

Prospects for light exotic scalar measurements at the e^+e^- Higgs factory

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While we are looking forward to the next update of the European Strategy for Particle Physics, it is clear that an electron-positron Higgs factory is the highest-priority next collider to be built. Its physics program will mainly focus on precision measurements of the 125 GeV Higgs boson, with the Higgs-strahlung process being the dominant production channel at 250 GeV. However, searches for possible deviations from the Standard Model (SM) predictions, for direct signatures of the “new physics” phenomena are also of primary interest. Production of new exotic light scalars is still not excluded by the existing experimental data, provided their coupling to the SM gauge bosons is sufficiently suppressed. This was selected as one of the focus topics of the ECFA Higgs/Top/EW factory study. Couplings of new exotic scalars could also be very different from the SM predictions, leading to non-standard decay patterns, so various search strategies have to be considered. Summarised in this contribution are results addressing the feasibility of direct light scalar observation at future e^+e^- collider experiments in different decay channels.

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1. Motivation

There is a general consensus in the particle physics community that the next-generation large infrastructure in the field should be an electron-positron Higgs factory. It was indicated as the highest-priority goal in the previous update of the European Strategy for Particle Physics (ESPP) [1] and the choice of the collider option is one of the main topics discussed in the ongoing ESPP 2026 update. While full exploitation of the Higgs boson physics requires running at collision energies up to the TeV range, most of the precision Higgs measurements can be already carried out at the collision energy of 250 GeV, maximizing the cross section for the Higgs boson production in the so called Higgsstrahlung process, $e^+e^- \rightarrow Z H$. However, existence of additional light scalar particles, with masses of the order of or below the mass of the 125 GeV state discovered at the LHC, is by far not excluded experimentally and is also well motivated from the theoretical point of view [2, 3]. Current experimental bounds and theoretical constraints set relatively weak limits on the light scalar production cross section in the scalar-strahlung process. Shown in Fig. 1 [4] are benchmark points allowed by all experimental and theoretical constraints, resulting from the parameter scan of the Two-Real-Singlet Model [5–7], Two Higgs-Doublet Model [8] and Minimal R-symmetric Supersymmetric SM [9]. Production cross sections in the $O(10)$ fb range are still not excluded.

Electron-positron Higgs factories should be sensitive to exotic scalar production even for very light states and small couplings, thanks to clean environment, measurement precision and hermeticity of the detectors. Still, prospects for such a measurements were hardly studied in the past. That is why this subject was included as one of the so called focus topics of the ECFA e^+e^- Higgs/EW/top factory study [10]. Two theoretical and phenomenological targets were defined:

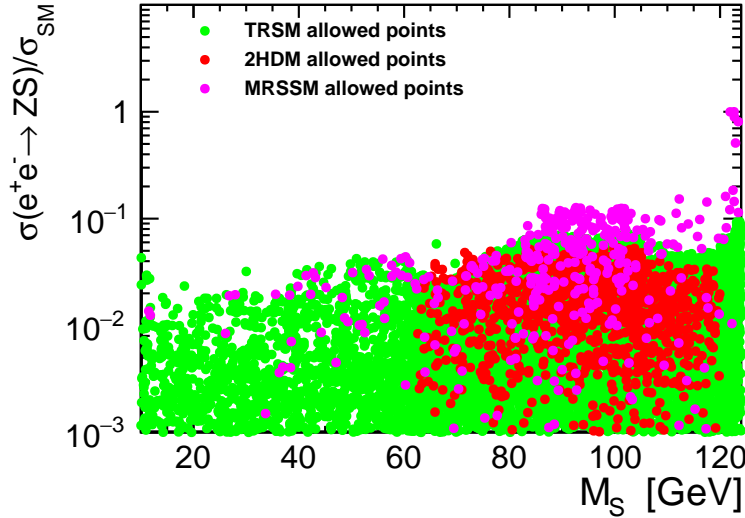


Figure 1: Allowed cross section values for the scalar-strahlung process at 250 GeV Higgs factory, relative to the SM predictions for the Higgs boson production at given mass, as a function of the new scalar mass. Shown are benchmark points consistent with current experimental and theoretical constraints for three selected models [4].

