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Who's the Real Neutrino? A QFT Deep Dive into Neutron Decay

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The *flavor/mass dichotomy* in mixed quantum fields remains one of the most debated topics in Quantum Field Theory (QFT). In this study, we address this issue by analyzing neutrino mixing in the context of the neutron β-decay process. We perform a comparative calculation of the transition amplitude using three distinct representations of neutrino states: *i*) Pontecorvo states, *ii*) mass eigenstates, and *iii*) exact QFT flavor states - defined as eigenstates of the flavor charge operator. To determine theoretical consistency, we invoke lepton charge conservation at the interaction vertex as a guiding criterion, reflecting its foundational role within the Standard Model at tree level. In the short-time regime, our results reveal that only the formalism based on QFT flavor states maintains full consistency with the Standard Model, while the other two approaches lead to violations of lepton charge. This finding provides critical insight into the nature of neutrino mixing in weak interactions and offers meaningful progress in resolving the longstanding flavor/mass ambiguity within QFT.

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1. Intoduction

Neutrino mixing and oscillations are among the most intriguing yet insufficiently understood phenomena in particle physics. First introduced by Pontecorvo in 1957 [1] by analogy with kaon behavior, the concept only reached its present theoretical maturity in the 1970s [2, 3], following the discovery of the second [4] and third [5] neutrino generations and the subsequent incorporation of matter effects [6, 7]. Experimentally, major milestones occurred between 1998 and 2002 with the detection of atmospheric and solar neutrino oscillations by the Super-Kamiokande [8, 9] and SNO [10, 11] collaborations, findings which were later corroborated by KamLAND [12].

Beyond their intrinsic interest, neutrino oscillations remain the only confirmed evidence of physics beyond the Standard Model (SM), spurring a broad and ongoing wave of theoretical and experimental investigations. Among these, increasing attention has been devoted to the influence of gravity and acceleration on flavor oscillations [13–21], while additional applications have emerged in diverse areas such as quantum information [22–24] and quantum optics [25].

Despite extensive work within quantum mechanics (QM), the corresponding analysis in Quantum Field Theory (QFT) remains comparatively underexplored. An early treatment of flavor mixing in QFT was proposed in [26], where the Fock space structure of mixed neutrino fields was constructed, thereby highlighting the conceptual limitations of the standard QM approach. Unlike the simple rotation structure of Pontecorvo transformations between mass and flavor eigenstates in QM, the QFT framework introduces a rotation composed with a Bogoliubov transformation [26]. This leads to orthogonal vacuum states for flavor and mass fields, with the flavor vacuum behaving as a condensate of entangled massive particle-antiparticle pairs [27, 28]. As a consequence, the associated Fock spaces are *unitarily inequivalent*, giving rise to physical phenomena beyond the QM domain [29–36]. For a discussion of different viewpoints, see [37–39].

More recently, interest in the QFT treatment of neutrino mixing has been revitalized through the study of weak interaction processes involving neutrinos. A particularly fertile setting for such investigations has been the weak decay of uniformly accelerated protons [40–42], a process permitted by the Unruh effect [43]. Within this framework, calculations of decay rates in both the laboratory and comoving frames revealed conflicting results depending on whether mass or flavor neutrino states are used [40–42], exposing the so-called *flavor/mass dichotomy* - a uniquely QFT phenomenon [44] that has no counterpart in QM owing to the Stone-von Neumann theorem. These discrepancies have reignited the fundamental question regarding the correct asymptotic description of neutrino states [45, 46].

Against this backdrop, the present study seeks to further investigate the structure of neutrino mixing within QFT by addressing the flavor/mass dichotomy in the context of *neutron β-decay*.[1] We compute the transition amplitude for the process $n \to p + e^- + \bar{\nu}$ using three distinct neutrino state representations: *i*) Pontecorvo states, *ii*) mass eigenstates, and *iii*) exact QFT flavor states, the latter defined as eigenstates of the flavor charge operator. Each representation leads to a different expression for the decay amplitude, and agreement between them is found only in the ultra-relativistic limit. This raises the critical question: which representation yields a physically consistent picture?

[1] This study is based on an extended analysis of the work presented in [47].

In the absence of direct experimental resolution, we adopt a phenomenological criterion grounded in theoretical consistency with the SM: specifically, the conservation of lepton charge at the interaction vertex, a symmetry that is both empirically supported and built into the SM at tree level [48]. Our analysis demonstrates that only the QFT flavor state framework preserves this conservation law under all conditions considered. By contrast, the Pontecorvo and mass state approaches result in lepton charge violation, indicating a breakdown in consistency with the SM.

The structure of the paper is as follows: in the next section we review the QFT formalism for neutrino mixing. Section 3 presents the computation of the tree-level β-decay amplitude using Pontecorvo and mass eigenstates, with emphasis on the short-time limit as a diagnostic for lepton charge conservation. Section 4 extends the analysis to exact QFT flavor states. To maintain clarity and focus on the essential physical insights, we neglect higher-order flavor-changing loop corrections, which are irrelevant to the central argument. Final remarks and future directions are provided in Section 5. Throughout the manuscript, natural units are used, with $\hbar = c = 1$.

2. Theoretical and Phenomenological Aspects of Neutrino Mixing in QFT

In this section, we review the formalism of neutrino mixing within the framework of QFT and highlight its principal differences from the traditional QM approach (see [26] for an in-depth treatment). For clarity, we consider a two-flavor scenario, noting that our conclusions remain valid under extension to a three-flavor framework.

It is well-known that Pontecorvo's formalism expresses the mixing of neutrino flavor states as

$$|\nu^r_{\mathbf{k},e}\rangle_P = \cos\theta\,|\nu^r_{\mathbf{k},1}\rangle + \sin\theta\,|\nu^r_{\mathbf{k},2}\rangle\,, \qquad |\nu^r_{\mathbf{k},\mu}\rangle_P = -\sin\theta\,|\nu^r_{\mathbf{k},1}\rangle + \cos\theta\,|\nu^r_{\mathbf{k},2}\rangle\,, \tag{1}$$

where $|\nu^r_{\mathbf{k},i}\rangle$ $(i = 1,2)$ denote mass eigenstates with masses m_i, and $|\nu^r_{\mathbf{k},\ell}\rangle_P$ $(\ell = e,\mu)$ represent flavor eigenstates. The index $r = 1,2$ labels the helicity states, and the subscript P indicates the Pontecorvo construction.

