy Physics (EPS-HEP2025)
7-11 July 2025
Marseille, France

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 ISSN 1824-8039 . Published by SISSA Medialab.

PoS(EPS-HEP2025)383

1. Introduction

The decay of Higgs boson to gluons plays a central role in precision Higgs phenomenology, providing sensitivity to QCD dynamics and potential physics beyond the Standard Model (SM). The Higgs boson dominantly decays to pair of bottom quarks in SM that is, $H \to \bar{b}b$(+hadrons), followed by second leading hadronic decay channel $H \to gg$, which is predominantly mediated by the top quark.

The hadronic processes such as $H \to gg$ are typically evaluated only to finite order in perturbative QCD, and as a result these predictions exhibit a pronounced sensitivity to the renormalization-group (RG) scale μ. In this work, we investigate the $H \to gg$ decay width in the renormalization-group summed perturbation theory (RGSPT) [1]-[15], which resums all RG-accessible logarithms through closed-form expressions derived from RG invariance. One of the salient features of the new RGSPT expansions is the reduced sensitivity to RG-scale μ in spite of the presence of large logarithms [2]. Additionally, we investigate the higher-order behaviour of $H \to gg$ decay width using the asymptotic Padé approximant (APAP) method and through the asymptotic Padé-Borel approximant (PBA) method. These complementary approaches yield consistent higher-order estimates and support the robustness of RGSPT for achieving precise predictions of the $H \to gg$ decay width.

2. Theoretical framework

The partial decay width of the Higgs boson to gluons $\Gamma_{H\to gg}$ decay is given as,

$$\Gamma_{H\to gg} = \frac{\sqrt{2}\, G_{\rm F}}{M_H} \, |C_1|^2 \, {\rm Im}\, \Pi^{GG}(-M_H^2 - i\delta), \tag{1}$$

where M_H is the mass of the Higgs boson, Π^{GG} represents the contribution to the self-energy of the Higgs boson due to its effective coupling to gluons, and δ is a positive real parameter, which is infinitesimally small. The coefficient C_1 is known up to $\mathrm{N^4LO}$ and the absorptive part of the vacuum polarization $\Pi^{GG}(q^2)$ is computed at $\mathrm{N^4LO}$ in reference [16], thus, making the Higgs-boson to gluons decay width complete at the $\mathrm{N^4LO}$ order.

In the fixed-order perturbation theory (FOPT) the partial decay width $\Gamma_{H\to gg}$ can be written as,

$$\Gamma_{H\to gg} = \left[\sqrt{2} G_F M_H^3/72\pi\right] x^2(\mu) S\left[x(\mu), L(\mu)\right]. \tag{2}$$

The perturbative expansion $S[x(\mu), L(\mu)]$ in the FOPT can be written as,

$$S_{\rm FOPT}[x(\mu), L(\mu)] = \sum_{n=0}^{\infty} \sum_{k=0}^{n} T_{n,k} x^n L^k, \tag{3}$$

where $x(\mu) = \frac{\alpha_s(\mu)}{\pi}$ and $L(\mu) = ln(\mu^2/m_t^2(\mu))$.

In the RGSPT, the FOPT expansion of the function $S[x(\mu), L(\mu)]$ is equivalent to writing the following new expansion,

$$S(x, L) = \sum_{n=0}^{\infty} x^n S_n(u), \tag{4}$$

where $u = xL$, and the function $S_n(u)$ is defined by,

$$S_n(u) \equiv \sum_{k=n}^{\infty} T_{k,k-n} u^{k-n}. \tag{5}$$

The main feature of the RGSPT is the explicit all-orders summations of all RG-accessible logarithms in the functions $S_n(u)$ [2,3]. Moreover, the functions $S_n(u)$ can be derived in a closed analytical form [2,3] using RG-invariance of $\Gamma_{H\to gg}$.

The decay width $\Gamma_{H\to gg}$ decay defined in equation 2 is scale independent and satisfies the RG equation,

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \left\{\Gamma_{H\to gg}\right\} = 0. \tag{6}$$

Substituting the perturbative expansion of 5 in the RG-equation 6, we get a system of linear equations as follows,

$$(1-\beta_0 u)\frac{\mathrm{d}S_n}{\mathrm{d}u} - u\sum_{\ell=0}^{n-1}\beta_{\ell+1}\frac{\mathrm{d}S_{n-\ell-1}}{\mathrm{d}u} + 2\sum_{\ell=0}^{n-1}\gamma_\ell \frac{\mathrm{d}S_{n-\ell-1}}{\mathrm{d}u} - \sum_{\ell=0}^{n}(n-\ell+2)\beta_\ell S_{n-\ell} = 0 \tag{7}$$

The above system of equations can be solved iteratively to get closed-form expressions of $S_n(u)$ [17].