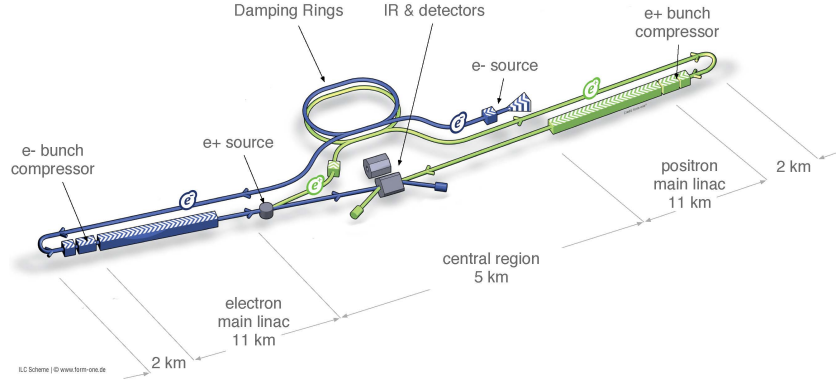


Figure 2: Schematic layout of the ILC in the 500 GeV configuration, as described in [12].

29 associated production of the new scalar with the Z boson, $e^+e^- \rightarrow Z S$ (scalar-strahlung process)
 30 and light scalar pair-production in 125 GeV Higgs boson decays, $H \rightarrow S S$. Summarised in this paper
 31 are results addressing the feasibility of direct light scalar observation at the International Linear
 32 Collider (ILC), in different decay channels, prepared within the ILD concept group as contributions
 33 to the ECFA study report [11].

34 2. ILC and its experiments

35 The International Linear Collider (ILC) was proposed as a future e^+e^- Higgs factory running
 36 at the energy of up to 500 GeV [12], see Fig. 2. The baseline running scenario assumes starting at a
 37 centre-of-mass energy of 250 GeV followed by a 500 GeV stage and 1 TeV considered as the possible
 38 upgrade [13]. In the assumed 22-year running period the ILC is expected to deliver the integrated
 39 luminosity of about 2 ab^{-1} at 250 GeV and 4 ab^{-1} at 500 GeV, with an additional 200 fb^{-1} collected
 40 at the top-quark pair-production threshold around 350 GeV. The design includes polarisation for
 41 both e^- and e^+ beams, of 80% and 30%, respectively, which is the unique feature of the ILC.
 42 Polarisation is crucial for many precision measurements, direct and indirect BSM searches, as well
 43 as for control of systematic effects. Two detector concepts, ILD and SiD, have been developed for
 44 the ILC [14, 15], both optimised for the Particle Flow reconstruction. While the baseline running
 45 scenario [13] was assumed for all studies described in this contribution, new running scenarios,
 46 with increased luminosity goals, are currently considered within the Linear Collider Vision [16]
 47 and proposed for the Linear Collider Facility at CERN [17].

48 3. Decay mode independent search

49 Electron-positron Higgs factory is assumed to run, for the precision Higgs boson measurements,
 50 at the collision energy of 240–250 GeV. This corresponds to the maximum of the 125 GeV scalar
 51 production cross section, see Fig. 3. At this energy, production is dominated by the Higgsstrahlung
 52 process, which can be tagged, independent of the Higgs decay channel, based on the recoil mass
 53 technique, see Fig. 4. Same approach can be used in the search for new exotic light scalar production
 54 in the corresponding scalar-strahlung process, $e^+e^- \rightarrow Z S$. For best recoil mass reconstruction Z
 55 decays to electron or muon pairs can be used. Decays to muons were exploited in the full simulation

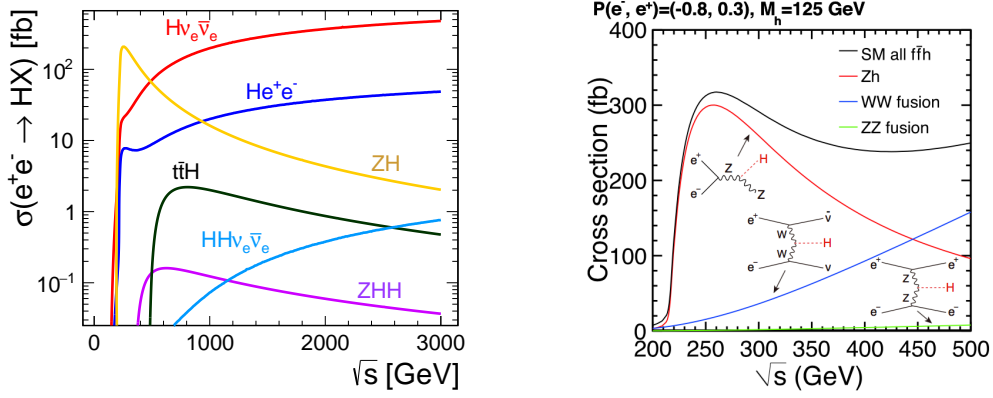


Figure 3: Left: cross section as a function of centre-of-mass energy for the main Higgs production processes at an e^+e^- collider. Right: cross sections comparison for the three major production processes relevant in the low energy range [18].

