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Approximate Minimal SU(5), Several Fundamental Scales, Fluctuating Lattice

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Having shortly reviewed our idea of the grand unified SU(5) being only exact in a classical limit, in a truly existing lattice, an ontological lattice, we go over to putting a series of different physical energy scales such the approximate unification scale for the SU(5) (without any SUSY), the Planck scale, and e.g. the scale of see-saw neutrino masses into a certain plot showing the energy scales on a straght line. This straight line of this plot supposed to result from such an ontological lattice, that fluctuates in link size a and lattice density in a very strong way according to a log normal distribution. The point is that different energy scales result from the link size of the lattice to different powers, like the averages $< a^n >$, where the power n depends on the type of scale considered. Since the n'th root of the average of the n'th power of the link size in the fluctuationg lattice is very strongly dependent on the power n, because of very huge fluctuations, the different types of physical energy scales can get very different.
With a Galton i.e. log normal distribution of the link size the various energy scales have their logarithms fall into a nice straight line versus the power of the link size, on which they depend.
Our model gives a surprisingly good number for the small deviations of the experimental electron- and muon-anomalous magnetic moment from the pure Standard Model value.

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1. Introduction

Many physicists would probably speculate, if the Planck scale [3] should be the fundamental energy scale of physics; but there are several other energy scales of completely different orders of magnitude, the scale of the see-saw masses e.g. is closer about $10^{11} GeV$, but although unknown by several orders of magnitude at least not at all equal to the Planck scale. Similarly even in supersymmetric theories the Grand Unification, say SU(5), scale only hardly reach the Planck scale, and without SUSY, just taken as the scale of closest meeting of the three running fine structure constants, the approximate unification energy scale becomes much smaller than the Planck scale. So a lot of regimes, with for us to guess new physics, in between these energy scales, of which we may have hints, seem to have to be present. (Different ways of looking for fundamental scales may be seen in [4, 5] and fluctuating or irregular lattices as we shall use as our model may be seen in [6, 9]. Our own motivation for fluctuating lattice is the expectation of fluctuation in the gauge of gravity, the reparametrization[65–67].)

In the present article we want to "unify" the several energy scales by proposing a strongly fluctuating lattice, that could provide effectively **several** order of magnitudewise different scales (by averaging different powers of the link size). Then the intermediate regimes would not be needed to connect different energy scales, in as far as rather it would all be described by the same cut off - let us consider it a lattice- which though in our model is fluctuating and have e.g. different linksizes in different places.

This proceeding is of a talk, that can be considered a review of the recent article [1], the point of which is that a series of various energy scales such as the Plack scale[3], the grand unified SU(5) unification scale[2, 12] taken as an only approximately unification scale, so that it is only the scale, where the running gauge theory couplings for the Standard Model- the fine structure constants - are closest to agree with the SU(5) relation, etc. can be fitted with only two parameters. In fact a series of 9 energy scales fall on a straight line, if you plot the logarithms of their energy-values versus the integer number giving, what power of the lattice link length of a truly existing (ontological fluctuating) lattice, that is related to the energy scale in question.

This article [1] came out as a follower of my finding in [2] (this work was based on a lot of old works of ours assuming that the gauge coupling constants were udjusted to be critical couplings for a lattice (at the Planck scale)[17, 19–23, 25, 26, 29–33], and the idea of an approximate SU(5) is found in Senjanovic and Zathedeski [51].) of that the deviation of the experimental fine structure constants from the GUT SU(5) predition could be explained as due to a quantum correction of a lattice theory, which in the classical approximation is SU(5) invariant - in the sense of giving the SU(5) relation between the fine structure constants -, though with the little trouble: The quantum correction should be just 3 times bigger than, what the calculation with a simple lattice would give. Another point of deviation from usual Grand Unification is that the true gauge **group** (according to O'Raifeartaigh [39] one can to some extend assign a meaning to a gauge Lie Group and not only to Lie algebra as usually, except for on a lattice) in our model is only, what we call the Standard Model **group** $S(U(2) \times U(3)) \subset SU(5)$. So SU(5) is broken from the very beginning. The idea to explain the factor 3 we had to put in should be, that there in Nature indeed is one lattice for each family of fermions in the Standard Model, meaning 3 layers of lattices. But the important point for the series of energy scales of the present article is, that with this interpretation of the deviation from SU(5) as a quantum correction (with somehow the extra factor 3 due to the layers) allows us to avoid supersymmetry and leads to a unification scale $5.3 * 10^{13} GeV$, very much (five and a half order of magnitudes) below the Plack scale, $2 * 10^{19} GeV$, so that several different energy scales are indeed needed!

An energy scale already proposed in the article with the approximate SU(5) unification is a scale, we call "fermion tip" scale, and which is the scale obtained by putting the fermions in the Standard Model in a number order after the mass, starting the enumeration at the top mass and going downward in mass (counting the number of Weyl fermions, say, so that top counts for 6 and τ counts for two) and extrapolating to the number 0 in this counting. We found that there is a good fit with a parabola touching the abscissa at the fermion number 0 at this "tip of the fermion spectrum" being an extrapolated mass $10^4 GeV$.

1.1 Main Philosophy

The main point of our work is to assume that we have a lattice - this shall then be fluctuating in tightness, being somewhere tight, somewhere rough with big links and net holes - and then the various physical energy scales are calculated each of them from some power of the length a of a link. While for a rather narrow distrbution of a variable a say it is so that whatever the power of the variable a you need for your purpose you get about the same value for the effective typical a size,

$$\sqrt[n]{< a^n >} \approx n\text{-independent for narrow distributions.} \quad (1)$$

$$\text{However Galton distribtion: } P(\ln a) d\ln(a) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{(\ln a - \ln a_0)^2}{2\sigma}) d\ln a \quad (2)$$

$$\text{gives rather } \sqrt[n]{< a^n >} = a_0 \exp(\frac{n}{2} * \sigma). \quad (3)$$

So for a large spread σ in the logarithm different typical sizes $\sqrt[n]{< a^n >}$ become very different! Imagining that the gravitational Einstein metric $g_{\mu\nu}$ is quantum fluctuating relative to some fundamental lattice, we take it to mean that the lattice fluctuate relative to the metric in a way making its link size flutuate even strongly in size relative to the metric tensor field. As an ansatz we take the link size to have the Galton distribution[13–15] or let us just call it the Log Normal distribution. If a distribution is composed of many **multiplicative** fluctuating effects it will by essentially the central limit theorem obtain this Log Normal distribution as pointed out by Gibrat[13–15]. If the spread σ of this Log Normal distribtion is large the different $\sqrt[n]{< a^n >}$ can be of different orders of magnitude i.e. quite different.

To get an idea of how we may derive the relevant average of a power $< a^n >$ let us for example think of a particle, a string, or a domaine wall being described by action of the Nambu-Goto types

$$\text{Particle action} S_{particle} = C_{particle} \int \sqrt{\frac{dX^\mu}{d\tau} g_{\mu\nu} \frac{dX^\nu}{d\tau}} d\tau \tag{4a}$$

$$= C_{particle} \int \sqrt{\dot{X}^2} d\tau \tag{4b}$$

$$\text{String action } S_{string} = C_{string} \int d^2\Sigma \sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2} \tag{4c}$$

$$= -\frac{1}{2\pi\alpha'} \int d^2\Sigma \sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2} \tag{4d}$$

$$\text{Domain wall action } S_{wall} = C_{wall} \int d^3\Sigma \tag{4e}$$

$$\sqrt{\det \begin{bmatrix} (\dot{X})^2 & \dot{X} \cdot X' & \dot{X} \cdot X^{(2)} \\ X' \cdot \dot{X} & (X')^2 & X' \cdot X^{(2)} \\ X^{(2)} \cdot \dot{X} & X^{(2)} \cdot X' & (X^{(2)})^2 \end{bmatrix}} \tag{4f}$$

Here of course these three extended structures are described by repectively 1, 2, and 3 of the parameters say τ, σ, β, the derivatives with respect to which are denoted by respectively $\dot{}, ', and^{(2)}$. So e.g. $d^3\Sigma = d\tau d\sigma d\beta$ and

$$X^{(2)} = \frac{\partial X^\mu}{\partial \beta} \tag{5}$$

$$X' = \frac{\partial X^\mu}{\partial \sigma} \tag{6}$$

$$\dot{X} = \frac{\partial X^\mu}{\partial \tau} \tag{7}$$

$$\tag{8}$$

Finally of course $\cdot$ is the Minkowski space inner product.

