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One-loop symmetry-restoring finite counter-terms in the Standard Model Effective Field Theory

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We discuss dimensional regularization in the Breitenlohner-Maison-'t Hooft-Veltman scheme. Our goal is to calculate within this framework finite counter-terms in chiral theories which reestablish the Slavnov-Taylor identities at one loop. These counter-terms are necessary due to the presence of evanescent symmetry-breaking terms in the classical Lagrangian, which are needed to regularize fermion propagators. We focus on dimension-6 four-fermion operators of the Standard Model Effective Field Theory, determining the required set of finite counter-terms. As expected, we find no obstructions to the Slavnov-Taylor identities which cannot be removed by the introduction of such category of counter-terms.

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The Standard Model Effective Field Theory (SMEFT) parameterizes new physics effects above the electroweak scale, which obey the local symmetries of the SM. There can be operators of different dimensionalities, which encode possibly different phenomenological aspects. There is a single operator of dimension 5, and many more at dimension 6, with a growing interest for operators of even higher dimensions. The renormalization of operators of dimension 6 has been fully completed, with finite, matching effects onto the low-energy EFT already calculated (in a scheme mostly consisting of naive dimensional regularization) [1].

The interest of moving to higher orders goes beyond the quest for precision. For instance, many operators do not mix at one loop, mixing first at two loops, such as in the case of many four-fermion operators mixing into dipole operators. Here, we would like to discuss one step towards such higher orders, consisting of implementing the one-loop renormalization by finite terms which are necessary in order to establish basic symmetry properties at the quantum level.

We discuss dimensional regularization, and so the algebra of γ-matrices is extended to an arbitrary dimension D. When projecting onto 4 or the remaining $D-4$ dimensions (by means of the metric $\bar{g}_{\mu\nu}$ or $\hat{g}_{\mu\nu}$, respectively), objects will hereafter be indicated with bars or hats. In vector-like theories it is straightforward to perform calculations with this regularization scheme, e.g., the possible forms of infinities are fixed by symmetry arguments. In chiral theories, however, the situation is more complicated, due to the need to manipulate chiral projectors. The only known mathematically consistent scheme to deal with γ_5 in dimensional regularization is the so-called Breitenlohner-Maison-'t Hooft-Veltman (BMHV) scheme [2–5], which leads to unambiguous results.[1] In particular, within this scheme γ_5 has both anti-commutation and commutation relations

$$\{\gamma_5, \bar{\gamma}^\mu\} = 0\,, \quad [\gamma_5, \hat{\gamma}^\mu] = 0\,. \tag{1}$$

The known problem with the BMHV scheme is the breaking of chiral invariance, which can be appreciated by analyzing the fermion kinetic term

$$i\bar{\psi}\partial\!\!\!/\psi = i\overline{\psi_R}\bar{\partial\!\!\!/}\psi_R + i\overline{\psi_L}\bar{\partial\!\!\!/}\psi_L + i\overline{\psi_L}\hat{\partial\!\!\!/}\psi_R + i\overline{\psi_R}\hat{\partial\!\!\!/}\psi_L\,. \tag{2}$$

We want to achieve the expression $ip\!\!\!/ /p^2$ for the propagator, which is correctly regularized. In the right-hand side of Eq. (2), one cannot stick to four-dimensional objects, because they lead to the un-regularized expression $i\bar{p\!\!\!/}/\bar{p}^2$. One has then to include the evanescent part, which consists of terms mixing left- and right-handed degrees of freedom. Therefore, it breaks chiral gauge invariance. Moreover, this evanescent expression would also break global chiral symmetries: e.g., in the SM left-handed degrees of freedom come in doublets, while right-handed ones in singlets. The solution to avoid the breaking of global chiral symmetries is to introduce "fictitious fields" that transform the same way as the physical fields under global symmetries. These fictitious fields only appear in the kinetic terms and do not couple to the gauge sector [6–8]; in other words, they are sterile in our adopted scheme. As it should be clear, the breaking of chiral gauge symmetries happens already at the tree level in dimensional regularization, and will leave footprints at the quantum level after taking the limit to 4 dimensions. The use of fictitious fields allows to control the form of these quantum effects, which must then respect global chiral symmetries.

[1] There however remain scheme choices, such as the subtraction of BRST invariant operators as it will be later mentioned.

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In order to study symmetries at the quantum level, one introduces the Slavnov-Taylor (ST) identities. *The symmetry we want to analyze is here the BRST transformation*, which follows from the gauge fixing procedure. This BRST symmetry has the inconvenience of being non-linear; however, its nilpotency greatly simplifies its analysis. In the case of a vector-like theory, one way of expressing these identities is as such

$$\int \mathrm{d}^D x \frac{\delta_R \Gamma\,[X, K]}{\delta K_n\,(x)} \frac{\delta_L \Gamma\,[X, K]}{\delta X^n\,(x)} = 0\,, \tag{3}$$

where Γ is the quantum effective action (QEA), built from connected 1PI amplitudes, and the (left or right) variational derivatives are made with respect to the classical fields X and the external sources K present in the action accompanying the BRST transformations of the fields. There is a more compact and practical way of formulating these identities, through the so-called Slavnov operator or the Zinn-Justin antibracket [6, 9], which however we will not introduce in this note.