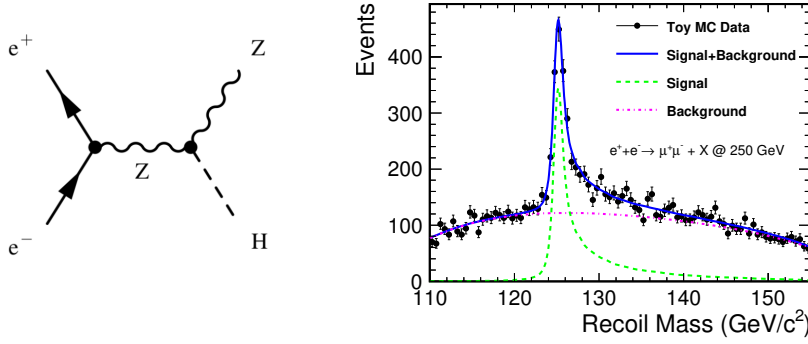


Figure 4: Left: the leading-order Feynman diagram for the Higgsstrahlung process. Right: recoil mass spectrum for signal of Higgs boson production and SM background at 250 GeV ILC, for $Z \rightarrow \mu^+\mu^-$ selection [19].

studies previously performed within ILD [20, 21]. Shown in Fig. 5 (left) is the recoil mass distribution expected for SM background processed at 250 GeV ILC together with expectations for different signal hypothesis. Expected limits on the scalar production cross section, relative to the SM scalar production cross section at given mass, are compared with LEP limits based on the similar approach in Fig. 5 (right). Expected sensitivity of the experiment at the future e^+e^- Higgs factory is an order of magnitude better than the existing LEP limit and the mass range probed can be significantly extended as well.

A new analysis based on this approach has been initiated for the ECFA study, with use of up-to-date simulation, reconstruction and analysis tools. Significant improvement in the expected limits is also expected from combining Z boson decays to muon and electron pairs. Example results of the new study, illustrating the decay-mode independent selection of the 125 GeV scalar production events, are presented in Fig. 6.

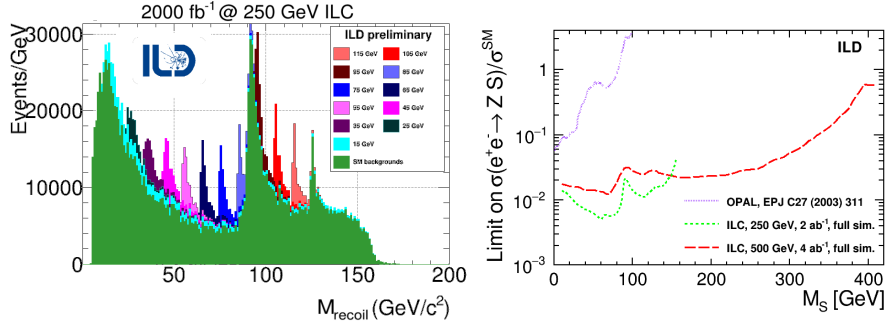


Figure 5: Left: the recoil mass distributions after the selection cuts for signal of light scalar production and SM backgrounds processes for ILC running at 250 GeV. Right: expected 95% C.L. limits on the scalar production cross section, relative to the SM scalar production cross section at given mass, for ILC running at 250 GeV and 500 GeV [20, 21].