Now imagine, that in the world with the ontological lattice, which we even like to take fluctuating, these tracks of objects, the particle track, the string track or the wall-track, should be identified with selections of in the praticle case a series of links, in the string case a suface of plaquettes, and in the wall-case a three dimensional structure of cubes, say. One must imagine that there is some dynamical marking of the lattice objects - plaquettes in the string case e.g.- being in an extended object. Now the idea is that we assume the action for the lattice to have parameters of order unity. In that case the order of magnitude of the effective tensions meaning the coefficients $C_{particle}, C_{string}, C_{wall}$ can be estimated in terms of the statitical distribution of the link length - for which we can then as the ansatz in the model take the Galton distribution (2) - by using respectively the averages of the powers 1, 2, 3, for our three types of extended objects. I.e. indeed we say that by order of magnitude, the mass of the particle, the square root of the string energy density or the string tension, and the cubic root of the domain wall tension are given as the inverses of averages of a like:

$$\text{Particle mass } m \sim < a >^{-1} \tag{9}$$

$$\text{Square root of string tension } \sqrt{\frac{1}{2\pi\alpha'}} \sim \sqrt{< a^2 >}^{-1} \tag{10}$$

$$\text{String tension itself } \frac{1}{2\pi\alpha'} \sim < a^2 >^{-1} \tag{11}$$

$$\text{Cubic root of wall tension } S^{1/3} \sim \sqrt[3]{< a^3 >}^{-1} \tag{12}$$

$$\text{wall tension itself } S \sim < a^3 >^{-1} . \tag{13}$$

Here the ~ approximate equalities are supposed to hold order of magnitudewise under the assumption that no very small or very big numbers are present in the coupling parameters of the lattice, so that it is the somehow averaged lattice that gives the order of magnitude for these energy densities or tensions.

1.2 Illustration of the idea

Although our speculations for the three energy scales - meaning numbers with dimension of energy - which we in my speculation attach to these three objects, the particle, the string, and the wall, are indeed very speculative only, and that we shall give a bit better set of such scales in next subsection, let us nevertheless as a pedagogical example consider these three first:

The speculative pedagogical example: The three speculative scales are chosen as:

- **The particle** Remembering that on a lattice with gauge particle degrees of freedom you may due to the very lattice get monopoles, the time tracks of which are described by series of cubes in the lattice, we propose to identify the "particle" described by an action of the type (4b) with a monopole caused by the lattice structure for the lattice gauge theory of the Standard Model.

  One of the very few new physics particles perhaps found in the LHC and even by studies of the data from LEP is a dimuon resonance with mass 27 GeV found in data selected with some b-meson activity[63, 64]. We speculate that this hopefully existing dimuon decaying particle is related to some monopoles for the Standard Model group - possibly a bound state by some sort of confinement of a couple of monopoles - with monopole mass of the order

  $$\text{"monopole mass"} \sim 27GeV. \tag{14}$$

  Taking it that all other parameters than the ones in the link distribution are of order unity, we get using (3, 9) up to order unity factors:

  $$27GeV \approx < a >^{-1} = a_0^{-1} \exp(-\frac{\sigma}{2}). \tag{15}$$

- **String** We shall identify the strings with the **hadronic strings**, if we dreamt of some realistic type of fundamental superstring, we would have no chanse to fit our fluctuating lattice with the other proposal we are making. So we take it that the moderate success of Veneziano models and string models[73] in hadron physics means, that the hadrons are at least in some crude approximation strings. This is true because confinement means, that a quark and an antiquark will be held together by a potentially long gluon structure, which functions like a string. In the string model for the hadrons the string tension or equivalently their energy density are given as $\frac{1}{2\pi\alpha'}$ where the α' is the Regge slope meaning the derivative of the angular momentum versus the square of the mass(this slope seems to vary rather little along the Regge trajectory, as well as from trajectory to trajectory). In our old paper[71] Ninomiya and I used the empirical value $\alpha' = 0.88GeV^{-2}$ and thus

$$\text{Energy density } \frac{1}{2\pi\alpha'} = \frac{1}{2\pi 0.88GeV^{-2}} \tag{16}$$
$$= 0.181GeV^2 \tag{17}$$
$$\text{or energy scale } \sqrt{\frac{1}{2\pi\alpha'}} = 0.43GeV. \tag{18}$$
$$\text{Also we thought of “Hagedorn temperature”} = 0.16GeV \tag{19}$$

So we get assuming other parameters than the ones in in link distribution being of order unity and using (3,10) up to order unity factors:

$$0.43GeV \approx \sqrt{< a^2 >}^{-1} = a_0^{-1} \exp(-2\frac{\sigma}{2}). \tag{20}$$

- **Domaine wall** Colin Froggatt and myself have for long developped a model for dark matter [69], in which there are (at least) two different phases of the vacuum and domaine walls in between them, where these phases meet each other. Fitting various informations on the dark matter we arrived in our latest work [69] in table 2 the sopposedly best values, 12 MeV and 4 MeV for the cubic root of the tension of the domaine wall. It was found by two different methods in line 2. and 3. respectively. The value 12 MeV were derived using the rate of the DAMA observations, while the value 4 MeV was based on the hypotesis, that the stopping length of our dark matter pearls, when hitting the Earth, were just so as to make the pearls stop at the depth 1400m of the DAMA experiment, favouring that just this experiment DAMA sees the dark matter, while no other ones see it. In our model radiation of the dark matter with its 3.5 keV radiation, which is with trouble seen astronomically[75], is supposed to be what underground experiments potentially see, but only DAMA saw it. The deviation between the two estimates of the cubic root of the wall, 12 MeV and 4 MeV, as only a factor 3 should be considered a sign of consistency or success of our model and a geometrically averaged cubic root of the tension, 7 MeV could be taken.

$$\text{cubic root of tension} = 7MeV = 0.007GeV \tag{21}$$
$$\text{tension} = (7MeV)^3 = 2 * 10^2 MeV^3 \tag{22}$$

Thus modulo order unity factors our fluctuating lattice here using (3, 12) gives:

$$7MeV \approx \sqrt[3]{< a^3 >}^{-1} = a_0^{-1} \exp(-\frac{\sigma}{2} * 3). \tag{23}$$

The check of our model - the fluctuating lattice with the Galton link size distribution (2) - means for these three energy scales that the following two ratios are equal to each other:

$$\exp(\frac{\sigma}{2}) = \frac{27GeV}{0.43GeV} = 63(\text{from "particle" and "string"}) \quad (24)$$
$$\exp(\frac{\sigma}{2}) = \frac{0.43GeV}{7MeV} = 61(\text{from "string" and "wall"}) \quad (25)$$
$$\text{Success with } \sigma = 8.3 \quad (26)$$
$$a_0^{-1} = 1.7 * 10^3 GeV. \quad (27)$$

It is indeed very strange, if the energy scale of the approximative string theory describing the hadron physics were connected with a fundamental lattice - such as our fluctuating one - because we know from QCD, that the scale of these hadronic strings is gives by the running strong Yang Mill coupling and not from any lattice a priori.

2. Our main set of Energy Scales

At the time of the talk in Corfu I had not yet thought upon the just presented three energy scales, but was rather occupied the **approximate SU(5) unification scale**, the (reduced) Planck scale, and a scale, which I invented myself called the **"fermion tip" scale**:

- **Fermion tip scale** In principle you can count all the fermions in the standard model after their mass - except that some are degenerate due to color symmetry - and then you can fit the number of the fermion counted from the highest mass downward versus the logarithm of the mass with a smooth curve. We claim that doing that a parabola like curve tounching the number zero at the mass $10^4 GeV$ fits well. In this sense we can claim that the "tip of the fermion spectrum" is at the "extrapolated mass" $10^4 GeV$. In the tables we have for each bunch of degenarate fermion states (counted as number of Weyl fermions) put the average number in the series counted from above and the logarithm (with basis 10 for simplicity) and from there constructed a number that should be constant in the sense of being the same for all the bunches of fermions provided the fermion number in the series indeed fitted the just mentioned parabola. That a curved fitting curve is favourable to a line fit of the number in the series versus the logarithm of the mass is seen from the figure 1. Our tables shown give for each bunch of fermions in the Standard model a construction of a number, that will be constant provided the fermions ordered this way indeed lies on the parabola, and indeed it is seen to be about 1.1 to 1.2 mostly.