In the case of chiral theories, one can find instead the following expression [10]

$$\int \mathrm{d}^D x \frac{\delta_R \Gamma\,[X, K]}{\delta K_n\,(x)} \frac{\delta_L \Gamma\,[X, K]}{\delta X^n\,(x)} = \lim_{Q\to 0} \int \mathrm{d}^D x \frac{\delta_R \Gamma\,[X, K, Q]}{\delta Q\,(x)}\,. \tag{4}$$

I.e., there is a so-called "obstruction" to the ST identities, which are not satisfied anymore in general. Here the new object is Q, which is the source for the BRST variation of the symmetry-breaking term $i\overline{\psi}\hat{\partial\!\!\!/}\psi$, introduced before when discussing Eq. (2). Therefore, the meaning of the right-hand side of Eq. (4) is that one has to compute those diagrams with single insertions of the variation of the evanescent term $i\overline{\psi}\hat{\partial\!\!\!/}\psi$; this is illustrated in Fig. 1 by vertices represented by a cross. Before further discussing this point, let us mention that "obstructions" in general can refer to two very different objects: true anomalies, which would then invalidate the consideration of the chiral gauge theory, or obstructions that, after renormalizing the QEA by a finite, non-evanescent amount, are compensated for, as we further discuss next.

From the reading of the obstruction to the ST identities in Eq. (4) one has to consider insertions of the BRST transformation of the symmetry-breaking evanescent piece $i\overline{\psi}\hat{\partial\!\!\!/}\psi$ of the kinetic expression for fermions. One ends up then with the following evanescent expression for a physical left-handed fermion paired with a fictitious right-handed fermion

$$s\left[i\overline{\psi}\hat{\partial\!\!\!/}\psi\right] = \omega^{\alpha}\,\overline{\psi}\,t^L_{\alpha}\left(\overrightarrow{\hat{\partial\!\!\!/}}\,P_R + \overleftarrow{\hat{\partial\!\!\!/}}\,P_L\right)\psi\,, \tag{5}$$

where ω refers to the set of ghosts of the theory, including the Abelian case(s). The situation of a physical right-handed fermion paired with a fictitious left-handed fermion leads to an analogous expression. If the obstruction is not coming from a true anomaly, one can consider shifting the QEA by a finite, non-evanescent amount S_{fct}, which at the level of the ST identities consists of having the BRST variation of a finite piece

$$s[S_{\mathrm{fct}}] = -\lim_{Q\to 0} \int \mathrm{d}^D x \frac{\delta_R \Gamma\,[X, K, Q]}{\delta Q\,(x)}\,. \tag{6}$$

If such a finite counter-term S_{fct} can be introduced in this way such that its variation is equal to minus the obstruction, then we can cancel the effect of the obstruction, thus restoring the ST identities.

We will illustrate this in the following. A more general framework can be established from the introduction of Batalin-Vilkovisky antifields [6, 9], which we will not do here.

There is a straightforward way of computing the obstructions to the ST identities [11, 12]. It relies on the fact that these obstructions come from ϵ/ϵ terms, where ϵ is the regulator in dimensional regularization. Without going into the details of the computation, it means that one can just identify $1/\epsilon$ terms, which is a much simpler task than fully calculating the QEA and determining from it the obstructions to the ST identities from a direct use of Eq. (4). Since these terms come from infinities, it means that the obstructions to the ST identities are local objects.

Let us then start discussing concrete cases. First we would like to mention that the calculation in, e.g., a renormalizable and in an EFT cases can be different, which means that the calculation has to be performed *for each theory considered*. These contributions are then needed when performing the matching between the two theories. The low-energy EFT is discussed in Ref. [13].