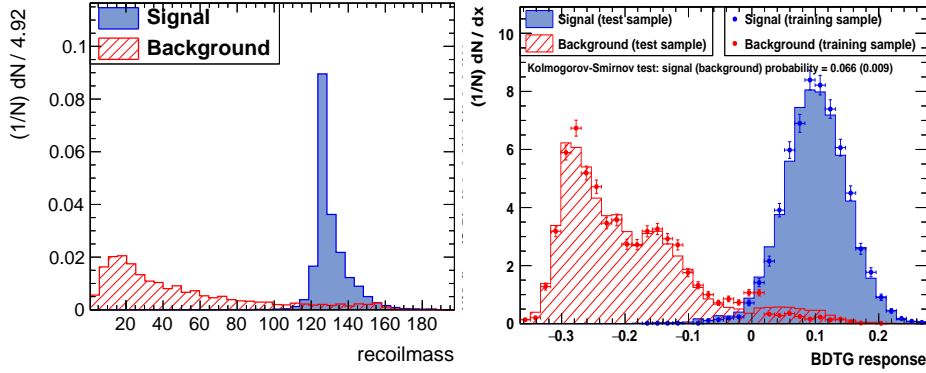


Figure 6: Example results from the new full simulation study on the light exotic scalar search at the ILD, in the decay-mode independent approach. Left: recoil mass reconstruction for the SM background and 125 GeV scalar production signal. Left: response distribution for the BDT classifier trained to differentiate between the scalar strahlung events and the two-fermion production background.

68 4. Search for $S \rightarrow \tau^+\tau^-$

69 Limits resulting from the decay independent search based on the recoil mass distribution only
70 are the most general ones, valid for all models, independent on the expected scalar decay channels.
71 Significant improvement of sensitivity, at the price of some model dependence, is expected when
72 particular decay channels of the new scalars are addressed. As some of the discrepancies from SM
73 predictions observed in LEP and LHC data suggested possible existence of the new scalar with mass
74 of about 95 GeV and enhanced branching ratio to the $\tau^+\tau^-$ final state [2], we decided to consider this
75 decay channel as the possible discovery scenario first. Event samples used for the presented study
76 were generated using WHIZARD [22, 23] version 3.1.2. Both background (including 125 GeV Higgs
77 boson production assuming nominal couplings) and light scalar signal production were modeled
78 using the built-in SM_CKM model. For signal sample, Higgs boson production events were generated
79 for sets of modified Higgs masses, followed by Higgs decay to tau lepton pair. Total luminosity of
80 2 ab^{-1} was assumed for ILC running at 250 GeV, as expected in the H-20 running scenario [13], with
81 $\pm 80\%$ and $\pm 30\%$ polarisation for electron and positron beams, respectively. The ILC beam energy

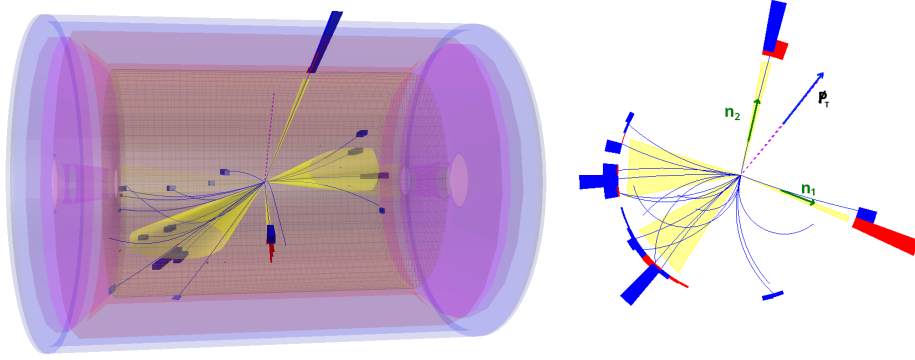


Figure 7: Left: example signal event, with hadronic decays of the two tau leptons produced in the light scalar decay. Right: same event in the transverse plane, missing transverse momentum \vec{p}_T and two unit vectors along tau jet directions (\vec{n}_1 and \vec{n}_2) are indicated.

Table 1: Event categories considered in the search for light scalar production with decay to tau lepton pair.

Event category	Isolated leptons	Selection requirements	
		tight selection	loose selection
hadronic	zero	4 jets, 2 with τ -tag	4 jets, 1 with τ -tag
semi-leptonic	one	3 jets, 1 with τ -tag	3 jets with no τ -tag
leptonic	two	two jets without τ -tag	

profile, as simulated with GuineaPig, was taken into account based on CIRCE2 parametrisation and hadronisation was simulated with the PYTHIA 6 [24]. The fast detector simulation framework DELPHES [25] was used to simulate detector response, with built-in cards for parametrisation of the ILC detector, `delphes_card_ILCgen.tcl` [26]. Example of signal event with hadronic final state, as simulated by DELPHES, is shown in figure 7 left.