On the other hand, in QFT the mixing is implemented at the level of fields as

$$\nu_e(x) = \cos\theta\,\nu_1(x) + \sin\theta\,\nu_2(x)\,, \qquad \nu_\mu(x) = -\sin\theta\,\nu_1(x) + \cos\theta\,\nu_2(x)\,, \tag{2}$$

where $\nu_\ell(x)$ $(\ell = e,\mu)$ are interacting Dirac fields with definite flavor, while $\nu_i(x)$ $(i = 1,2)$ denote free fields with definite mass, i.e.,

$$\nu_i(x) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k},r}\left[u^r_{\mathbf{k},i}\,\alpha^r_{\mathbf{k},i}(x^0) + v^r_{-\mathbf{k},i}\,\beta^{r\dagger}_{-\mathbf{k},i}(x^0)\right]e^{i\mathbf{k}\cdot\mathbf{x}}\,, \qquad i = 1,2\,. \tag{3}$$

Here, $\alpha^r_{\mathbf{k},i}(x^0)$ and $\beta^{r\dagger}_{-\mathbf{k},i}(x^0)$ represent the time-dependent annihilation and creation operators, respectively, with $\omega_{\mathbf{k},i} = \sqrt{\mathbf{k}^2 + m_i^2}$ (for further details on the explicit forms of the spinors $u^r_{\mathbf{k},i}$, $v^r_{-\mathbf{k},i}$, and the Dirac γ-matrices, we refer the reader to [47]).

For the fields with definite mass, the vacuum (referred to for brevity as the *mass vacuum*) is defined as usual by $\alpha^r_{\mathbf{k},i}|0\rangle_m = \beta^r_{\mathbf{k},i}|0\rangle_m = 0$, with $|0\rangle_m \equiv |0\rangle_1 \otimes |0\rangle_2$. Furthermore, canonical anti-commutation relations yield $\left\{\alpha^r_{\mathbf{k},i}, \alpha^{s\dagger}_{\mathbf{q},j}\right\} = \delta_{\mathbf{kq}}\,\delta_{rs}\,\delta_{ij}$, with similar relations for $\beta^r_{\mathbf{k},i}$. For reasons that will become clear below, Eq. (3) presents the finite-volume expansion of free fields. The infinite-volume limit is formally obtained by applying the usual prescription [47].

To describe mixing at the operator level, one introduces the generator [26]

$$G_\theta(x^0) = \exp\left\{\theta \int d^3\mathbf{x} \left[\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)\right]\right\}, \tag{4}$$

which allows us to recast mixing transformations in the form

$$\nu_\ell^\alpha(x) = G_\theta^{-1}(x^0)\, \nu_i^\alpha(x)\, G_\theta(x^0), \qquad (\ell, i) = (e,1), (\mu,2). \tag{5}$$

In turn, the *flavor vacuum* $|0(x^0)\rangle_f$ at time x^0 is obtained by acting $G_\theta^{-1}(x^0)$ on the mass vacuum, i.e.,$|0(x^0)\rangle_f = G_\theta^{-1}(x^0)|0\rangle_m$.

It is important to emphasize that, while $G_\theta(x^0)$ is unitary at finite volume, it exhibits a non-unitary character in the infinite-volume limit (i.e., for systems with infinitely many degrees of freedom, such as fields), leading to the orthogonality of the flavor and mass vacua, i.e., ${}_f\langle 0(x^0)|0\rangle_m = 0, \forall x^0$.

By acting with the generator $G_\theta(x^0)$ on the mass eigenfields $\nu_i(x)$ in Eq. (5), the flavor fields can be expressed as

$$\nu_\ell(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},r} \left[u^r_{\mathbf{k},i}\, \alpha^r_{\mathbf{k},\nu_\ell}(x^0) + v^r_{-\mathbf{k},i}\, \beta^{r\dagger}_{-\mathbf{k},\nu_\ell}(x^0)\right] e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{6}$$

with $(\ell, i) = (e,1), (\mu,2)$. The associated flavor annihilation operators at x^0 are given by:

$$\alpha^r_{\mathbf{k},\nu_e}(x^0) = \cos\theta\, \alpha^r_{\mathbf{k},1}(x^0) + \sin\theta \sum_s \left[u^{r\dagger}_{\mathbf{k},1} u^s_{\mathbf{k},2}\, \alpha^s_{\mathbf{k},2}(x^0) + u^{r\dagger}_{\mathbf{k},1} v^s_{-\mathbf{k},2}\, \beta^{s\dagger}_{-\mathbf{k},2}(x^0)\right], \tag{7}$$

and similarly for other operators. This expression reveals that the flavor operators are constructed via a combination of the standard Pontecorvo rotation and an additional *Bogoliubov transformation*.

Without loss of generality, we may choose the reference frame such that $\mathbf{k} = (0, 0, |\mathbf{k}|)$. In this configuration, only the wavefunction products with $r = s$ are non-vanishing, allowing Eqs. (7) to be rewritten in the simplified form

$$\alpha^r_{\mathbf{k},\nu_e}(x^0) = \cos\theta\, \alpha^r_{\mathbf{k},1}(x^0) + \sin\theta \left(|U_\mathbf{k}|\, \alpha^r_{\mathbf{k},2}(x^0) + \epsilon^r |V_\mathbf{k}|\, \beta^{r\dagger}_{-\mathbf{k},2}(x^0)\right), \tag{8}$$

where $\epsilon^r = (-1)^r$ and the Bogoliubov coefficients $|U_\mathbf{k}|$ and $|V_\mathbf{k}|$ satisfy $|U_\mathbf{k}|^2 + |V_\mathbf{k}|^2 = 1$ (see [26] for their explicit expression). In the ultra-relativistic limit $\Delta m/|\mathbf{k}| \ll 1$, such coefficients simplify to $|U_\mathbf{k}| \approx 1$ and $|V_\mathbf{k}| \approx \Delta m/(2|\mathbf{k}|)$, thereby recovering the Pontecorvo formalism as $\Delta m/|\mathbf{k}| \to 0$.