The new RGS expansions now can be written as,

$$S^{N^4LO}_{RGSPT} = S_0(xL) + xS_1(xL) + x^2 S_2(xL) + x^3 S_3(xL) + x^4 S_4(xL) \tag{8}$$

The above RGSPT expansions exhibit good stability and reduced sensitivity to RG scale μ.

3. Asymptotic Padé approximant improved $H \to gg$ decay rate

The Padé approximant for a generic perturbative expansion of the form,

$$S \equiv 1 + R_1 x + R_2 x^2 + R_3 x^3 + R_4 x^4 + \cdots, \tag{9}$$

where the coefficients $\{R_1, R_2, R_3, R_4\}$ are known and the coefficients $\{R_5, \cdots\}$ are unknown, is denoted by,

$$\begin{aligned} S_{[N|M]} &\equiv \frac{1 + a_1 x + a_2 x^2 + \cdots + a_N x^N}{1 + b_1 x + b_2 x^2 + \cdots + b_M x^M} \\ &= 1 + R_1 x + R_2 x^2 + R_3 x^3 + R_4 x^4 + \cdots + R_{N+M+1}\, x^{N+M+1} + \cdots. \end{aligned} \tag{10}$$

The asymptotic error in the Padé approximant prediction is given by,

$$\frac{R^{Padé}_{N+M+1} - R_{N+M+1}}{R_{N+M+1}} = -\frac{M! A^M}{[N + M + aM + b]^M} \tag{11}$$

where $R^{Padé}_{N+M+1}$ is the prediction obtained from $[N|M]$ Padé approximant . This asymptotic error allows us to estimate an improved value of the true value R_{N+M+1} of the coefficient, which is referred to as APAP prediction in literature [18]. In this work [17] , we choose $a = b = 0$ which provide the best predictions. Our APAP estimate of R_5 is,

$$R_5 = \frac{8R_2^2(R_3^3 - 2R_2R_3R_4 + R_1R_4^2)}{(R_1^4 - 2R_1^2R_2 - 7R_2^2)(R_2^2 - R_1R_3)}. \tag{12}$$

4. Asymptotic Padé-Borel approximant improved $H \to gg$ decay rate

In addition to APAP formalism, we also estimate the higher-order contribution to $\Gamma_{H\to gg}$ using PBA method. Under this formalism we apply the generalized Borel transform 13 to the perturbative expansion of $\Gamma_{H\to gg}$.

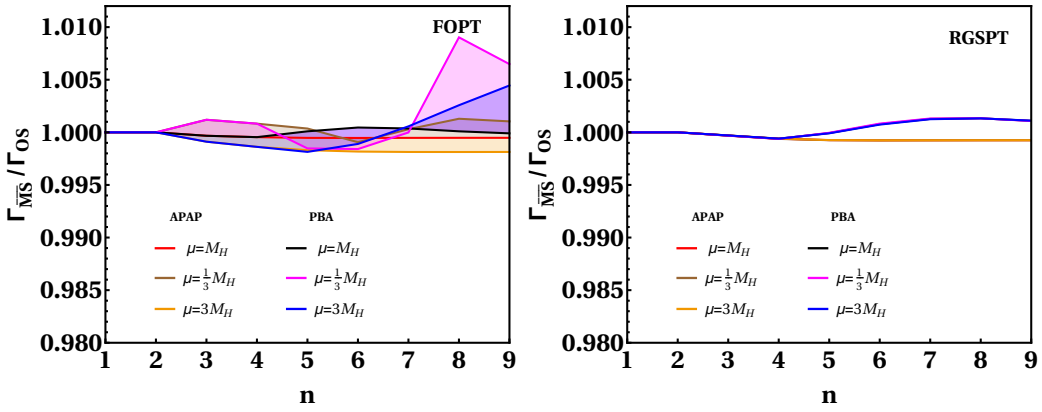
We define the generalized Borel transform of the perturbative expansion 9 as,

$$B[S](u) = \sum_{n}^{\infty} \left(\frac{d_1}{n!} + \frac{d_2}{n!^2} + \frac{d_3}{n!^3} + \frac{d_5}{n!^5} \right) R_n u^n, \tag{13}$$

where $d_{1,2,3,5}$ are the scheme-dependent real constants given in table 1.