Depending on the decays of the two tau leptons, three decay channels can be considered for the signal events: hadronic (with both taus decaying hadronically), semi-leptonic (with one leptonic tau decay) and leptonic (with leptonic decays of both taus). As a tight selection, we require each tau candidate to be identified either as an isolated lepton (and missing p_T) or hadronic jet with τ -tag. In addition to two tau candidates from the decay of the scalar, we also require reconstruction of two (untagged) hadronic jets from the hadronic decay of the Z boson. However, as the efficiency of tau jet tagging implemented in DELPHES is relatively poor (at most 70%), we also consider loose selection of hadronic and semi-leptonic events, when we require only one identified tau candidate (isolated lepton or τ -tagged jet) and three untagged hadronic jets, and take the jet with smallest invariant mass as the second tau candidate. At the pre-selection stage we thus select five event categories, as summarised in table 1. One of the challenges in the search for scalar decays into tau leptons is to properly reconstruct the invariant mass of the produced scalar, which can be significantly underestimated due to the escaping neutrinos. To correct for the neutrino energy, we use the so called collinear approximation [27]. For high energy tau leptons, decay products are highly boosted in the initial lepton direction and one can therefore assume that the initial tau lepton, escaping

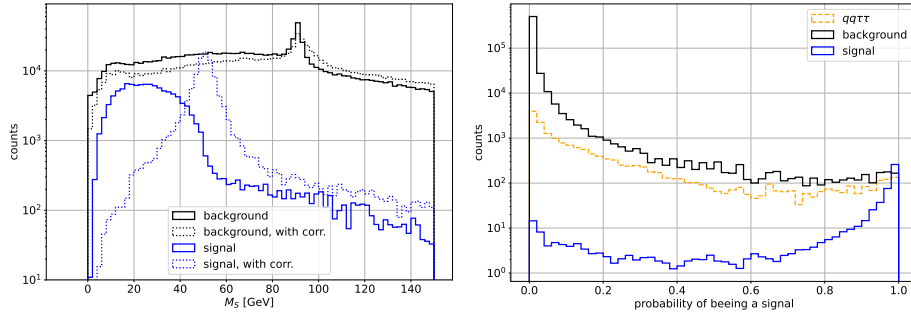


Figure 8: Left: reconstructed invariant mass of the two tau candidates after tight selection for SM background (black) and signal of 50 GeV scalar production (blue) before (solid) and after (dashed) collinear correction. Right: example of the BDT response distribution for 50 GeV scalar signal (blue) and SM background (black) events, for tight semi-leptonic event selection and ILC running with $e_L^- e_R^+$ polarisation combination. The signal cross section is normalized to 1% of the SM Higgs boson production cross section at given scalar mass. Orange dashed lines indicated the contribution of the leading background process, $e^+e^- \rightarrow q\bar{q}\tau^+\tau^-$.

neutrino and the observed tau candidate are collinear. With this assumption neutrino energies, and thus also corrected tau candidate energies, can be found from the transverse momentum balance. While more advanced reconstruction methods exist, based on the reconstruction of secondary decay vertex position, the advantage of this method is that it can be applied to all events and the solution is unique. Shown in Fig. 8 (left) are the mass distributions of the tau candidate pairs in signal and background events, before (solid) and after (dashed) collinear correction. After the correction, the scalar mass can be reconstructed with about 5 GeV precision, also for semi-leptonic and leptonic events.

For best event classification, we consider each event category and each beam polarisation combination separately, resulting in 20 independent BDTs trained for event classification, for each scalar mass considered. Example of BDT response distribution is presented in Fig. 8 (right). Expected exclusion limits, assuming no deviation from SM predictions are observed, are presented in Fig. 9. As expected, the semi-leptonic event selection results in the strongest limits, combining high event statistics (about 47% of decays) with background lower than for the hadronic channel. Including loose selection categories improves the expected limits by 20-30% for the whole considered mass range. When comparing to the decay independent limits, we can state that the targeted analysis results in more than an order of magnitude increase in sensitivity. However, as the limit includes the branching ratio, we can conclude that the di-tau channel is more sensitive only if the branching ratio is of the order of 10% or above.