In the framework described above, neutrino states are defined as the result of the action of flavor creation operators on the flavor vacuum $|0\rangle_f$ at $x^0 = 0$, namely [26]:

$$|\nu^r_{\mathbf{k},\ell}\rangle \equiv \alpha^{r\dagger}_{\mathbf{k},\nu_\ell}(0)|0\rangle_f, \tag{9}$$

which consistently reduce to the standard Pontecorvo states in the ultra-relativistic limit [26, 47].

These findings equip us with the theoretical tools needed to compute the transition amplitude for neutron β-decay using a fully consistent QFT approach. In the following sections, we employ the expansion (6) and the relation (8) as the foundation for our analysis.

3. Exploring Neutron β-Decay Through Pontecorvo and Mass Eigenstates

Let us apply the above considerations to the paradigmatic neutron β-decay

$$\text{a)} \quad n \longrightarrow p + e^- + \bar{\nu}_e \,, \tag{10}$$

which serves as a representative scenario for evaluating the compatibility of neutrino mixing with lepton charge conservation at the tree-level vertex. Alongside the process (10), we also examine the flavor-violating channel

$$\text{b)} \quad n \longrightarrow p + e^- + \bar{\nu}_\mu, \tag{11}$$

which, while not forbidden outright due to the phenomenon of neutrino oscillations, poses important theoretical constraints. Specifically, in the regime where the elapsed time is much greater than the neutron lifetime τ_n, yet still significantly shorter than the characteristic neutrino oscillation period T_{osc}, the amplitude for this process at tree level should vanish to maintain consistency with lepton charge conservation. This regime will be referred to as the "short-time" limit [45].

In this work, we are concerned with the short-time regime of the decay processes described by Eqs. (10) and (11). For this reason, we apply first-order perturbation theory and restrict our attention to time intervals $\Delta x^0 = x^0_{\text{out}} - x^0_{\text{in}}$ that are much shorter than the typical neutrino oscillation timescale T_{osc}. Within the interaction picture, and labeling the initial and final states as $|\psi_i\rangle$ and $|\psi_f\rangle$, respectively, the amplitude for the transition is written as $\langle e^{iH_0x^0}\psi_f|U_I(x^0)|\psi_i\rangle$. At leading order, the time evolution operator $U_I(x^0)$ takes the form $U_I(x^0) \simeq 1 - i\int_0^{x^0} dx^{0\prime}\, H_{\text{int}}(x^{0\prime})$, where the interaction Hamiltonian is defined by $H_{\text{int}}(x^0) = e^{iH_0x^0} H_{\text{int}} e^{-iH_0x^0}$.

The interaction Lagrangian corresponding to the process in Eq. (10) can be effectively described by the Fermi theory as [48]:

$$\mathscr{L}_{\text{int}}(x) = -\frac{G_F}{\sqrt{2}} \left[\bar{e}(x)\,\gamma^\mu(\mathbb{1}-\gamma^5)\,\nu_e(x)\right] \left[V_{ud}\,\bar{q}_u(x)\,\gamma_\mu\,(f-\gamma^5 g)\,q_d(x)\right],$$

where G_F denotes the Fermi coupling constant and V_{ud} is the CKM matrix element governing transitions between up and down quarks. The symbols f and g represent the relevant hadronic form factors. The fermionic fields are labeled as follows: $e(x)$ for the electron, $\nu_e(x)$ for the electron neutrino, and $q_u(x)$, $q_d(x)$ for the up and down quarks, respectively. The above form of $\mathscr{L}_{\text{int}}$ is a low-energy approximation derived from Fermi's theory of weak interactions. Although the full electroweak Lagrangian is more elaborate, these higher-order details do not significantly affect our present analysis and can be safely ignored [47].

With this framework in place, we proceed to evaluate the decay amplitudes for channels (a) and (b). Initially, we will carry out the computation using Pontecorvo flavor states $|\nu_{\mathbf{k},\ell}\rangle_P$ defined in Eq. (1), and then switch to mass eigenstates $|\nu_{\mathbf{k},i}\rangle$ as the fundamental basis for neutrinos. Subsequently, we will analyze the behavior of the amplitudes in the short-time limit and examine the domain of validity of our approximations.

3.1 Pontecorvo States

We begin by examining decay channel a). At tree level, the transition amplitude reads

$$A^{a)}_P = {}_P\langle \bar{\nu}^r_{\mathbf{k},e},\, e^s_{\mathbf{q}},\, p^{\sigma_p}_{\mathbf{p}_p}| \left[-i\int_{x^0_{\text{in}}}^{x^0_{\text{out}}} d^4x\, \mathscr{L}_{\text{int}}(x)\right] |n^{\sigma_n}_{\mathbf{p}_n}\rangle, \tag{12}$$

where the momenta and helicities of the neutron, proton, and electron are denoted by $\mathbf{p}_n$ (σ_n), $\mathbf{p}_p$ (σ_p), and $\mathbf{q}$ (s), respectively. The outgoing antineutrino is described using a Pontecorvo state. For simplicity, the time dependence of external states is not displayed explicitly. Clearly, in standard interaction picture formalism, in- and out-states are conventionally evaluated at $x^0 = x^0_{\rm in}$ and $x^0 = x^0_{\rm out}$, respectively[2].

After some algebra, the amplitude in Eq. (12) becomes [47]

$$
\begin{aligned}
A_P^{a)} &= i\mathcal{N}G_F\,\delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, e^{-i\omega^e_{\mathbf{q}} x^0_{\rm out}}\, \bar{u}^s_{\mathbf{q},e}\,\gamma^\mu(\mathbb{1}-\gamma^5) \\
&\times \int_{x^0_{\rm in}}^{x^0_{\rm out}} dx^0\, \Big[\cos^2\theta\, v^r_{\mathbf{k},1}\, e^{-i\omega_{\mathbf{k},1}x^0_{\rm out}} e^{-i(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},1})x^0} \\
&\quad + \sin^2\theta\, v^r_{\mathbf{k},2}\, e^{-i\omega_{\mathbf{k},2}x^0_{\rm out}} e^{-i(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},2})x^0}\Big] \\
&\times V_{ud}\,\bar{u}^{\sigma_p}_{\mathbf{p}_p}\gamma_\mu(f-\gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\, e^{-i(\omega_{\mathbf{p}_p}x^0_{\rm out}-\omega_{\mathbf{p}_n}x^0_{\rm in})},
\end{aligned} \tag{13}
$$

where $\mathcal{N} = \left[\sqrt{2}(2\pi)^3\right]^{-1}$ contains normalization factors.