  We want to assume, that in the region of masses below the tip point - which we call m_{mnl} - the number of fermions which can still at a certain scale be considered massless shall be proportional to the number of links the inverse size of which equals that scale. With such philosophy - to be explained more in the articles we review [1] - the tip point m_{mnl} should be the maximal probality point for the (logarithmic) distribution of the inverse link size. I.e. we should take

$$m_{mnl} = 1/a_0. \quad (28)$$

$$\textbf{Quark for } \mathbf{m_{mnl}} = \mathbf{10^4 GeV} \tag{29}$$

Name	number n	Mass m	$log_{10\,GeV} m$	$diff$ = 4-$logm$	$diff^2$	$diff^2/n$
top	3 ± 1	172.76 ± 0.3 GeV	2.2374 ± 0.0008	1.7626	3.1066 ± 0.003	$1.0355 \pm 0.001 \pm 0.4$
bottom	9 ± 0.3	4.18 ± 0.0079 GeV	0.6212 ± 0.001	3.3788	11.416 ± 0.01	$1.268 \pm 0.001 \pm 0.03$
charm	17 or 15	1.27 ± 0.02	0.10382 ± 0.009	3.8962	15.180 ± 0.07	$0.893 \pm 0.004 \pm 0.06$
strange	25 or 23	0.095 ± 0.006 GeV	-1.0223 ± 0.003	5.0223	25.223 ± 0.03	$1.009 \pm 0.001 \pm 0.1$
down	31	4.79 ± 0.16 MeV	-2.3197 ± 0.01	6.3197	39.939 ± 0.06	1.288 ± 0.002
up	37	2.01 ± 0.14 MeV	-2.6968 ± 0.03	6.6968	44.847 ± 0.4	1.212 ± 0.01

$$\textbf{Leptons for } \mathbf{m_{mnl}} = \mathbf{10^4 GeV} \tag{30}$$

Name	number n	Mass m	$log_{10\,GeV} m$	$diff$ = 4-$logm$	$diff^2$	$diff^2/n$
τ	13 or 19	1.77686 ± 0.00012	0.2496 ± 0.00003	3.7503	14.065 ± 0.0003	$1.082 \pm 0.00002 \pm 0.4$
mu	21 or 27	$105.6583745 \pm 2.4 * 10^{-6}$ MeV	-0.9761 ± 10^{-8}	4.9761	24.761 ± 10^{-7}	$1.179 \pm 4 * 10^{-9} \pm 0.3$
electron	41	$0.51099895069 \pm 1.6 * 10^{-10}$	$-3.2915 \pm 4 * 10^{-10}$	7.2916	53.167 ± 10^{-8}	1.297 ± 10^{-11}

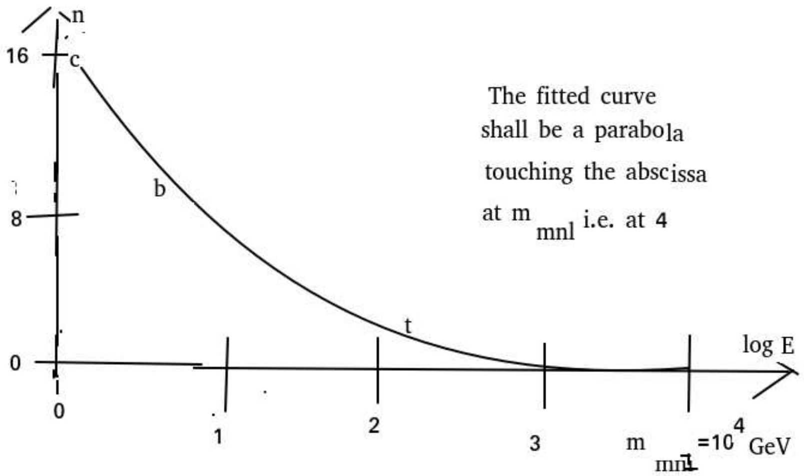

Figure 1: Because the distribtuion of log links after size has a maximum at the fermion tip point, we suggest that also the density of activ fermions (=fermions with lower mass than the scale E, at which you ask for the density of links) should behave this way, meaning with a **parabola** behavior with maximum at the point m_{nml} (=the fermion tip point). If we take it that there are 45 or 44 chiral fermions per family the number of families will at chiral fermion number n counted from the top downwards in mass, be "Number of active families" = $(3 * 45 - \text{n})/45$. Note that the three shown point, c, b, and t, fit better on the parobola than on a straight line, and so the fitting form chosen is somewhat empirically supported.

This "Fermion tip" scale is to be thought of as the analogue to the other scales with their form $\sqrt[n]{< a^n >}$ but with $n = 0$. However $n = 0$ does not quite make a good sense in as far as it would only give $\sqrt[0]{1}$, which is nonsense. So instead we must take the best we can:

$$\text{"Fermion tip"-scale} = \exp(< \ln(a) >) \tag{31}$$
$$\text{formally n=0}$$

- **Approximate unification scale** This is the scale at which the running (inverse) fine structure constants meet in the ratios predicted by grand unfied SU(5). It is wellknown that without susy[**?**] or some similar modification the running fine structure constants do not meet as they should in grand unified SU(5); but there is a little interval of energy scales in which $U(1)$ fine structure constant (corrected with its 3/5 for SU(5) notation) lies in between the two non-abelian fine structure constants, and one can say that in this region the three fine structure constants are more concentrated than at other energy scales. So we can take this rather small interval as "the approximate unification scale". In [2] we propose that with a 3 times magnified quantum corrections to a classically SU(5) symmetric action we get after the tripled quantum correction understanding of the running couplings as such a quantum corrected (with 3 times too much) classical SU(5) set of couplings. This gives a rather accurate unification inside the just mentioned interval of scales.

Now as is easily seen the effective Lagrangian density for the gauge theories evaluated in a lattice theory is given by the coulings on the single plaquattes multiplied by a number of the order a^{-4} due to that the Langian density per $(length\ unit)^4$ is of course proportional to the number of hyper-cubes in such a 4-volume. This number is scaled up of course by the a^{-4}. The very crossing of the running couplings has character of a logarithmic relation, and we believe it will not change how the unification scale is to be extracted from the link-distribution. So we take it:

$$\text{"Approximate unification" scale } m_U \approx \sqrt[-4]{< a^{-4} >} \tag{32}$$

$$= \frac{1}{\sqrt[4]{< a^{-4} >}}. \tag{33}$$

$$\text{I.e. } n = -4. \tag{34}$$

Let us evaluate this value $n = -4$ (again) by noticing the way the various factors in the action for a Yang Mills theory -or just electro dynamics - behave as functions of the link length a when one latticize this Yang Mills theory:

$$\text{Action } S = \int \frac{-1}{4e^2} F_{\mu\nu} F^{\mu\nu} d^4x \tag{35}$$

$$\text{or with metric } S = \int \frac{-1}{4e^2} g^{\mu\rho} g^{\nu\tau} F_{\mu\nu} F_{\rho\tau} \sqrt{g} d^4x \tag{36}$$

$$\text{where say for EM: } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{37}$$

In "latticising" we get the a dependences:

$$
\begin{aligned}
&\textbf{Without metric:} && && \\
\int ...d^4x &\rightarrow && < a^4 > \Sigma_{sites}... && (38)\\
\partial_\mu &\propto && a^{-1} && (39)\\
A_\mu &\propto && a^{-1} && (40)\\
\text{"Space-time-volume"}V_4 &= && \int d^4x \rightarrow < a^4 > \Sigma_{sites} 1 && (41)\\
\text{So action } S &\propto && \text{"}integral\ of\text{"} a^{-4} < a^4 > && (42)\\
\text{so } \frac{S}{V_4} &\propto && < a^{-4} > && (43)\\
\text{while } &\textbf{With metric} && && \\
\int ...d^4x &\rightarrow && < a^4 > \Sigma_{sites} \frac{1}{\sqrt{g}}... && (44)\\
\text{or } \int ...\sqrt{g} d^4x &\rightarrow && < a^4 > \Sigma_{sites}... && (45)\\
g_{\mu\nu} &\propto && a^2 && (46)\\
g^{\mu\nu} &\propto && a^{-2} && (47)\\
\partial_\mu &\propto && 1 && (48)\\
A_\mu &\propto && 1 && (49)\\
\text{"Space-time-volume"}V_4 &= && \int \sqrt{g} d^4x \rightarrow < a^4 > \Sigma_{sites} 1 && (50)\\
\text{So that } S = \int g^{\mu\nu} g^{\rho\tau} F_{\mu\rho} F_{\nu\tau} \sqrt{g} d^4x &\propto && \text{"}integral\ of\text{"} a^{-4} < a^4 > && (51)\\
\text{and thus } \frac{S}{V_4} &\propto && < a^{-4} > && (52)
\end{aligned}
$$

In our article [2] we found by our model of SU(5) symmetry being exact in a lattice theory in the classical approximation but being corrected by a quantum correction 3 times the a priori true one that the unification scale for this only classicially SU(5) unification was

$$m_U = 5.3 * 10^{13} GeV, \tag{53}$$

which is matching with just looking for the scale, at which the running couplings in the minimal SU(5) notation are closets to each other.

(Had we believed in susy the unification energy scale would have been rather

$$\text{susy unification} = 10^{16} GeV, \tag{54}$$

but that would *not* fit well in our scheme. So our phenomenolgical line fit, disfavours susy!)