5. Search for $S \rightarrow b\bar{b}$

If the structure of new light scalar couplings to SM particles is similar to that of the SM Higgs boson then the decay to $b\bar{b}$ is expected to dominate down to the masses of the order of 10 GeV. As huge hadronic background is expected from pair production of W bosons, leptonic Z boson decays are explored in this search channel. The initial study was based on the event samples generated in WHIZARD and processed with DELPHES framework for detector response, as described in Sec. 4. Conservation of transverse momentum could again be used to improve invariant mass reconstruction

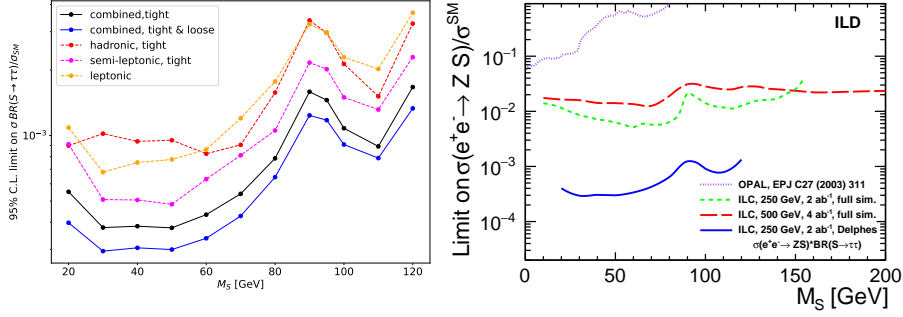


Figure 9: Expected 95% C.L. cross section limits on the light scalar production cross section times di-tau branching ratio for ILC running at 250 GeV. Left: comparison of combined limits with limits obtained for different event categories. Right: combined limits compared with limits resulting from decay independent study.

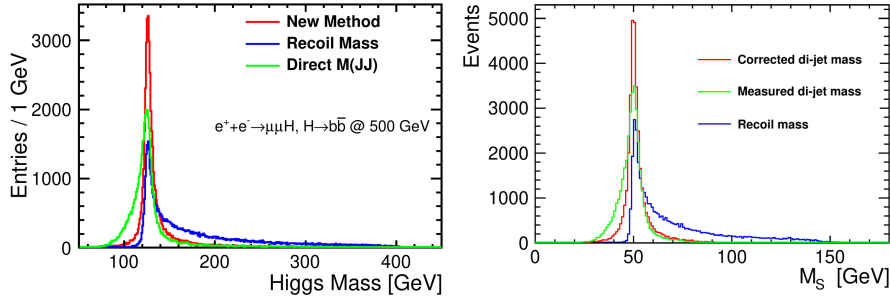


Figure 10: Invariant mass of two jets reconstructed using raw jet energies, corrected jet energies and recoil method. Left: full simulation results for the 125 GeV Higgs boson production at 500 GeV ILC [28]. Right: DELPHES simulation results for the signal of 50 GeV scalar production at 250 GeV ILC.

for the new scalar based on the well measured leptonic final state and jet angles. This approach was first proposed for Higgs mass measurement at the ILC [28], but it works very well also for light scalar production. This is shown in Fig. 10, where reconstructed di-jet mass distributions are compared before and after jet energy correction for 125 GeV Higgs boson [28] and new scalar of 50 GeV. Mass reconstructed with the recoil method (based on the Z decay measurement only) is also included for comparison. One can clearly see that jet energy correction based on the transverse momentum conservation significantly improves scalar mass measurement and allows to obtain precision much better than with uncorrected jet energies or the recoil method.

As the next step, for inclusion in the ECFA study report, the analysis is to be repeated with the full simulation event samples and detailed event reconstruction based on ILCSOFT [29].

6. Search for $S \rightarrow$ invisible

The fast simulation framework based on WHIZARD and DELPHES, as described in Sec. 4, was also used to study prospects for observing new scalar production in scenarios when its decays to invisible final states (e.g. dark sector) dominate. Hadronic Z decays were considered for highest sensitivity.