To give a concrete, simple example, consider an SM 1PI diagram in which an $SU(2)_L$ gauge boson of gauge coupling g_L is internally exchanged by a fermion open line of left-handed chirality, to which a $U(1)_Y$ ghost λ is attached; the left-handed fermion ψ^i_L carries y_L charge under the latter gauge symmetry. We find a non-vanishing result proportional to

$$\frac{y_L g_L^2}{16\pi^2}\overline{\psi^i_L}\bar{\gamma}^\mu\psi^i_L\bar{\partial}_\mu\lambda\,, \tag{7}$$

which requires the introduction of a counter-term proportional to (up to BRST-vanishing structures)

$$\frac{y_L g_L^2}{16\pi^2}\overline{\psi^i_L}\bar{\not{A}}\psi^i_L\,, \tag{8}$$

where only four-dimensional parts were kept; see also Ref. [8], where a dependence on the ξ-gauge is explicitly shown. No term proportional to $y_R g_L^2\overline{\psi^i_L}\bar{\gamma}^\mu\psi^i_L\bar{\partial}_\mu\lambda$ can be produced in our approach, where y_R is the charge under $U(1)_Y$ of a right-handed physical degree of freedom.[2] Diagrams with an extra gauge boson being radiated do not contribute, as the superficial degree of divergence is zero, and as explained in the Appendix B of Ref. [10] (by also considering the structure of the coupling of the symmetry-breaking vertex) no obstruction can come from such one-loop cases when the symmetry-breaking vertex is attached to an internal fermion line; as also discussed therein, the renormalization of four-fermion Green's functions does not require symmetry-restoring counter-terms at one loop. The flavors i of the fermion and anti-fermion are the same in the above expressions, and the effect is universal across the three generations; in the massive case, the unitary mixing matrix in charged currents satisfies orthogonality relations, i.e., the GIM mechanism prevents a flavor changing structure (such as $b \to s$) at the one-loop order, which is also connected to the ϵ/ϵ local terms we are interested in.

[2]We find difficulties in our implementation of the so-called spurion method of Ref. [14]. Namely, in this case we find in the numerator of the term proportional to y_L

$$P_R\gamma_{\bar{\mu}_1}P_L\left[\gamma^{\bar{\nu}}p_{\bar{\nu}} + \left(\Omega_0 P_L + \Omega_0^\dagger P_R\right)\gamma^{\hat{\nu}}p_{\hat{\nu}}\right]P_R\gamma_{\bar{\mu}}P_L\left[\gamma^{\bar{\rho}}p_{\bar{\rho}} + \left(\Omega_0 P_L + \Omega_0^\dagger P_R\right)\gamma^{\hat{\rho}}p_{\hat{\rho}}\right]P_R\gamma_{\bar{\mu}_2}P_L\,, \tag{9}$$

when calculating a similar SM 1PI diagram as previously, with the $U(1)_Y$ ghost being replaced by the corresponding gauge boson A^μ. This expression does not produce terms quadratic in the spurion $\Omega_0^{(\dagger)}$: terms in $\Omega_0 P_L$ (or $\Omega_0^\dagger P_R$) vanish due to the P_R (respectively, P_L) projectors on their right (respectively, left).

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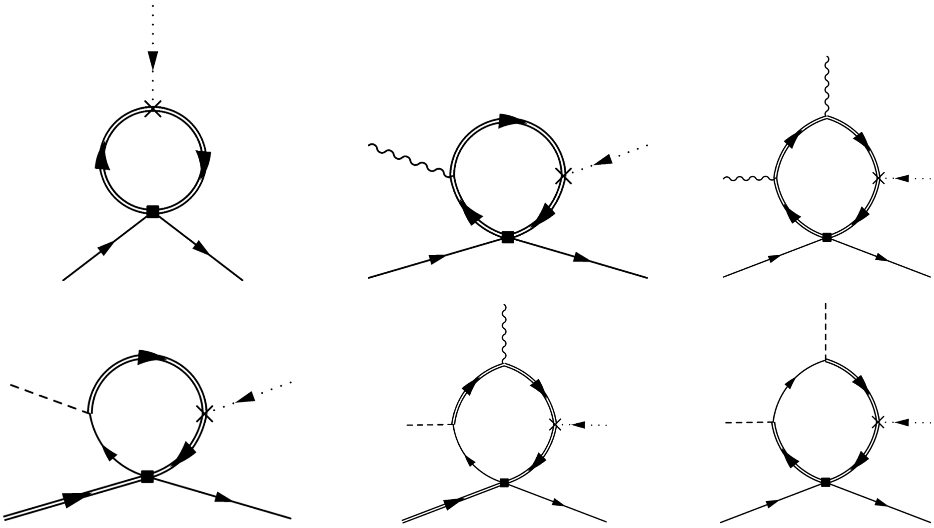

Figure 1: Examples of 1PI diagrams with single insertions of SMEFT four-fermion operators (represented by filled squares) at one loop. Insertions of the symmetry-breaking operator are indicated by a cross, and ghosts are represented by a dotted line. Scalars are indicated by dashed lines, and gauge bosons by wiggly lines. The single (double) solid lines represent physical right-handed (left-handed) fermions; analogous diagrams with opposite chiralities are also possible.