- **Planck energy scale** The Einstein field equation looks

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \tag{55}$$

$$\text{or in trace reversed form } R_{\mu\nu} = \kappa (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}), \tag{56}$$

where then the left hand side is the curvature while the right hand side represent the energy momentum tensor. Thinking of a single hyper cube in the lattice the curvatur is to be represented as a discreticed second derivative of some variables defined on the lattice. Therefore on a single hypercube the ratio

$$\begin{aligned} \text{second derivative at hypercube} &\propto a^{-2} \quad (57)\\ \text{number hypercubes per four-volume} &\propto a^{-4} \quad (58)\\ \text{so naively } \frac{1}{\kappa} \propto a^{-2}. & \quad (59) \end{aligned}$$

But this was naive because we should have the second derivative and the energy or momentum sitting on the **same** hypercube so that the special gravitational interaction between them comes in. When the a is fluctuating and one shall average over, the common place for a second derivative and a piece of energy momentum This would mean inside a unit length hypercube the second derivative providing hypercube should be one among of the order of a^{-4} ones. Such a restricted second derivative would be a factor a^{-4} lower in probability of being found. So:

$$\begin{aligned}
\text{Restricted } \partial^2 = (\partial_\mu \partial_{nu})_{at\ given\ hypercube} &\propto a^{-2}/a^{-4}/\text{“unit four space time vloume”} && (60a)\\
&= a^2/\text{“unit 4-volume”} && (60b)\\
\text{Taking it } g_{\mu\nu} & \quad \text{independent of } a && (60c)\\
\text{and Einstein equation of form:} \partial^2 \text{“} g_{\mu\nu}{}' s'' &\propto \kappa a^{-4} * \text{“} g_{\mu\nu}{}' s'' && (60d)\\
\text{or } \partial^2 &\propto \kappa a^{-4} (\text{mod } g_{\mu\nu} \text{ factors, allowing diemensions different}) && (60e)\\
\text{so that with } \partial^2 &= \text{Restricted } \partial^2 \propto a^2/\text{“unit 4-volume”} && (60f)\\
\text{we get } \frac{1}{\kappa} &\propto a^{-6} * \text{“unit 4-volume”} && (60g)\\
\text{If relevant power is for } \frac{1}{\sqrt{\kappa}} & \ is \ a^{-6} && (60h)\\
\text{and we just use dimensionality } \frac{1}{\sqrt{\kappa}} &\sim \sqrt[6]{< a^{-6} >} = a_0 \exp(\frac{\sigma}{2} * 6); && (60i)
\end{aligned}$$

but this last step sounded dangerous.

This should mean that order of magnitude of the coefficient κ in the Einstein field equation, provided the lattice parameters are of order unity, should be given by averaging a sixth power so that we expect

$$\text{Reduced Planck constant} \frac{1}{\sqrt{\kappa}} \approx \sqrt[6]{< a^{-6} >} \quad (61)$$

(This calculations of the relevant powers deserves some further development, and a slightly different calculation is found in [1], and is actually alluded to in the table.)

In the notation with $\hbar = c = 1$

$$\begin{aligned}
\kappa = 8\pi G &= 2.07665 * 10^{-43} N && (62)\\
&= 2.07665 * 10^{-43} N * 8.19 * 10^5 N/GeV^2 && (63)\\
&= 1.7008 * 10^{-37} GeV^{-2} && (64)\\
\text{so that "Reduced Planck energy"} = \frac{1}{\sqrt{\kappa}} &= \frac{1}{1.7008 * 10^{-37} GeV^2} && (65)\\
&= (4.124 * 10^{-19} GeV)^{-1} && (66)\\
&= 2.425 * 10^{18} GeV. && (67)
\end{aligned}$$

With ignoring the 8π we get the unreduced Planck scale

$$\text{Planck scale(not reduced)} = 1.22 * 10^{19} GeV. \qquad (68)$$

Now fitting to (3) we have

$$\begin{aligned}
\frac{\text{"Reduced Planck scale"}}{\text{"Unification scale"}} &= \frac{2.424 * 10^{18} GeV}{5.3 * 10^{13} GeV} && (69)\\
&= 4.58 * 10^4 && (70)\\
\text{so } \exp(\sigma/2) &= \sqrt{4.58 * 10^4} && (71)\\
&= 214; && (72)\\
\text{to be compared with:} & && (73)\\
\frac{\text{"Unification scale"}}{\text{"Fermion tip scale"}} &= \frac{5.3 * 10^{13} GeV}{10^4 GeV} && (74)\\
&= 5.3 * 10^9 && (75)\\
\text{so that } (\exp(\sigma/2))^4 &= 5.3 * 10^9 && (76)\\
\text{giving } \exp(\sigma/2) &= \sqrt[4]{5.3 * 10^9} && (77)\\
&= 270 && (78)
\end{aligned}$$

Order-of-magniudewise at least 214 and 270 are very close, so the fit to the Galton distribution is good.

If we accept that the at first gotten 61 to 63 are also of the order of 214 to 270, then we have now about 6 different energy scales fitting on the straight line of the logarithm of the energy versus the power n to which the link size a should be taken before the average and the for dimensionality needed nth root of the average is taken. We had namely approximately same value for the a_0^{-1}, namely $1.7 * 10^3 GeV$ versus $10^4 GeV$.(See (27, 29, 30)).

Indeed we have invented/found in litterature a couple or three more such energy scales, believe it or not, we can claim they fit also into the same line in the logarithm versus n plot, so that we now have about 8 to 9 such energy scales. When I give the number we reached to as 8 to 9 it is because we can count the two scales connected with inflation of the universe, "the infation time Hubble-Lemaitre expansion rate H" and the "fourth root of the inflaton potential $V_{inflation}$ i.e. $\sqrt[4]{V_{inflation}}$" as two different ones or as essentially the same.(The information used for these cosmological energy scales was extracted from [58–60].)

The problem with many of the extra scales found is that they are typically like the "scale of the see-saw neutrino masses" not only dependent on the phenomenologically accessible information, as for the see-saw scale the neutrino oscillation data, but on some theoretically guessed model. This makes the true uncertainty of this see-saw-scale several orders of magnitude, but the value, which come from most detailed theoretical models of "See saw mass scale" = $10^{11} GeV$, fits wonderfully our straight line from the Galton distribtuion.

3. The Full Table

For completeness we include the table of all the 8 to 9 energy scales from [1], and to follow the integer number q in the reference is related to our power n to which the link size is raised when the average is taken by

$$q = n + 4. \quad (79)$$

3.1 Short Explanation of the table

For most details of the table now to be given we refer to the long article [1] or the description after the table; but shortly you should notice the symbols of the form a^n in the second column and the third line inside each of the blocks; this is the expression - a power, the nth power, of the link length a - over which the average under the lattice fluctuation - with the Galton distribution - is to be taken, when evaluating the (our) lattice theory prediction of the order of magnitude of the energy scale, whos name is put in the first line inside its block in the first column. Of course for dimenional reasons the energy scale must then be

$$\text{“}the\ energy\ scale\text{''} \approx \sqrt[n]{< a^n >}^{-1} \quad (80)$$

$$\text{when we use } a^n. \quad (81)$$

The second thing to note in the table are the two numbers in line 1 and 2 in the blocks in column 3. The uppermost number inside the block is the phenomenologically or experimentally given number for the energy scale, while the number below is the prediction of our fit - fitting the numbers $1/a_0$ and $\exp(\sigma/2)$ - for the energy scale energy in question. It is thus the order of magnitude agreement of these two numbers in column number 3, that express the success of our model for the energy scale of the block in question.

For several of the energy scales there are small variations in how the exact definiton of the scale should be taken or the like, and therefore some scales have gotten a couple of blocks, then seprated by a single horizonatal line, while one energy scale and the next are seperated by a double horizontal line.

3.2 Short mention of those Scales, we Essentially Skipped in this article

After looking at the articles[27, 45, 54–57] we decided the guess

$$\text{“see-saw scale”} \approx 10^{11} GeV, \quad (82)$$

and similarly after looking at [58–60] we decided to take for the situation at (end of) infaltion:

$$\text{“Infl. Hubble const.”}\ H_{infl.} \approx 10^{14} GeV \quad (83)$$
$$\sqrt[4]{\text{“Infl. potential”}} V_{infl.}^{1/4} \approx 10^{16} GeV \quad (84)$$

Concerning these cosmological scales about inflation, I do not feel safe as to what should be our power n, but believe we got it right with $n = -5$ for the $V_{infl.}^{1/4}$ and $n = -4$ for the $H_{infl.}$. (The information on the energies were extracted from [58–60]).

Finally since the “scalars scale” is only my phantasy, there is no direct observation of it; but the speculation is that there exist with the mass of this “scalars scale” a lot of scalar bosons, some of which have vacuum expectation values breaking spontaneously some symmetries of importance for making the masses of Standard Model Fermions having big mass rations (so called small hierarchy problem), and since the fermions from the “see-saw-scale” are supposed to be involved as intermediate fermions in the masses for the Standard Model ones, the ratio of the of the “see-saw scale” to the “scalars scale” provides the small number making the breakings of the symmetries small. From this kind of model for weak breaking of symmetries and thus large mass ratios among the Standard Model Fermions we expect such ratios to be of the order of $\frac{\text{“}scalars\ scale''}{\text{“}see\text{-}saw\ scale''} = 251$ from our straight line, to be compared with a typical ratio of Standard Model Fermions 43 (or corrected by a $\sqrt{4\pi}$ rather 152.)