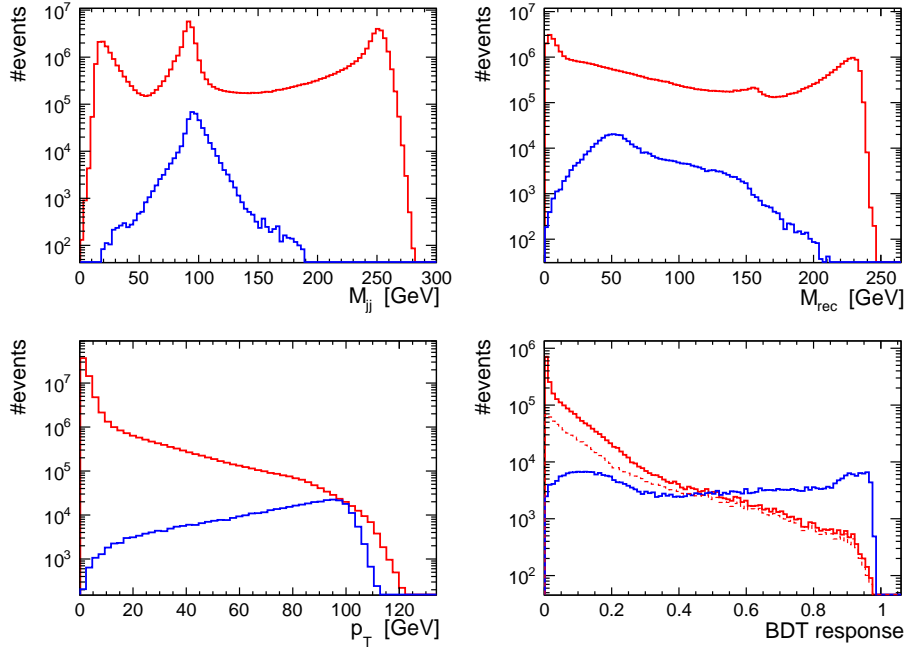


Figure 11: Example results from the fast simulation study on the light exotic scalar search in the invisible decay mode, for the signal of 50 GeV scalar production and SM background: distributions of the reconstructed Z boson candidate (di-jet) mass (top left), reconstructed recoil mass (top right), Z boson transverse momentum (bottom left) and the BDT classifier response after pre-selection cuts (bottom right). Dashed line in the BDT classifier response distribution indicates the leading contribution from the $q\bar{q} \ell \nu$ background.

Only events with purely hadronic final state, with no with isolated leptons or photons reconstructed in the detector (including also forward calorimeters, LumiCal and BeamCal) were included in the analysis. All reconstructed final state objects were clustered into two jets (using Durham algorithm in exclusive mode) assumed to form the Z boson candidate. Selected for the final classification were events with reconstructed di-jet invariant mass in the ± 20 GeV window around the Z mass and the missing transverse momentum greater than 10 GeV. Distributions of the two cut variables and of the invariant mass recoiling against the Z boson candidate, for the signal of 50 GeV scalar production and SM background, are compared in Fig. 11. Also show is the distribution of the BDT classifier response, which demonstrates that the kinematic distribution for the signal is expected to be very different from the known SM background processes, allowing us to select high purity signal sample.

7. Search for $S \rightarrow W^+W^-$

Assuming SM-like coupling structure of the new scalar, convolution of the production cross section and the expected branching ratio indicates that this channel could only be considered in a very small mass range between 125 and 150 GeV for the 250 GeV Higgs factory. However, it could be a promising discovery channel when running at higher energies.

As the first step, the generator level study based on the Two-Real-Singlet-Model (THDM) [30] was undertaken to establish the proper procedure for generation of signal event in WHIZARD. While

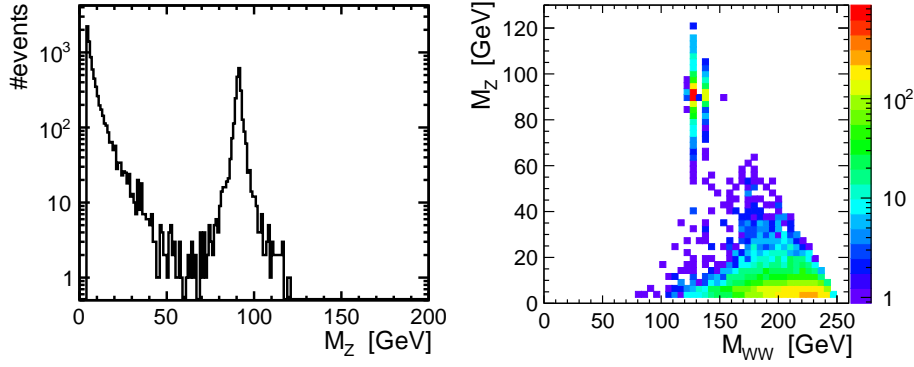


Figure 12: Example results from the generator level study on the light exotic scalar search in the W^+W^- decay mode, for the signal of 140 GeV scalar production and SM background, at 250 GeV ILC. Left: invariant mass distribution for the muon pair (Z boson candidate). Right: correlation of the reconstructed Z boson and scalar (W pair) candidate masses.

two-fermion final states are expected to dominate TRSM scalar decays at low masses (including $b\bar{b}$ and $\tau^+\tau^-$ channels addressed in previous sections), considered in the study was only the feasibility for observing $S \rightarrow W^+W^-$ decay at higher scalar masses. Parameter space of the TRSM has already been restricted by the 125 GeV Higgs boson measurements at the LHC, but the model is far from being excluded, even for additional scalar masses down to masses of the order of 10 GeV [5, 7]. Sizable production cross sections at a future Higgs factory are still not excluded.