Similarly, for decay channel $b)$ in Eq. (11), the amplitude is:

$$
A_P^{b)} = {}_P\langle \bar{\nu}^r_{\mathbf{k},\mu},\, e^s_{\mathbf{q}},\, p^{\sigma_p}_{\mathbf{p}_p}|\left[-i\int_{x^0_{\rm in}}^{x^0_{\rm out}} d^4x\, \mathscr{L}_{\rm int}(x)\right]|n^{\sigma_n}_{\mathbf{p}_n}\rangle\,, \tag{14}
$$

which gives

$$
\begin{aligned}
A_P^{b)} &= i\mathcal{N}G_F\, \sin\theta\cos\theta\,\delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, e^{-i\omega^e_{\mathbf{q}} x^0_{\rm out}} \bar{u}^s_{\mathbf{q},e}\,\gamma^\mu(\mathbb{1}-\gamma^5) \\
&\times \int_{x^0_{\rm in}}^{x^0_{\rm out}} dx^0\, \Big[v^r_{\mathbf{k},2} e^{-i\omega_{\mathbf{k},2}x^0_{\rm out}} e^{-i(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},2})x^0} \\
&\quad - v^r_{\mathbf{k},1} e^{-i\omega_{\mathbf{k},1}x^0_{\rm out}} e^{-i(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},1})x^0}\Big] \\
&\times V_{ud}\,\bar{u}^{\sigma_p}_{\mathbf{p}_p}\gamma_\mu(f-\gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\, e^{-i(\omega_{\mathbf{p}_p}x^0_{\rm out}-\omega_{\mathbf{p}_n}x^0_{\rm in})}.
\end{aligned} \tag{15}
$$

As expected, this expression vanishes when the mixing angle tends to zero ($\theta \to 0$).

We now turn to the evaluation of lepton charge conservation at the tree-level vertex in the short-time limit. To this end, we define the time integration limits as $x^0_{\rm in} = -\Delta t/2$ and $x^0_{\rm out} = \Delta t/2$, where the interval Δt is chosen to be significantly longer than the neutron mean lifetime τ_n, to allow for the decay to occur, yet still shorter than the neutrino oscillation time $T_{\rm osc}$, in accordance with the reasoning following Eq. (11).

Let us first consider channel $a)$. Substituting the new integration limits into Eq. (13) and evaluating the time integral, we obtain [47]

$$
\begin{aligned}
A_P^{a)} &= 2i\,\mathcal{N}G_F\,\delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, e^{-i\omega^e_{\mathbf{q}}\Delta t/2}\, \bar{u}^s_{\mathbf{q},e}\gamma^\mu(\mathbb{1}-\gamma^5) \\
&\times \left\{\cos^2\theta\, v^r_{\mathbf{k},1}\, e^{-i\omega_{\mathbf{k},1}\Delta t/2}\,\frac{\sin[\tilde{\omega}_1\Delta t/2]}{\tilde{\omega}_1} + \sin^2\theta\, v^r_{\mathbf{k},2}\, e^{-i\omega_{\mathbf{k},2}\Delta t/2}\,\frac{\sin[\tilde{\omega}_2\Delta t/2]}{\tilde{\omega}_2}\right\} \\
&\times V_{ud}\,\bar{u}^{\sigma_p}_{\mathbf{p}_p}\gamma_\mu(f-\gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\, e^{-i(\omega_{\mathbf{p}_p}+\omega_{\mathbf{p}_n})\Delta t/2},
\end{aligned} \tag{16}
$$

[2] As discussed in [45], amplitudes involving exact QFT neutrino states should be consistently defined as $\langle\psi_\ell(x^0_{\rm out})|e^{-iH(x^0_{\rm out}-x^0_{\rm in})}|\psi_\ell(x^0_{\rm in})\rangle = \langle\psi_\ell(x^0_{\rm in})|U_I(x^0_{\rm out},x^0_{\rm in})|\psi_\ell(x^0_{\rm in})\rangle$, due to the nontrivial time evolution of orthogonal Hilbert spaces. This will be explicitly considered in Sec. 4.

where we introduced $\tilde{\omega}_i = \omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} - \omega_{\mathbf{k},i}$ for $i = 1, 2$. It is worth pointing out that in the limit $\Delta t \to \infty$, each factor of the form $\sin(\tilde{\omega}_i \Delta t/2)/\tilde{\omega}_i$ approaches a Dirac delta distribution, thus enforcing energy conservation. At finite Δt, these terms reflect the allowed energy uncertainty due to the finite interaction time window, consistent with the time-energy uncertainty principle [49].

In the short-time limit, the dominant contributions in Eq. (16) arise when $\tilde{\omega}_i \approx 0$. To the leading order, the amplitude simplifies to:

$$\begin{aligned} A_P^{a)} \simeq\, & i\,\mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5) \left(\cos^2\theta\, v^r_{\mathbf{k},1} + \sin^2\theta\, v^r_{\mathbf{k},2}\right) \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n} . \end{aligned} \tag{17}$$

Since neutrinos observed in experiments are highly relativistic, it is natural to examine the behavior of Eq. (17) in the relativistic limit. Using the identity $v^r_{\mathbf{k},1}|U_{\mathbf{k}}| - \epsilon^r\, u^r_{-\mathbf{k},1}|V_{\mathbf{k}}| = v^r_{\mathbf{k},2}$, the amplitude can be rewritten as:

$$\begin{aligned} A_P^{a)} \simeq\, & i\,\mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5) \\ & \times \left\{ v^r_{\mathbf{k},1} \left[1 - \sin^2\theta (1 - |U_{\mathbf{k}}|)\right] - \sin^2\theta\, \epsilon^r\, u^r_{-\mathbf{k},1}\, |V_{\mathbf{k}}| \right\} \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n} . \end{aligned} \tag{18}$$

Applying the approximations for U_k and V_k below Eq. (8), and keeping terms up to first order in $O(\Delta m/2|\mathbf{k}|)$, we get:

$$\begin{aligned} A_P^{a)} \simeq\, & i\,\mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5) \\ & \times \left(v^r_{\mathbf{k},1} - \sin^2\theta\, \frac{\Delta m}{2|\mathbf{k}|}\, \epsilon^r\, u^r_{-\mathbf{k},1} \right) \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n} . \end{aligned} \tag{19}$$

In the ultra-relativistic limit, where $\Delta m/|\mathbf{k}| \to 0$, the second term becomes negligible, and the amplitude reduces to:

$$\begin{aligned} A_P^{a)} \simeq\, & i\,\mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5)\, v^r_{\mathbf{k},1} \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n} . \end{aligned} \tag{20}$$

This result is structurally identical to the amplitude for emission of a free antineutrino with mass m_1, as expected. Further interpretation of this outcome will follow in the next section.