3.3 The very table

Name comming from status	[Coefficient] Eff. q in m^q term	"Measured" value Our Fitted value	Text ref. Lagangian dens.
Planck scale Gavitational G wellknown	$[mass^2]$ in kin.t. q $=-2$ a^{-6}	$1.22 * 10^{19}$ GeV $2.44 * 10^{18}$ GeV	(68) $\frac{R}{2\kappa}$ $\kappa = 8\pi G$
Redused Planck Gravitational $8\pi G$ wellknown	$[mass^2]$ in kin.t. q $=-2$ a^{-6}	$2.43 * 10^{18}$ GeV $2.44 * 10^{18}$ GeV	(67) $\frac{R}{2\kappa}$ $\kappa = 8\pi G$
Minimal $SU(5)$ fine structure const.s α_i only approximate	[1] q $=0$ a^{-4}	$5.3 * 10^{13}$ GeV $3.91 * 10^{13}$ GeV	(53) $\frac{F^2}{16\pi\alpha}$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
Susy $SU(5)$ fine structure const.s works	[1] q $=0$ a^{-4}	10^{16} GeV $3.91 * 10^{13}$ GeV	(54) $\frac{F^2}{16\pi\alpha}$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
Inflation H CMB, cosmology "typical" number	[1] ? q $=0$? a^{-4}	10^{14} GeV $3.91 * 10^{13}$ GeV	(83) $\lambda\phi^4$ $V = \lambda\phi^4$
Inflation $V^{1/4}$ CMB, cosmology "typical"	concistence ? q $=-1$? a^{-5}	10^{16} GeV $9.96 * 10^{15}$ GeV	(84) consistency $V = \lambda\phi^4$?
See-saw Neutrino oscillations modeldependent	$[mass]$ *in non-kin.* q $=1$ a^{-3}	10^{11} GeV $1.56 * 10^{11}$ GeV	(82) $m_R\bar{\psi}\psi$ m_R right hand mass
Scalars small hierarchy invented by me	$[mass^2]$ *in non-kin.* q = 2 a^{-2}	$\frac{seesaw}{44\ to\ 560}$ $\frac{1.56*10^{11}\ \text{GeV}}{250}$	(149) and (150) $m_{sc}^2\vert\phi\vert^2$ breaking $\frac{seesaw}{scalars}$
Fermion tip fermion masses extrapolation	"$[mass^4]$ *in non-kin.*" q $=4$ $a^0 = 1$	10^4 GeV 10^4 GeV	(30) and (29) "1" quadrat fit
Monopole dimuon 28 GeV invented	"$[mass^5]$ *in non-kin.*" q $=5$ a	28 GeV 40 GeV	(14) $m_{monopol}\int ds$ $S \propto a$
String $1/\alpha'$ hadrons intriguing	"$[mass^6]$ *in non-kin.*" q $=6$ a^2	1 Gev 0.16 GeV	(18) Nambu Goto $S \propto a^2$
String $T_{hagedorn}$ hadrons intriguing	" $[mass^6]$ in non-kin." q $=6$ a^2	0.170 GeV 0.16 GeV	(19) Nambu Goto
Dom. wall dark matter intriguing	$[mass^7]$ in non kin. q=7 a^3	0.007 GeV 0.00064 GeV	(12, 21) Goto 3 dim.

3.3.1 Content in the Different Coloumns

- **The first column (from left)** contains the three items:

  1. A name, we just acsribe to the scale in question.
  2. An allusion to, from which data the energy scale number is determined.
  3. What we call "status", an estimation of how good the story of the scale in question is.

- In **the second column** the items are:

1. $[coefficient]$: It is the dimensionality of the coefficient to a term in the Lagrangian relevant for the interaction giving the scale in question. In the table is added either " ìn kin.t. " or "in non-kin.", meaning respectively, that the term with the coefficient of the dimension given is the kinetic term or the non-kinetic term repectively.
2. In next line inside this column 2 is written the quantity q and its value for the scale in question, or some effective value for this q. For non-kin. $q = dim_{energy}(coefficient)$, while for " kin.t. " it is $q = -dim_{energy}(coefficient)$ because we have in all of cases (by accident), that the other coefficient is dimensionless.
3. The power n of the link size a the average of which is supposed to give the size of the (coupling constant for) energy scale in question expressed by the symbol a^n. It means that the energy scale is given as

$$\text{" energy scale"} = \sqrt[n]{< a^n >}^{-1} = \frac{1}{a_0} \exp(-n\frac{\sigma}{2}). \tag{85}$$

- Then in the **third column**: comes the numbers, the energy scale.

1. In the top line inside the blocks comes the experimental or rather best theoretical estimate from the experimental data.(the type of data was mentioned in second line in column 1).
2. In the next line we have put the fit to the straight line of the logarith of the energy versus $q = n + 4$. It is given by the fitting formula:

$$\begin{aligned} \text{"fitted value"} &= 10^4 \text{ GeV} * 250^{-n} && (86)\\ &= 10^4 \text{ GeV} * (250)^{4-q} && (87)\\ &= 3.91 * 10^{13} \text{ GeV} * 250^{-q} && (88)\\ &= 3.91 * 10^{13} \text{ GeV} * 250^{-n+4} && (89) \end{aligned}$$

- In **fourth column**:

1. In first line is a reference to the formula in the text representing the decicion as to, what value to take for the scale in question. (Usually at best trustable to order-of magnitude).
2. In the second line is the Lagrangian density used in determining the dimension of its coefficient, $[coefficiient]$.
3. In the third line we put some formula or remark supposed to make it recognizable, what the Lagrangian density in the line 2 means.

Note in generel that the main point of our paper is, that the **two numbers in the third column agree** for most of the scales. The agreement for the susy unification is though not so impressive. This means that our idea with the fluctuating lattice does **not agree well with supersymmetric SU(5) unification !** However, the minimal SU(5) unification and the inflation Hubble constant $H_{inflation}$, seems to fit better. The to the Hubble Lemaitre expansion rate at inflation associated $V^{1/4}$ we only get into our scheme by using its relation to this infaltion Hubble Lemaitre expansion in the inflation time by LFRW relation. Therefore we wrote "consistency" for this $V^{1/4}$ case.

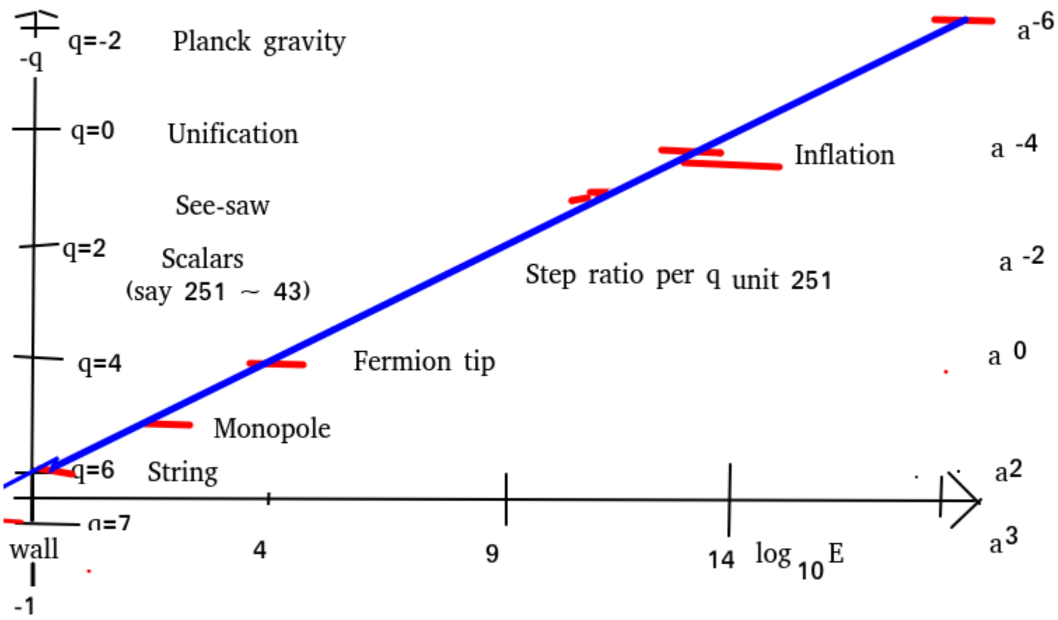

Figure 2: Here we see how the energy scales E, along the abscissa, the logarithms of them, (for easiness of calculation we used basis 10 logartihm). They all fit within expected accuracy except as it is put here the "inflation scale" for which is it not so obviuos what to take for $q = 4 + n = 4 +$ "the power for a relevant for averaging". In the table we decided that there were two scales involved with inflation, the Hubble-Lemaitre expansionrate H and the fourth root of the potential for the inflaton. For the first a^{-4} and for the second a^{-5} was chosen, and then it fits well. The ordinate is the number $q = n + 4$ or the power n for the to be averaged a^n. The assumed Galton distribution for the link length a gives the straight line as prediction. For the energy scale called "scalars", which is my invention just as a scale at which there would be many scalar particles having their masses and therefore also some vacuum expectation values of scalar fields like the Higgs (but the known Higgs is *not* there ! (see subsubsection 3.3.2) of the order of this "scalars" scale. The insertion on the plot of "(251 ∼ 43)" alludes to that the step in energy E per step in the power n is 251 while a typical suppression of fermion masses by an extra charge seperating the left and the right Weil components is estimated to 43.(see subsection 3.4 or [1] for details)

3.3.2 Standard Model Higgs needs a finetuning somehow or another

One needs to have finetuning making the known Higgs so light as the 125 Gev, and that must overwrite, that the scalars should have had the "scalars scale" masses. Personally I would have to think of our theory with complex action and influence from the future[40, 41, 49, 50] as giving some fine tuning mechanism which could overwritting the suggestion that the scalrs should have mass at the "scalars scale" give the much smaller Standard Model Higgs mass 125 GeV.