Simulation of the signal events has to be done with care, as on-shell production of the three gauge bosons ($Z W^+ W^-$), expected from the scalar production in the scalar-strahlung process with subsequent scalar decay to W^+W^- , is not possible at 250 GeV and at least one boson need to be virtual. Boson widths also need to be properly taken into account and this can only be achieved by simulating the full process, with six-fermion final state, $e^+e^- \rightarrow 6 f$. We simulated the process $e^+e^- \rightarrow \mu^+\mu^-e^-\bar{\nu}_e q\bar{q}$ in TRSM model with additional 140 GeV scalar as a test scenario. For this particular final state, matching of the final state fermions to (real or virtual) Z, W^+ and W^- bosons is unique, simplifying the analysis. Contributing to the process is the new scalar production, but also SM-like 125 GeV Higgs boson production and all other SM diagrams with the same final state, with possible interference effects also properly taken into account in WHIZARD. Shown in 12 are the invariant mass distribution for the Z candidate (muon pair) and the correlation of the reconstructed masses of the Z boson and the scalar candidate (fermions forming the W^+W^- pair). Scalar-strahlung events can be selected by a very distinct peak in the di-muon invariant mass (left plot). Contributions of the two scalars, with 125 GeV and 140 GeV mass, are even better visible in the correlation of Z and W^+W^- invariant masses (right plot), showing that the scalar-strahlung and other SM background processes are well separated in the kinematic space. For the scalar-strahlung process, the Z boson is produced on-shell while at least one of the W bosons is off-shell. The dominant background is the on-shell W^+W^- pair production associated with a muon pair that comes from a virtual Z or photon exchange. While the detector resolution will clearly blur this picture, the separation of scalar-strahlung process in the W^+W^- decay channel should still be possible with high efficiency. Also, one can conclude that the contribution from the possible interference term between the signal

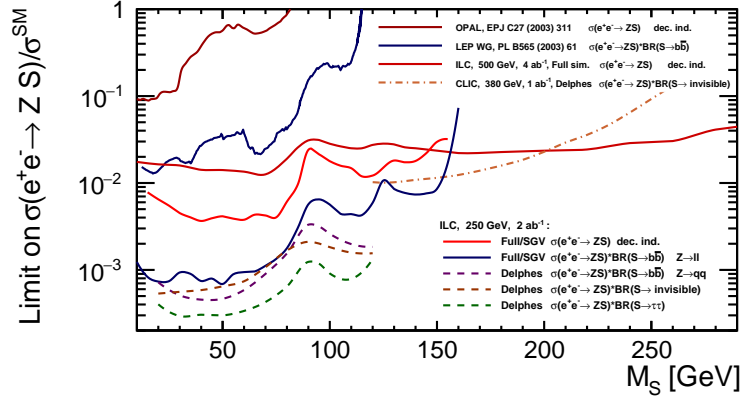


Figure 13: Comparison of expected exclusion limits for exotic light scalar production searches in the scalar-strahlung processes, for different search strategies and scalar decay channels considered. When particular decay channel is indicated, limits are set on the cross section ratio times the branching fraction [11].

and irreducible SM background processes is negligible. This is a crucial observation and simplifies a lot the actual analysis of the experimental sensitivity in this decay channel.

8. Conclusions

BSM scenarios involving light scalars, with masses accessible e^+e^- Higgs factories, are still not excluded by existing data. Sizable production cross sections for new scalars can also coincide with non-standard decay patterns, so different decay channels should be considered. Strong limits are already expected from decay independent searches, based on recoil mass reconstruction. More than an order of magnitude higher search sensitivity is expected for light scalar decays to tau pairs, if this is a dominant decay channel. Included in this contributions are results presented at the Corfu 2024 workshop, most of them reflecting the status of the studies progressing towards the final workshop of the ECFA e^+e^- Higgs/EW/top factory study. By the time of writing this contribution, most of these studies had already been finalized and their results included in the study report [11]. Comparison of the expected exclusion limits from different decay modes considered, as included in the ECFA report, is shown in Fig. 13.

Acknowledgments

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