Schemes	d_1	d_2	d_3	d_5
$\overline{\text{MS}}$	0.5	1.5	0	1.2
OS	1	0	1.623	0
SI	0.87	0	1.6	0

Table 1: The numerical values of the constants $d_{1,2,3,5}$.



(a) (b)

Figure 1: The variation of $\Gamma_{\overline{\text{MS}}}/\Gamma_{\text{OS}}$ at RG scales $\mu = \frac{1}{3}M_H, M_H$, and $3M_H$ in the 1(a) FOPT and 1(b) RGSPT up to order $n = 9$.

The higher-order behaviour of the ratio of the Higgs to gluons decay width in the $\overline{\text{MS}}$ and OS schemes, as predicted by the APAP and PBA methods in the FOPT framework, is shown in Fig. 1(a). The two formalisms exhibit very good agreement. The corresponding predictions in the RGSPT framework, displayed in Fig. 1(b), likewise show consistent behaviour across the three RG scales. As expected, the RGSPT expansions remain remarkably stable with respect to the RG scale even at higher orders.

5. Higgs to gluons decay width

Our final prediction of the $\Gamma_{H\to gg}$ decay width at the order N^{5}LO in the $\overline{\text{MS}}$, SI and OS schemes are given by 14 in the FOPT and by 15 in the RGSPT ,

$$\begin{aligned}
\Gamma^{\overline{\text{MS}}}_{\text{N}^5\text{LO}} &= \Gamma_0\Big(1.837 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0009_{\text{P}} \pm 0.007_{\text{s}}\Big), \qquad (14)\\
\Gamma^{\text{SI}}_{\text{N}^5\text{LO}} &= \Gamma_0\Big(1.837 \pm 0.046_{\alpha_s(M_Z),1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0026_{\text{P}} \pm 0.007_{\text{s}}\Big),\\
\Gamma^{\text{OS}}_{\text{N}^5\text{LO}} &= \Gamma_0\Big(1.838 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0023_{\text{P}} \pm 0.007_{\text{s}}\Big),
\end{aligned}$$

$$\begin{aligned}
\Gamma^{\overline{\text{MS}}}_{\text{RGSN}^5\text{LO}} &= \Gamma_0\Big(1.840 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0007_{\text{P}}\Big), \qquad (15)\\
\Gamma^{\text{SI}}_{\text{RGSN}^5\text{LO}} &= \Gamma_0\Big(1.841 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0018_{\text{P}}\Big),\\
\Gamma^{\text{OS}}_{\text{RGSN}^5\text{LO}} &= \Gamma_0\Big(1.842 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0019_{\text{P}}\Big).
\end{aligned}$$

Here, P stands for the uncertainty due to the PBA predictions, and s denotes the uncertainty due to the series expansion. The central value of the $\Gamma_{H\to gg}$ decay width is evaluated at RG scale $\mu = M_H$.

We find that the sensitivity of the $\Gamma_{H\to gg}$ decay width to the top-quark mass is extremely small: a 4 GeV variation in M_t leads to only a 0.02% change in the decay width in the $\overline{\text{MS}}$, SI and OS schemes. The dependence on the Higgs boson mass is likewise mild, contributing about 0.36% to the uncertainty across these schemes. In contrast, the dominant source of uncertainty arises from the strong coupling constant: a 1% variation in $\alpha_s(M_Z^2)$ induces a $(2.5-2.6)$% shift in the predicted $\Gamma_{H\to gg}$ decay width [17].

6. Conclusion

In this article we have examined the RG-scale and scheme dependence of the $\Gamma_{H\to gg}$ decay width using both the FOPT and the RGSPT. By resumming all the RG-accessible logarithms, the RGSPT framework yields markedly improved stability and a substantial reduction in scale dependence across the $\overline{\text{MS}}$, SI and OS schemes [17]. We have also employed the APAP and PBA methods to estimate higher-order effects, finding consistent and stable behaviour at N^{5}LO. The truncation error is significantly reduced from 0.6% at N^{4}LO to 0.4% at N^{5}LO [17]. The dominant uncertainty remains due to $\alpha_s(M_Z^2)$. These results enhance the precision of the theoretical prediction for $H \to gg$, relevant for future high-precision Higgs measurements.

References

[1] C. J. Maxwell, *Complete renormalization group improvement: Avoiding scale dependence in QCD predictions*, *Nucl. Phys. B Proc. Suppl.* **86** (2000) 74–77 [hep-ph/9908463].