3.4 On the Scalars and the Seesaw scales and the Little hierarchy problem

[1]

[1]The words "little hierarchy problem" is used as the left over of the hierarchy problem after helped by susy, but we here (miss)use the words in the meaning of the wondering: Why are the quarks and leptons not having masses of the same order of magnitude?

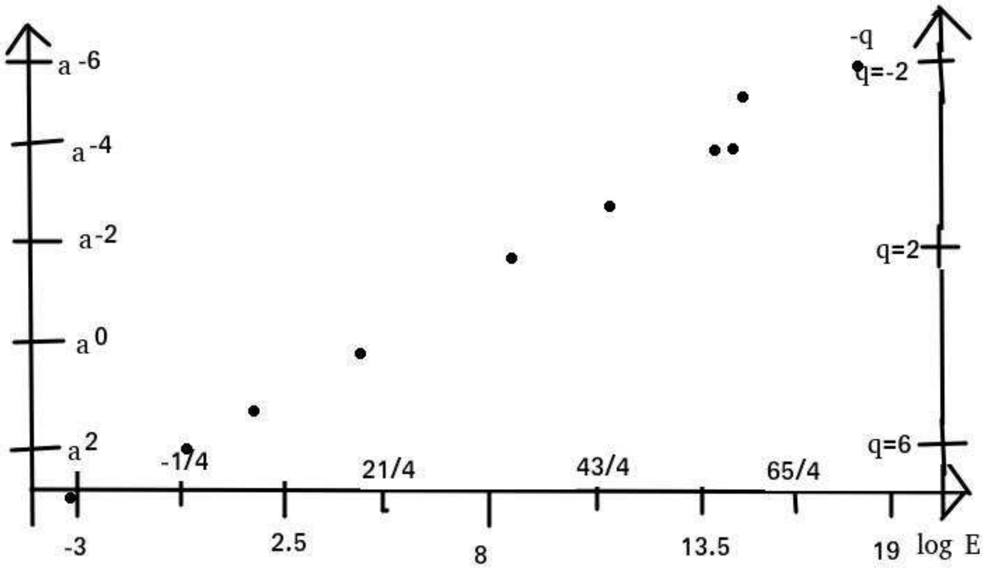

Figure 3: Similar figure as figure 2 but only with reduced Planck scale for the Planck scale, and written with just dots to let us enjoy how ell it fits the straight line. The cosmological entries looks worst and susy grand unification was left out.

The see-saw- scale is simply the mass order of magnitude for the see-saw neutrinos which in the see-saw model cause the neutrino oscillations of the observed Standard model neutrinoes. There are probably several such see-saw-neutrinoes and in the picture we shall take here they have order of magnitudewise the same masses, namely the see-saw-energy-sclale as mass. To determine this mass scale requires trusting some detailed theory for the see-saw neutrinoes and their couplings, but looking at the most trustable theoretical physicists proposals, we extract a scale for the see-saw masses of the order of 10^{11} GeV, which fit our straight line wonderfully. By analogy we now propose, that there should be a similar energy scale called “scalars”, at which there are several by order of magnitude similar scalar boson masses. Because of the mass square going into the Lagrangian for the scalar bosons while it is the mass itself that goes analogous in for the fermions, we find that in our scheme of energy scales the mass scale for scalars should be one factor 251 lower than that for the fermions, the see-saw- neutrinoes. This 251 is the step in energy scale corresponding to one unit in q or equivalently in the power n in a^n. But now in the philosophy of all couplings being a priori of order unity scalar field having vacuum expectation values will have them of the order of the “scalar scale” .

But then from the point of view of the see-saw fermions - which have masses of the see-saw scale - the vacuum expectation values of the scalar bosons possible spontaneously breaking various symmetries would look like, that the **very weak breakings** because the scalar scale is lower and the breakings are of the scalar scale order of magnitude. So here popped out of our scale hierarchy a mechanism for getting weak symmetry breakings without having to put in further small numbers. All the couplings we still assume of order unity. But such **weak** symmetry breakings is exactly, what can be usefull to make the hierarchy of fermion masses in the Standard Model by claiming the right and left Weyl components of the various Standard Model fermions being typically diffeerent with respect to some of these weakly broken charge types. It is exactly such weakly boken charges that may explain the so called small hierarchy problem[46, 74] of explaning the relativly large mass ratios among the Standard Model fermions.

But now in [1] and in the present article we predict the typical suppression of a fermion mass by having an extra charge needed to be broken to have an effective mass term to be a factor equal to the ratio of the see-saw scale over the scalar scale. In estimating this typical suppression factor of the masses in the Standard Model fermion spectrum in the article [1] we got something of order 43, while the ratio of the two energy scales see-saw versus scalars scale should be about 251 from our fitting. But the deviation between 43 and 251 is not great, the uncertainties of such estimates taken into account. So we consider this story of there being scalar scale as successful.

4. The anomalous magnetic moment deviations

As a little progress in the last moment before submission of the manuscript to the Corfu-proceedings let us make an attempt to see, if there is any chanse for our scheme with fluctuating lattice to give good numbers for the deviations of the experimental anomalous moments for muon and electron from the theoretically calculated values. It should namely be had in mind, that while one might usually expect, that the genuine cuttting off new physics might wait till the Planck scale, then the situation in our present scheme gives much better chanse for seeing indeed the new physics providing the cut off effect, that is at the end needed for the quantum field theories to converge, to make sense. Our scheme suggests the most fundamental scale to be the "fermion tip" one,and that is formally already inside the experimental reach 10 TeV!.

4.1 Plan and Simplification to come through at first.

For simplicity we shall not truly calculate the corrections from our fluctuating lattice, since even the lowest order diagram is quite complicated. Instead we shall choose a typical integral, that must occur in the evaluation of the finite lowest order anomalous magnetic moment contribution, and look for choosing such a representative integral so as to arrange our calculation of the correction by our (lattice) cut off as easily calculable as possible.

4.2 Choice of Representative Integral

We basically choose the form of the integral we would get by ignoring in the lowest order diagram - or putting to zero - the external four momenta and keeping only the denominators in the propagators. Since we are going to look for a relative correction, we do not have to care for the overall normalizaton. The divergent integrals in the true calculation of the $g-2$, i.e. anomalous magnetic moment, should cancel out, so we coose our representative integral to be convergent.

Indeed we choose to imagine, that we have already made the Wick rotation and denoting the numerical magnitude of the loop 4-momentum by q. Then we propose the following "representative integral":

$$\int_0^\infty \frac{q^3 dq}{(q^2+m^2)^2 q^2} = \frac{1}{2}\int_0^\infty \frac{d(q^2)}{(q^2+m^2)^2} \quad (90)$$

$$= \frac{1}{2}\int_{m^2}^\infty \frac{dY}{Y^2} \quad (91)$$

$$= \frac{1}{2}[-\frac{1}{Y}]_{m^2}^\infty \quad (92)$$

$$= \frac{1}{2m^2} \quad (93)$$

4.3 Introducing our Lattice Cut off

Now we shall a bit crudely introduce a cut off very similar to that of a fluctuation lattice. But really we do not believe in the cut off in Nature being exactly a lattice cut off; so rather we would like to just modify the propagators in the perturbation theory in a nice smooth way, and admit that we do not really know exactly how this modification should be, except that we replace the q or q^2 in the denominators of the propagators by some smooth - analytical - function, which is not very special, so that its Taylor expansion has coefficients of order of magnitude unity. In order to think of a concrete modification we replace the size of the four-momentum q (thought after the weak rotation, so that this size q is real and positive) by $\frac{1}{a}sinh(aq)$, which has the same property of being odd under sign change for q as q itself has, and which has such a sign of the Taylor expansion coefficient for q^3 that the propagator immediately begins to deviate from the usual un-cut-offed propagator $\frac{1}{m^2+q^2}$ in the right direction to make the propagator $\frac{1}{m^2+\frac{1}{a^2}\sinh^2(aq)}$ rather quickly disappear, as it has to at high energies in order to work as a cut off. We make thus the replacement of representative $sinh$-type:

$$q \rightarrow \frac{1}{a}\sinh(aq) \quad (94a)$$

$$\approx q + q\frac{1}{6}(aq)^2 + \ldots \quad (94b)$$

$$\approx q(1+\frac{1}{6}(aq)^2) \quad (94c)$$

$$\text{thus } q^2 \rightarrow q^2(1+\frac{1}{3}(aq)^2) \quad (94d)$$

$$\text{and } m^2+q^2 \rightarrow (m^2+q^2)(1+\frac{1}{3}(aq)^2 * \frac{q^2}{m^2+q^2}) \quad (94e)$$

$$\frac{1}{q^2(q^2+m^2)^2} \rightarrow \frac{1}{q^2(q^+m^2)^2} * \left(1-\frac{1}{3}(aq)^2(1+2\frac{q^2}{m^2+q^2})\right) \quad (94f)$$

$$\text{but integration measure } q^3dq \rightarrow q^3dq \text{ is NOT changed} \quad (94g)$$

$$\text{So } \int \frac{q^3dq}{q^2(m^2+q^2)^2} \rightarrow \int \frac{q^3dq}{q^2(m^2+q^2)^2}\left(1-\frac{1}{3}(aq)^2(1+2\frac{q^2}{m^2+q^2})\right) \quad (94h)$$

$$\approx \int \frac{q^3dq}{q^2(m^2+q^2)^2}\left(1-(aq)^2\right) \text{ for large } q \text{ in correction}$$

We here took the attitude that it is only the propagators which are modified by the cut off, while the integration measure $\int \ldots d^4q \propto \int \ldots q^3dq$ is just kept. Had we made a true lattice cut off, we should have made a sharp cut off in the integration region, but with our *sinh*-type we have gotten an exponential cut off at high q values, and we can let it take over the role of the cut off in the integration region, and thus truly keep the unchanged integration measure. We have indeed replaced the true lattice cut off by a more general type of cut off, which is easy by the q integration measure being kept undisturbed (i.e. no sharp cut off).