We now repeat the short-time analysis for channel $b)$. Starting from Eq. (15), we obtain the leading-order approximation:

$$\begin{aligned} A_P^{b)} \simeq\, & i\,\mathcal{N} G_F\, \sin\theta\cos\theta\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5)\, (v^r_{\mathbf{k},2} - v^r_{\mathbf{k},1}) \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n} . \end{aligned} \tag{21}$$

This amplitude is evidently nonzero unless $m_1 = m_2$, since $v^r_{\mathbf{k},1} \neq v^r_{\mathbf{k},2}$ in general.

To gain more insight, let us consider the relativistic regime. Following the same calculations as before, we find after simplification [47]:

$$\begin{aligned} A_P^{b)} \simeq\, & -i\,\mathcal{N} G_F\, \sin\theta\cos\theta\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \frac{\Delta m}{2|\mathbf{k}|}\, \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5)\, \epsilon^r u^r_{-\mathbf{k},1} \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n} . \end{aligned} \tag{22}$$

This result reveals a significant issue: even in the short-time limit, the amplitude for the flavor-violating process $n \longrightarrow p + e^- + \bar{\nu}_\mu$ is nonzero, despite the absence of oscillation effects. This explicitly violates lepton charge conservation at the vertex. The origin of this anomaly lies in the use of Pontecorvo states, which are not eigenstates of the flavor charge operator in QFT. As we will demonstrate in the next section, a consistent treatment using exact QFT states resolves this inconsistency. In particular, Eq. (22) shows that the amplitude vanishes in the ultra-relativistic limit $\Delta m/|\mathbf{k}| \to 0$, where Pontecorvo states approximate the exact QFT neutrino states accurately [26].

3.2 Mass Eigenstates

Following the same procedure as in the previous subsection, we now evaluate the neutron β-decay amplitude using the mass eigenstates $|\nu^r_{\mathbf{k},i}\rangle$, with $i = 1, 2$, as the fundamental representation for neutrinos. This point of view is the one adopted in [42]. In this framework, we consider the two decay channels $n \longrightarrow p + e^- + \bar{\nu}_i$, $i = 1, 2$.

Accordingly, the expression for the transition amplitude, previously given in Eq. (12) (or equivalently Eq. (14)) for flavor states, must now be reformulated to reflect the use of mass eigenstates. For each process, the amplitude takes the form:

$$A_i = \langle \bar{\nu}^r_{\mathbf{k},i},\, e^s_{\mathbf{q}},\, p^{\sigma_p}_{\mathbf{p}_p}| \left[-i \int_{x^0_{\rm in}}^{x^0_{\rm out}} d^4x\, \mathscr{L}_{\rm int}(x) \right] |n^{\sigma_n}_{\mathbf{p}_n}\rangle\,, \qquad i = 1, 2\,, \tag{23}$$

where the index i labels the mass eigenstate of the outgoing antineutrino.

The amplitude A_1 can be directly obtained from Eq. (13) by applying the inverse transformation in Eq. (1), effectively rotating the flavor state back into the mass basis. This yields:

$$\begin{aligned} A_1 = {} & i\, \mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, e^{-i(\omega^e_{\mathbf{q}} + \omega_{\mathbf{k},1}) x^0_{\rm out}} \cos\theta\, \bar{u}^s_{\mathbf{q},e}\, \gamma^\mu (\mathbb{1} - \gamma^5) \\ & \times \int_{x^0_{\rm in}}^{x^0_{\rm out}} dx^0\, v^r_{\mathbf{k},1}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} - \omega_{\mathbf{k},1}) x^0} \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p}\, \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\, e^{-i(\omega_{\mathbf{p}_p} x^0_{\rm out} - \omega_{\mathbf{p}_n} x^0_{\rm in})}\,. \end{aligned} \tag{24}$$

Taking the short-time limit of this expression gives:

$$\begin{aligned} A_1 \simeq {} & i\, \mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \cos\theta\, \bar{u}^s_{\mathbf{q},e}\, \gamma^\mu (\mathbb{1} - \gamma^5)\, v^r_{\mathbf{k},1} \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p}\, \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\,. \end{aligned} \tag{25}$$

As expected, this result is consistent with Eq. (20) in the ultra-relativistic limit, where the effects of the mass difference can be neglected.

In a similar manner, the amplitude A_2 corresponding to the second mass eigenstate in the short-time limit is found to be:

$$\begin{aligned} A_2 \simeq {} & i\, \mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, \Delta t\, \sin\theta\, \bar{u}^s_{\mathbf{q},e}\, \gamma^\mu (\mathbb{1} - \gamma^5)\, v^r_{\mathbf{k},2} \\ & \times V_{ud}\, \bar{u}^{\sigma_p}_{\mathbf{p}_p}\, \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\,. \end{aligned} \tag{26}$$

To compare with previous results in the literature, it is instructive to examine the decay rates corresponding to the two channels $n \longrightarrow p + e^- + \bar{\nu}_i$, which can be obtained by squaring the amplitudes A_i. According to standard treatments [37, 50], the total decay rate for the processes (10) and (11) is computed as an incoherent sum over contributions from the different mass eigenstates. Each contribution is weighted by $\cos^2\theta$ or $\sin^2\theta$, depending on whether the final antineutrino corresponds to the electron or muon flavor. Applying this incoherent summation prescription and using Eqs. (25) and (26), one immediately observes that the decay probability for the process (11) remains non-zero even in the short-time limit. This again implies a violation of lepton flavor conservation at the tree level, suggesting that mass eigenstates cannot serve as the fundamental physical states for mixed neutrinos if one aims to maintain consistency within the theoretical framework.