Moving to SMEFT, here we discuss only single insertions of four-fermion operators. We illustrate some of the diagrams that have to be computed in Fig. 1. One has to consider a complete set of possible finite structures, perform their BRST transformation and then equate their linear combination of unknown coefficients to the result of all the possible 1PI diagrams; ambiguities appear if the basis is non-minimal, e.g., the set of finite counter-terms is defined up to a BRST-invariant combination. It means that we face a calculation similar to the one of renormalizing divergences in SMEFT at the one loop.

Let us consider a concrete example in SMEFT. All four-fermion operators of the Warsaw basis can be considered. We pick one case for illustration, namely, $Q_{qx}^{(8)}$, $x = u, d$, which turned out being amongst the most challenging ones. We have then a very long expression for the obstruction to the ST identities, of which we display one single element

$$\propto d_{ABC}\, (\overline{\xi_R}\gamma_\mu T^A \xi_R)\, \partial_\rho \mathbf{G}^B_\nu \partial_\sigma \mathbf{g}^C \epsilon^{\mu\nu\rho\sigma} \,, \tag{10}$$

where $\xi_R = u, d$, capital $\mathbf{G}$ is the gluon field, the lower case $\mathbf{g}$ its ghost, and $d_{ABC} = 2\,\mathrm{Tr}\{T^A(T^B T^C + T^C T^B)\}$, which is a totally symmetric structure in A, B, C. We look for a linear combination of operators of dimension 6, which can serve as counter-terms. In this example, the structure in Eq. (10) can be compensated for by the first piece of the BRST transformation of the counter-term proportional to $d_{ABC}\, (\overline{\xi_R}\gamma_\mu T^A \xi_R)\, \partial_\rho \mathbf{G}^B_\nu \mathbf{G}^C_\sigma \epsilon^{\mu\nu\rho\sigma}$ in the following expression

$$\begin{aligned} & s[d_{ABC}\, (\overline{\xi_R}\gamma_\mu T^A \xi_R)\, \partial_\rho \mathbf{G}^B_\nu \mathbf{G}^C_\sigma \epsilon^{\mu\nu\rho\sigma}] \\ &\quad = d_{ABC}\, (\overline{\xi_R}\gamma_\mu T^A \xi_R)\, \left(\partial_\rho \mathbf{G}^B_\nu \partial_\sigma \mathbf{g}^C + C_{BEF} \mathbf{G}^C_\sigma \mathbf{G}^E_\nu \partial_\rho \mathbf{g}^F\right) \epsilon^{\mu\nu\rho\sigma} \,. \end{aligned} \tag{11}$$

The operator being BRST-transformed in Eq. (11) is an example of a non-evanescent, symmetry-breaking operator needed to restore the ST identities at one loop. Note that, although QCD is a vector-like theory, left-handed and right-handed degrees of freedom carry different quantum numbers, so symmetry-breaking terms in presence of QCD can be generated. This renormalization program has been executed for all four-fermion operators, and as expected all obstructions could be cured by appropriate finite counter-terms.[3]

In the course of our analysis, Ref. [16] was released. They employ a different approach, based on the so-called spurion external field, while we employ here the more traditional algebraic renormalization method [8, 17, 18]. An advantage of the spurion approach is that counter-terms are found more straightforwardly, without the need to consider ("inverse") BRST transformations. Compared to Ref. [14], which first introduces it, Ref. [16] employs fictitious fields (called evanescent fields by them), and functional integration. A detailed comparison is currently ongoing.

Before concluding, let us mention a physical application to higher orders. Consider for instance a heavy scalar sector, which when integrated out leads to four-fermion contact interactions, see Refs. [19, 20]. At the level of two loops, one has to consider one-loop insertions of one-loop finite counter-terms, in order to correctly determine the anomalous dimension matrix elements. In such two-loop endeavor, the SM one-loop renormalization is required in general. Some other multi-loop calculations have already been considered in the case of renormalizable interactions, e.g., Ref. [21]. As illustrated therein, non-finite gauge non-invariant pieces are found at higher order.

This brings us to the conclusions. We have discussed the finite renormalization of SMEFT operators in the BMHV scheme of dimensional regularization. This finite renormalization is needed in order to restore Slavnov-Taylor identities. We have been focusing on four-fermion operators. In the coming future, we plan to discuss other SMEFT operators. The computation of some two-loop anomalous dimension matrix elements is also on the horizon [22].

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[3]The structure in Eq. (10) is similar to the one of lower mass dimensionality found when computing the allowed possible forms of true anomalies in the SM, namely, $\propto d_{ABC} \mathbf{G}^A_\mu \partial_\rho \mathbf{G}^B_\nu \partial_\sigma \mathbf{g}^C \epsilon^{\mu\nu\rho\sigma}$. Although similar, a crucial difference is that in SMEFT the obstruction in Eq. (10) can be compensated for by finite counter-terms. Indeed, the fermion content of the SM is such that no anomalies to its local symmetries are generated. See Ref. [15] for further discussion.

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