Let us illustrate the by our choice a very easy calculation first: the large q approximation for the correction due to the cut off:

$$\text{"correction"} = -\int \frac{q^3 dq}{q^2(q^2+m^2)^2} * (aq)^2 \tag{95a}$$
$$= -\frac{1}{2}\int \frac{d(q^2)}{(m^2+q^2)^2} * (aq)^2 \tag{95b}$$
$$= -\frac{a^2}{2}\int \frac{((q^2+m^2)-m^2)d(q^2+m^2)}{(m^2+q^2)^2} \tag{95c}$$
$$= -\frac{a^2}{2}\int_{m^2}^{\infty} \frac{(Y-m^2)dY}{Y^2} \tag{95d}$$
$$= -\frac{-a^2}{2} + \frac{a^2}{2}\int_{m^2}^{\infty} \frac{1}{Y}dY \tag{95e}$$

We found here a logarithmically divergent bit, but that was because we used the approximation that q was much smaller than the genuine cut off at aq of order unity. So we shall simply cut the logarithmic divergence off at

$$Y = q^2 + m^2 = (\frac{O(1)}{a})^2 + m^2 \tag{96}$$
$$\approx \frac{O(1)}{a^2} \text{ for high cut off} \tag{97}$$

If we which to calculate as accurately as we can in a primive way, we we could say, that the order of magnitude quantity, $O(1)$ denoted here, should be chosen so, that at the point $aq = O(1)$ the cut off correction factor $\frac{\sinh(aq)}{aq}$ be of order say 2. i.e.

$$\frac{\sinh(\text{"}O(1)\text{"})}{\text{"}O(1)\text{"}} \approx 2, \tag{98}$$

so that the propagator or square root of it will be damped to about the 1/2.

Since

$$\frac{\sinh(2.18)}{2.18} = 2.00, \tag{99}$$

we should then take the suggestive order of unity quantity to

$$\text{"}O(1)\text{"} = 2.18. \tag{100}$$

So the logarithmic term becomes

$$\text{"log-term"} = -\frac{a^2}{2}\int_{m^2}^{\infty} \frac{dY}{Y} \tag{101}$$
$$= -\frac{a^2}{2} * \ln(\frac{2.18}{(am)^2}) \tag{102}$$

The finite term just deviates from original integral $\frac{1}{2}\int \frac{d(q^2)}{(m^2+q^2)^2}$ by an extra factor m^2a^2, so that

$$\begin{aligned} \text{“ finite-term”} &= \text{“original”} * (m^2a^2) && (103)\\ &= \frac{1}{2m^2} * (m^2a^2) && (104)\\ &= \frac{1}{2a^2}. && (105) \end{aligned}$$

Since we have been completely careless about the normalization, we have now to concentrate on the relative correction due to the cut off:

$$\begin{aligned} \text{“Relative correction”} &= \frac{\text{“Log-term”+ “finite-term”}}{\text{“original”}} && (106)\\ &= -(am)^2 \ln(\frac{2.18}{(am)^2}) - (am)^2 && (107)\\ &= -(am)^2 \ln(\frac{2.18}{(am)^2 * e}). && (108) \end{aligned}$$

It is obviously seen, that this expression in first approximation goes as a^2 as function of the link size a, and thus according to our table the value to be used for the cut off energy $\sqrt{< a^2 >}^{-1}$ about the string-scale, which we interprete as hadrons being strings, and the hadronic scale

$$\begin{aligned} \text{the string energy scale} &= 0.16GeV(\text{fitted}) && (109)\\ * - * - * - * - *- &\; * \; - * - * - * - * - * \\ \text{For muon } \mu\text{: } \sqrt{< (a^2m_\mu^2) >} &= \frac{106MeV}{160MeV} && (110)\\ &= 0.66 && (111)\\ \ln(0.66) &= -0.41 && (112)\\ < (am_\mu)^2 > &= 0.66^2 = 0.44 && (113)\\ * - * - * - * - *- &\; * \; - * - * - * - * - * \\ \text{for electron } e\text{; } \sqrt{< a^2m_e^2 >} &= \frac{0.51MeV}{160Mev} && (114)\\ &= 0.0032 && (115)\\ \ln(0.0032) &= -5.75 && (116)\\ < (am_e)^2 > &= 0.0000102 = 1.02 * 10^{-5} && (117)\\ * - * - * - * - *- &\; * \; - * - * - * - * - * \\ \text{for tau } \tau \text{ ; } \sqrt{< a^2m_\tau^2 >} &= \frac{1777MeV}{160MeV} && (118)\\ &= 11.1 \text{ (not much sense)} && (119)\\ \ln(11.1) &= 2.41 && (120)\\ < (am_\tau)^2 > &= 11.1^2 = 123 && (121) \end{aligned}$$

But this argumentation is only true provided the the two factors a in the correction causing quantities $(am)^2$ are indeed the same.**So we shall not trust the just above equations**.

4.4 Alternative to just using $<(am)^2>$, namely $<am>^2$

If we use the $<(am)^2>$ as suggested above without fluctuating lattice, it means that we assume, that the two a-factors in the expression $<(am)^2>$ really are the local lattice scales a from **the same point in the fluctuating lattice**. But is that really true? In the just above discussion we got the appearance of the second power in a by insisting on a rotational invariant propagator, so that especially we should end up with a propagator, which after having been made of the form $\frac{1}{m^2+q^2}$, should keep the denominartor being of **even** order in q. We therefore imposed our choice for the modification of the propagator to be so that it replaces odd order in q expressions with odd order in q and even order ones by even order. This, however, likely is a too strong restriction of, what the cut off could do enforced only cut-off caused modifications of second or at least even order in in q and thus in aq to come in.

But that may mean imposing the fluctuating lattice too much symmetry. Especially if we think of the q as a true four momentum, rather than just as the norm of such a four vector, a cut off caused replacement by a lattice would not even keep translational invariance a priori, but only after averaging out at the end, it would be the simple propagator $\frac{1}{q^2+m^2}$.

In fact we could think something like this:

$$q \quad \rightarrow \quad q(1+aq) \tag{122}$$

$$\frac{1}{m^2+q^2} \quad \rightarrow \quad \frac{1}{m^2+q^2(1+2aq)} \tag{123}$$

$$\text{With averaging} \quad \approx\rightarrow \quad \frac{1}{m^2+q^2+2q^2<aq>} \tag{124}$$

If you require the end result to be both translational and rotational invariant, all the terms linear in $<aq>$ disappear and only by going to second order in $<aq>$ you hat any correction from the cut off. You must calculate with second order terms in $<aq>$ included and discard by averaging under rotations all the fisrt order terms in $<aq>$.

Order of magnitudewise you get at the end of having imposed rotational invariance much the same as before, but now we got the change

$$<(aq)^2> \quad replaced\ by \quad <aq>^2 . \tag{125}$$

For a narrow distribution of a this would make no difference, but for the Galton distribution (2) we get a widely different result.

In fact with the $<aq>^2$ we have the typical energy scale to use to be the "monopole scale" according to our table. That is to say

$$<a>^{-1} \quad \approx \quad 40GeV(\text{the fitted value}) \tag{126}$$

(The last paragraphs I should excuse for not being convincing nor pedagogical, but even if you just take it to mean: Now we allow ourselves to adjust which of the two energy scales, the "string scale" $\sim 0.16GeV$ or the "monopole scale" $40GeV$ or $27GeV$, to use at the two different places in the very simple formula (137), the overall factor and the place in the logarithm.

Even then, it would mean, that we fitted the two anomalous magnetic moment deviation by selecting one possibility among $2\times 2=4$ possibilities. If we take into account, that we also has chosen, whether to believe the finestructure constant from the Cs or the Rb measurement, we would have chosen among 8 possibilities, but still it is not so much!)