4. Neutron β-Decay with Exact Flavor States

We now re-express the calculations presented in the previous sections within the framework of QFT using exact flavor states. These states are rigorously defined as eigenstates of the flavor charge operators [26]. Based on the definition in Eq. (9), one can readily verify [26] that

$$:: Q_{\nu_e}(0) :: \ |\nu^r_{\mathbf{k},e}\rangle = |\nu^r_{\mathbf{k},e}\rangle\,, \qquad :: Q_{\nu_\mu}(0) :: \ |\nu^r_{\mathbf{k},\mu}\rangle = |\nu^r_{\mathbf{k},\mu}\rangle\,, \tag{27}$$

$$:: Q_{\nu_e}(0) :: \ |\nu^r_{\mathbf{k},\mu}\rangle =:: Q_{\nu_\mu}(0) :: \ |\nu^r_{\mathbf{k},e}\rangle = 0\,, \qquad :: Q_{\nu_\ell}(0) :: \ |0\rangle_f = 0\,, \tag{28}$$

where the flavor charge operators are defined as

$$Q_{\nu_\ell}(x^0) \equiv \int d^3\mathbf{x}\, \nu^\dagger_\ell(x)\, \nu_\ell(x)\,, \qquad \ell = e, \mu\,. \tag{29}$$

The colons $:: Q_{\nu_\ell} ::$ denote normal ordering with respect to the flavor vacuum $|0\rangle_f$.

In this exact QFT framework, the decay amplitude for channel $a)$ is given by

$$A^{a)} = \langle \bar{\nu}^r_{\mathbf{k},e},\, e^s_{\mathbf{q}},\, p^{\sigma_p}_{\mathbf{p}_p} | \left[-i \int_{x^0_{\rm in}}^{x^0_{\rm out}} d^4x\, \mathscr{L}_{\rm int}(x) \right] |n^{\sigma_n}_{\mathbf{p}_n}\rangle. \tag{30}$$

The key difference lies in the expectation value involving the neutrino field, which now reads [47]:

$$\begin{aligned}\langle \bar{\nu}^r_{\mathbf{k},e}(x^0_{\rm in}) | \nu_e(x) | 0(x^0_{\rm in})\rangle_f \simeq e^{-i\mathbf{k}\cdot\mathbf{x}} \Big\{ &\cos^2\theta\, v^r_{\mathbf{k},1}\, e^{i\omega_{\mathbf{k},1}(x^0 - x^0_{\rm in})} \\ &+ \sin^2\theta \Big[\left(v^r_{\mathbf{k},1}\, |U_\mathbf{k}| - \epsilon^r\, u^r_{-\mathbf{k},1}\, |V_\mathbf{k}| \right) |U_\mathbf{k}|\, e^{i\omega_{\mathbf{k},2}(x^0 - x^0_{\rm in})} \\ &+ \left(v^r_{\mathbf{k},1}\, |V_\mathbf{k}| + \epsilon^r\, u^r_{-\mathbf{k},1}\, |U_\mathbf{k}| \right) |V_\mathbf{k}|\, e^{-i\omega_{\mathbf{k},2}(x^0 - x^0_{\rm in})} \Big] \Big\}.\end{aligned} \tag{31}$$

By employing the identities $v^r_{\mathbf{k},1}|U_\mathbf{k}| - \epsilon^r\, u^r_{-\mathbf{k},1}|V_\mathbf{k}| = v^r_{\mathbf{k},2}$ and $v^r_{\mathbf{k},1}|V_\mathbf{k}| + \epsilon^r\, u^r_{-\mathbf{k},1}|U_\mathbf{k}| = \epsilon^r\, u^r_{-\mathbf{k},2}$, the expectation value in Eq. (31) can be recast as

$$\begin{aligned}\langle \bar{\nu}^r_{\mathbf{k},e}(x^0_{\rm in}) | \nu_e(x) | 0(x^0_{\rm in})\rangle_f \simeq e^{-i\mathbf{k}\cdot\mathbf{x}} \Big\{ &\cos^2\theta\, v^r_{\mathbf{k},1}\, e^{i\omega_{\mathbf{k},1}(x^0 - x^0_{\rm in})} \\ &+ \sin^2\theta \left[v^r_{\mathbf{k},2}\, |U_\mathbf{k}|\, e^{i\omega_{\mathbf{k},2}(x^0 - x^0_{\rm in})} + \epsilon^r\, u^r_{-\mathbf{k},2}\, |V_\mathbf{k}|\, e^{-i\omega_{\mathbf{k},2}(x^0 - x^0_{\rm in})} \right] \Big\},\end{aligned} \tag{32}$$

which can be further manipulated to give

$$
\begin{aligned}
A^{a)} = i\,\mathcal{N} G_F\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, e^{-i\omega^e_{\mathbf{q}} x^0_{\text{out}}} \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5) \\
\times \int_{x^0_{\text{in}}}^{x^0_{\text{out}}} dx^0 \Big\{ \cos^2\theta\, v^r_{\mathbf{k},1}\, e^{-i\omega_{\mathbf{k},1} x^0_{\text{in}}}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} - \omega_{\mathbf{k},1}) x^0} \\
+ \sin^2\theta \Big[v^r_{\mathbf{k},2}\, |U_{\mathbf{k}}|\, e^{-i\omega_{\mathbf{k},2} x^0_{\text{in}}}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} - \omega_{\mathbf{k},2}) x^0} \\
+ \epsilon^r\, u^r_{-\mathbf{k},2}\, |V_{\mathbf{k}}|\, e^{i\omega_{\mathbf{k},2} x^0_{\text{in}}}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} + \omega_{\mathbf{k},2}) x^0} \Big] \Big\} \\
\times V_{ud} \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\, e^{-i(\omega_{\mathbf{p}_p} x^0_{\text{out}} - \omega_{\mathbf{p}_n} x^0_{\text{in}})}.
\end{aligned}
\tag{33}
$$

This result exhibits a markedly different structure from Eq. (13), due to the appearance of Bogoliubov coefficients and the explicit contribution from antiparticle modes (specifically, the term involving the spinor $u^r_{-\mathbf{k},2}$). Nonetheless, in the ultra-relativistic limit where $|U_{\mathbf{k}}| \to 1$ and $|V_{\mathbf{k}}| \to 0$, Eq. (33) smoothly reduces to Eq. (13) (up to overall phase factors), and the exact flavor states effectively reproduce the behavior of Pontecorvo states. As we will demonstrate in the following, the additional terms present in Eq. (33) are essential to restoring lepton charge conservation in the short-time limit.