[2] M. R. Ahmady, F. A. Chishtie, V. Elias, A. H. Fariborz, N. Fattahi, D. G. C. McKeon, T. N. Sherry and T. G. Steele, *Closed form summation of RG accessible logarithmic contributions to semileptonic B decays and other perturbative processes*, *Phys. Rev. D* **66** (2002) 014010

[hep-ph/0203183].
[3] M. R. Ahmady, F. A. Chishtie, V. Elias, A. H. Fariborz, D. G. C. McKeon, T. N. Sherry, A. Squires and T. G. Steele, *Optimal renormalization group improvement of the perturbative series for the e^+e^- annihilation cross-section*, *Phys. Rev. D* **67** (2003) 034017 [hep-ph/0208025].
[4] G. Abbas, B. Ananthanarayan, I. Caprini and J. Fischer, *Perturbative expansion of the QCD Adler function improved by renormalization-group summation and analytic continuation in the Borel plane*, *Phys. Rev. D* **87** (2013) 014008 [arXiv:1211.4316].
[5] G. Abbas, B. Ananthanarayan and I. Caprini, *Determination of $\alpha_s(M_{\tau^2})$ from Improved Fixed Order Perturbation Theory*, *Phys. Rev. D* **85** (2012) 094018 [arXiv:1202.2672].
[6] G. Abbas, B. Ananthanarayan, I. Caprini and J. Fischer, *Expansions of tau hadronic spectral function moments in a nonpower QCD perturbation theory with tamed large order behavior*, *Phys. Rev. D* **88** (2013) 034026 [arXiv:1307.6323].
[7] G. Abbas, B. Ananthanarayan and I. Caprini, *Determination of the strong coupling from hadronic tau decays using renormalization group summed perturbation theory*, *Mod. Phys. Lett. A* **28** (2013) 1360004 [arXiv:1306.1095].
[8] I. Caprini, *Strong coupling from the tau hadronic width by non-power QCD perturbation theory*, *Mod. Phys. Lett. A* **28** (2013) 1360003 [arXiv:1306.0985].
[9] I. Caprini, J. Fischer, G. Abbas and B. Ananthanarayan, *Perturbative Expansions in QCD Improved by Conformal Mappings of the Borel Plane*, in *Perturbation Theory: Advances in Research and Applications*, Nova Science Publishers (2018) 211–254 [arXiv:1711.04445].
[10] B. Ananthanarayan and D. Das, *Optimal renormalization and the extraction of the strange quark mass from moments of the tau-decay spectral function*, *Phys. Rev. D* **94** (2016) 116014 [arXiv:1610.08900].
[11] B. Ananthanarayan, D. Das and M. S. A. Alam Khan, *QCD static energy using optimal renormalization and asymptotic Pade-approximant methods*, *Phys. Rev. D* **102** (2020) 076008 [arXiv:2007.10775].
[12] B. Ananthanarayan, D. Das and M. S. A. Alam Khan, *Renormalization group improved m_s and $|V_{us}|$ determination from hadronic tau decays*, *Phys. Rev. D* **106** (2022) 114036 [arXiv:2207.00754].
[13] M. S. A. Alam Khan, *Renormalization group improved determination of light quark masses from Borel-Laplace sum rules*, *Phys. Rev. D* **108** (2023) 094016 [arXiv:2306.10266].
[14] M. S. A. Alam Khan, *Renormalization group summation and analytic continuation from spacelike to timelike regions*, *Phys. Rev. D* **108** (2023) 014028 [arXiv:2306.10262].
[15] M. S. A. Alam Khan, *Renormalization group improved determination of α_s, m_c, and m_b from the low energy moments of heavy quark current correlators*, *Phys. Rev. D* **108** (2023) 074029 [arXiv:2306.10323].
[16] F. Herzog, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, *On Higgs decays to hadrons and the R-ratio at N^4LO*, *JHEP* **08** (2017) 113 [arXiv:1707.01044].
[17] G. Abbas, A. Jain, V. Singh and N. Singh, *Renormalization-group improved Higgs to two gluons decay rate*, *Eur. Phys. J. Plus* **139** (2024) 114 [arXiv:2205.06061].
[18] J. R. Ellis, M. Karliner and M. A. Samuel, *A prediction for the four loop beta function in QCD*, *Phys. Lett. B* **400** (1997) 176–181 [hep-ph/9612202].