Turning now to the flavor-violating decay channel in Eq. (11), the amplitude (14) becomes

$$
A^{b)} = \langle \bar{\nu}^r_{\mathbf{k},\mu},\, e^s_{\mathbf{q}},\, p^{\sigma_p}_{\mathbf{p}_p} | \left[-i \int_{x^0_{\text{in}}}^{x^0_{\text{out}}} d^4x\, \mathscr{L}_{\text{int}}(x) \right] | n^{\sigma_n}_{\mathbf{p}_n} \rangle. \tag{34}
$$

The expectation value of the neutrino field in this case reads:

$$
\begin{aligned}
\langle \bar{\nu}^r_{\mathbf{k},\mu}(x^0_{\text{in}}) | \nu_e(x) | 0(x^0_{\text{in}}) \rangle_f \simeq e^{-i\mathbf{k}\cdot\mathbf{x}} \sin\theta\cos\theta \Big[(v^r_{\mathbf{k},1}\, |U_{\mathbf{k}}| - \epsilon^r\, u^r_{-\mathbf{k},1}\, |V_{\mathbf{k}}|)\, e^{i\omega_{\mathbf{k},2}(x^0 - x^0_{\text{in}})} \\
- v^r_{\mathbf{k},1}\, |U_{\mathbf{k}}|\, e^{i\omega_{\mathbf{k},1}(x^0 - x^0_{\text{in}})} + \epsilon^r\, u^r_{-\mathbf{k},1}\, |V_{\mathbf{k}}|\, e^{-i\omega_{\mathbf{k},1}(x^0 - x^0_{\text{in}})} \Big].
\end{aligned}
\tag{35}
$$

Using again the identity $v^r_{\mathbf{k},1}|U_{\mathbf{k}}| - \epsilon^r\, u^r_{-\mathbf{k},1}|V_{\mathbf{k}}| = v^r_{\mathbf{k},2}$, this expression simplifies to:

$$
\begin{aligned}
\langle \bar{\nu}^r_{\mathbf{k},\mu}(x^0_{\text{in}}) | \nu_e(x) | 0(x^0_{\text{in}}) \rangle_f \simeq e^{-i\mathbf{k}\cdot\mathbf{x}} \sin\theta\cos\theta \Big[v^r_{\mathbf{k},2}\, e^{i\omega_{\mathbf{k},2}(x^0 - x^0_{\text{in}})} - v^r_{\mathbf{k},1}\, |U_{\mathbf{k}}|\, e^{i\omega_{\mathbf{k},1}(x^0 - x^0_{\text{in}})} \\
+ \epsilon^r\, u^r_{-\mathbf{k},1}\, |V_{\mathbf{k}}|\, e^{-i\omega_{\mathbf{k},1}(x^0 - x^0_{\text{in}})} \Big].
\end{aligned}
\tag{36}
$$

Substituting this into Eq. (34), the final amplitude becomes:

$$
\begin{aligned}
A^{b)} = i\,\mathcal{N} G_F\, \sin\theta\cos\theta\, \delta^3(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q} - \mathbf{k})\, e^{-i\omega^e_{\mathbf{q}} x^0_{\text{out}}} \bar{u}^s_{\mathbf{q},e} \gamma^\mu (\mathbb{1} - \gamma^5) \\
\times \int_{x^0_{\text{in}}}^{x^0_{\text{out}}} dx^0 \Big[v^r_{\mathbf{k},2}\, e^{-i\omega_{\mathbf{k},2} x^0_{\text{in}}}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} - \omega_{\mathbf{k},2}) x^0} \\
- v^r_{\mathbf{k},1}\, |U_{\mathbf{k}}|\, e^{-i\omega_{\mathbf{k},1} x^0_{\text{in}}}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} - \omega_{\mathbf{k},1}) x^0} \\
+ \epsilon^r\, u^r_{-\mathbf{k},1}\, |V_{\mathbf{k}}|\, e^{i\omega_{\mathbf{k},1} x^0_{\text{in}}}\, e^{-i(\omega_{\mathbf{p}_n} - \omega_{\mathbf{p}_p} - \omega^e_{\mathbf{q}} + \omega_{\mathbf{k},1}) x^0} \Big] \\
\times V_{ud} \bar{u}^{\sigma_p}_{\mathbf{p}_p} \gamma_\mu (f - \gamma^5 g)\, u^{\sigma_n}_{\mathbf{p}_n}\, e^{-i(\omega_{\mathbf{p}_p} x^0_{\text{out}} - \omega_{\mathbf{p}_n} x^0_{\text{in}})}.
\end{aligned}
\tag{37}
$$

As in the previous case, the ultra-relativistic limit reproduces the result (15), modulo overall phases.

PoS(CORFU2024)093

We now analyze the short-time behavior of the amplitudes given in Eqs. (33) and (37). As in Sec. 3, we adopt the interval definition $x^0_{\rm in} = -\Delta t/2$ and $x^0_{\rm out} = \Delta t/2$, where Δt satisfies the constraints discussed earlier.

For the amplitude $A^{a)}$, integration over x^0 yields:

$$
\begin{aligned}
A^{a)} &= 2i\,\mathcal{N}G_F\,\delta^3(\mathbf{p}_n-\mathbf{p}_p-\mathbf{q}-\mathbf{k})\,e^{-i\omega^e_{\mathbf{q}}\Delta t/2}\,\bar{u}^s_{\mathbf{q},e}\gamma^\mu(\mathbb{1}-\gamma^5)\\
&\times\Bigg\{\cos^2\theta\,v^r_{\mathbf{k},1}\,e^{i\omega_{\mathbf{k},1}\Delta t/2}\,\frac{\sin\left[(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},1})\Delta t/2\right]}{\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},1}}\\
&\quad+\sin^2\theta\left[v^r_{\mathbf{k},2}\,|U_{\mathbf{k}}|\,e^{i\omega_{\mathbf{k},2}\Delta t/2}\,\frac{\sin\left[(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},2})\Delta t/2\right]}{\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}-\omega_{\mathbf{k},2}}\right.\\
&\quad\left.+\epsilon^r\,u^r_{-\mathbf{k},2}\,|V_{\mathbf{k}}|\,e^{-i\omega_{\mathbf{k},2}\Delta t/2}\,\frac{\sin\left[(\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}+\omega_{\mathbf{k},2})\Delta t/2\right]}{\omega_{\mathbf{p}_n}-\omega_{\mathbf{p}_p}-\omega^e_{\mathbf{q}}+\omega_{\mathbf{k},2}}\right]\Bigg\}\\
&\times V_{ud}\,\bar{u}^{\sigma_p}_{\mathbf{p}_p}\,\gamma_\mu(f-\gamma^5 g)\,u^{\sigma_n}_{\mathbf{p}_n}\,e^{-i(\omega_{\mathbf{p}_p}+\omega_{\mathbf{p}_n})\Delta t/2}.
\end{aligned}
\tag{38}
$$

In the short-time regime, retaining only the leading-order terms in Δt, the amplitude simplifies to:

$$
A^{a)} \simeq i\,\mathcal{N}G_F\,\delta^3(\mathbf{p}_n-\mathbf{p}_p-\mathbf{q}-\mathbf{k})\,\Delta t\,\bar{u}^s_{\mathbf{q},e}\gamma^\mu(\mathbb{1}-\gamma^5)\,v^r_{\mathbf{k},1}V_{ud}\,\bar{u}^{\sigma_p}_{\mathbf{p}_p}\gamma_\mu(f-\gamma^5 g)\,u^{\sigma_n}_{\mathbf{p}_n}. \tag{39}
$$

We now consider the flavor-violating channel described by Eq. (11). In the short-time limit, the amplitude (37) becomes:

$$
\begin{aligned}
A^{b)} \simeq i\,\mathcal{N}G_F\,\sin\theta\cos\theta\,\delta^3(\mathbf{p}_n-\mathbf{p}_p-\mathbf{q}-\mathbf{k})\,\Delta t\,\bar{u}^s_{\mathbf{q},e}\gamma^\mu(\mathbb{1}-\gamma^5)&\\
\times\left[v^r_{\mathbf{k},2}-v^r_{\mathbf{k},1}\,|U_{\mathbf{k}}|+\epsilon^r\,u^r_{-\mathbf{k},1}\,|V_{\mathbf{k}}|\right]&\\
\times V_{ud}\,\bar{u}^{\sigma_p}_{\mathbf{p}_p}\gamma_\mu(f-\gamma^5 g)\,u^{\sigma_n}_{\mathbf{p}_n}.&
\end{aligned}
\tag{40}
$$

By comparing Eq. (40) with its counterpart (21) derived using Pontecorvo states, one notices a key difference: the contribution associated with the mode of mass m_1 is now split into two distinct terms, each weighted by the Bogoliubov coefficients $|U_{\mathbf{k}}|$ and $|V_{\mathbf{k}}|$, respectively. The first term survives in the ultra-relativistic limit and corresponds to the Pontecorvo-like result in Eq. (21), while the second term is unique to the QFT framework and vanishes when $|V_{\mathbf{k}}| \to 0$. This additional component—absent in the Pontecorvo treatment—is precisely the term that restores consistency with the SM expectations at tree level. Indeed, by invoking the identity $v^r_{\mathbf{k},1}|U_{\mathbf{k}}| - \epsilon^r\,u^r_{-\mathbf{k},1}|V_{\mathbf{k}}| = v^r_{\mathbf{k},2}$, it is straightforward to verify that the entire expression within the square brackets in Eq. (40) vanishes identically, leading to $A^{b)} = 0$. This outcome demonstrates that the use of exact QFT states ensures conservation of lepton charge at the production vertex, in full agreement with SM.

To summarize the present stage of the discussion, we have focused on the short-time behavior of the β-decay amplitude. Naturally, a complete analysis should also account for the complementary limit $\Delta t \to \infty$. This regime has been investigated in [46], where it was shown that employing exact QFT flavor states yields results in agreement with SM expectations for relativistic neutrinos.

5. Conclusions and Outlook

In this work, we have explored the longstanding issue of the flavor/mass dichotomy in the context of neutrino mixing within QFT. To this end, we considered the neutron β-decay process

$n \to p + e^- + \bar{\nu}_e$ and its flavor-violating counterpart $n \to p + e^- + \bar{\nu}_\mu$ as a diagnostic framework. For both processes, we computed the transition amplitudes at tree level using standard scattering theory, focusing in particular on the short-time regime, corresponding to small distances from the interaction vertex. Our analysis was conducted by employing three different representations for neutrino states: *(i)* Pontecorvo states, *(ii)* mass eigenstates, and *(iii)* exact QFT flavor states, the latter defined as eigenstates of the flavor charge operators. The results demonstrate that only the exact QFT flavor states yield amplitudes consistent with lepton charge conservation at the vertex, in line with the SM predictions at tree level. This reinforces the view that QFT flavor states should be regarded as the correct fundamental representation for mixed neutrinos.

Several extensions to the present analysis are possible. A fully realistic treatment should include all three neutrino generations and the potential effects of CP violation. Additionally, a wave packet approach could more accurately model the finite localization of neutrino production and detection events. However, we do not expect these generalizations to invalidate our main conclusions. We also observed that, in the ultra-relativistic limit, the amplitudes obtained from both QFT and Pontecorvo flavor states converge. Since most neutrinos observed in current experiments are highly relativistic, this makes it challenging to experimentally distinguish between the two frameworks under typical conditions. Nevertheless, future experiments like PTOLEMY [51], designed to detect the Cosmic Neutrino Background (CNB) via tritium capture, may offer critical insights. Ongoing research in these directions is underway and will be the subject of future publications.

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