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4.5 The Observed Deviations

We here give a table with calculations meant to check a couple versions of our model(calculation) for the anomalies in the anomalous magnetic moments $a = \frac{g-2}{2}$, for the three charged lepton, but of course the τ magnetic moment is essentially not measured and of course not with the needed accuracy for seeing deviations - here denoted Δa from the in Standard Model predicted value. We have here taken the sign of this deveiation Δa as **the experimental value minus the theoretical one.**

1.	text	electron	muon	tau
2.	Experimental anomalous MM.	a_e^{exp}=0.00115965218059(13)	a_μ^{exp} = 0.0011659209(6)	a_τ^{exp}= 0.0009 ± 0.0032
3.	SM theoretical anomalous MM.	a_e^{SM}=0.001159652181643(764)	a_μ^{SM}=0.00116591804(51)	a_τ^{SM}=0.00117721(5)
4.	Deviation, exp - SM :	$\Delta a_e(Cs) = (-101 \pm 27) * 10^{-14}$ $\Delta a_e(Rb) = (34 \pm 16) * 10^{-14}$	$\Delta a_\mu = 249(22)(43) \times 10^{-11}$	
5.	Relative deviation	$\frac{\delta a_e(Cs)}{a_e} = (-87 \pm 23) * 10^{-11}$ $\frac{\Delta a_e(Rb)}{a_e} = (29 \pm 14) * 10^{-11}$	$\frac{\Delta a_\mu}{a_\mu} = 214(19)(37) * 10^{-8}$	
6.	Our $< (am)^2 >$	$< (am_e)^2 >= 1.05 * 10^{-5}$	$< (am_\mu)^2 >= 0.44$	$< (am_\tau)^2 >= 123$ (nonsense ?)
7.	$\frac{\text{Relative dev.}}{<(am)^2>}$	$\frac{\Delta a_e(Cs)}{a_e<(am_e)^2>} = (-83 \pm 22) * 10^{-6}$ $\frac{\Delta a_e(Rb)}{a_e<(am_e)^2>} = (28 \pm 15) * 10^{-6}$	$\frac{\Delta A_\mu}{a_\mu * <(am_\mu)^2>} = 486 * 10^{-8}$	
8.	Alternativ: $< am >$ $< am >^2$	$< am_e >=1.27 * 10^{-5}$ $< am_e >^2= 1.63 * 10^{-10}$	$< am_\mu > 2.7 * 10^{-3}$ $< am_\tau >^2= 7.0 * 10^{-6}$	$< am_\tau >= 4.5 * 10^{-2}$ $< am_\tau >^2= 2.0 * 10^{-3}$
9.	$\frac{Relative\ dev.}{<am>^2}$	$\frac{Rel.\ dev._e(Cs)}{<am_e>^2} = -5.34 \pm 1.41$ $\frac{Rel.\ dev._e(RB)}{<am_e>^2} = 1.78 \pm 0.85$	$\frac{Rel.dev._\mu}{<am_\mu>^2} = 0.306(21)(53)$	

4.5.1 The different lines in the table:

1. line The headings for the columns.

2. line The experimental values of the anomalous magnetic moments of the charged leptons.

3. line pair The theoretical Standard Model anomalous magnetic moment values

4.line pair (Here we include two lines formally intoline 3.) The deviation of the experimental anomalous magnetic moments from the Standard Model calculation one. But now an extra line not counted is sneaked in because the electron anomalous magnetic moment is measured so accurately that a 5 s.d. tension in the determination of the fine structure constant between measuring using Cs and Rb becomes important, so we must give the deviation for the twodifferent values of the fine structure constant.

5. line pair The deviations from 3. Δa divided by the value of the anomalous magnetic moment itself for the lepton in question.

6. line Our from our in the table fitting to $< a^2 >= \frac{1}{\text{“string energy scale”}^2}$ multiplied by the square of the mass m of the lepton announced in the top line. Since this mass m is just a constant of course

$$< (am)^2 > \;\; = \;\; < a^2 > *m^2 = \frac{m^2}{(0.16GeV)^2}. \tag{127}$$

7. line pair In this line pair we divided the relative deviations from line pair 4. by the averages in line 5., because if it really had the right thing to do have these $< (am)^2 >$ from line 5. provide the small size of the cut off effect, so that it was correct in (108) to put $(am)^2$ equal to $< (am)^>$,

we should have gotten these ratios in this line pair 6. close to unity. But that seems not at all to be right.

8. line We then attempt to instead put the over all factor in (108) $(am)^2$ equal to $< am >^2$, which is rather

$$< am >^2 \quad = \quad < a >^2 * m^2 = \frac{m^2}{\text{“monopole scale”}^2} = \frac{m^2}{(40GeV)^2}. \tag{128}$$

9. line pair When we now divide line pair 4., the relative deviations by the line 7. then we get indeed numbers of order unity. So if we can argue that the two a factors in front in (108)arenot correlated and shall be averaged separately, then these numbers in this line pair 8. being of order unity is a success of our model, (the log factor naturally shall be of order unity)

4.6 Backward fitting

If we take the philosophy, that the momentum scale, at which the cut off roughly sets in is given by the momentum scale, at which

$$< (aq)^2 > \quad \approx \quad 1 \tag{129}$$

$$\text{meaning that at } q = q_{cut\ off} \quad \approx \quad \frac{1}{\sqrt{< a^2 >_{formal}}} = \text{“string energy scale”} = 0.16GeV \tag{130}$$

$$\text{(as if the two } a\text{'s at same point),} \tag{131}$$

we might argue for using the two a factors in $< (aq)^2 >$ to be at the same point/hypercube for the cut off in the momentum going into the logarithm by thinking of it as a higher order term due to the lattice of the lattice Lagrangian term for the kinetic energy tern $F_{\mu\nu}F^{\mu\nu}$ say. Both factors a have to be at the hypercube of the contribution to the $F_{\mu\nu}F^{\mu\nu}$. This should give that the $< (am)^2 >$ in the logarithms in (137) should from the “string scale”.

But if we imagine, that the lattice takes up even momentum for small moments, although it delivers it back to keep phenomenologically/at the end momentum conservation, a term like $(am)^2$ would after being average to $< (am)^2 >_{true}$, we call it, really have two uncorrelated factors a, meaning from different sites, and the value would be rather $< (am)^2 >_{true} = < am >^2$ (meaning use “monopole scale”).

$$\text{Thus truly rather } < a^2 > = < a^2 >_{true} \quad \approx \quad < a >^2 \text{ (when } a\text{s at different points)}$$

$$\text{because the two } a\text{'s are often different} \tag{132}$$

$$\text{and thus } \frac{(< a^2 >_{formal})^{-1}}{(< a^2 >_{true})^{-1}} \quad \approx \quad \left(\frac{\text{“string energy scale”}}{\text{“monopole energy scale”}}\right)^2 \tag{133}$$

$$\approx \quad (\frac{0.16GeV}{40GeV})^2, \tag{134}$$

we might absorbing our order of unity factors into the “string energy scale”, by putting

$$(\text{“}string''\text{)}^2 \quad = \quad \frac{2.18}{e} * (\text{“string energy scale”})^2, \tag{135}$$

get the very simple form of our prediction for the deviation of the anomalous magnetic moment from the Standard model value:

$$\frac{\Delta a_l}{a_l} = -\frac{m_l^2}{\text{“monopole energy scale”}^2} * \ln(\frac{\text{“string”}^2}{m_l^2}) \quad (136)$$

$$= 2\frac{m_l^2}{\text{“monopole energy scale”}^2} \ln(\frac{m_l}{\text{“string”}}). \quad (137)$$

Of course having only measured two anomalous magnetic moments, it is not so great to fit with two parameters, “string” and “ monopole energy scale”, except if we find that the fitting values agree remarkably well with these scales as found by our fitting in the table above.

We may simply extract the fitting parameters by using

$$\frac{\Delta a_e}{a_e * m_e^2} - \frac{\Delta a_\mu}{a_\mu m_\mu^2} = \frac{2\ln(m_e/m_\mu)}{\text{“monopole energy scale”}^2}.$$

meaning for, say “Cs”:

$$\frac{(-87 \pm 26\%) * 10^{-11}}{(0.000511GeV)^2} - \frac{214(9\%)(17\%) * 10^{-8}}{(0.105658GeV)^2} = \frac{2\ln(\frac{0.000511GeV}{0.105658GeV})}{\text{“monopole energy scale”}^2}$$

$$\text{or } (-3.332 \pm 26\%) * 10^{-3}GeV^{-2} - 1.917(9\%)(17\&) * 10^{-4}GeV^{-2} = \frac{2 * (-5.332)}{\text{“monopole energy scale”}^2},$$

$$\text{so “monopole energy scale”} = \sqrt{\frac{-10.664}{(-3.523 * 10^{-3} \pm 26\%)GeV^{-2}}}$$

$$= 55.0GeV \pm 13\% \quad (138)$$

$$\text{to compare with} \quad 40GeV \text{ or } 27GeV. \quad (139)$$

Next we use say

$$\frac{m_\mu}{\text{“monopole energy scale”}} = 1.920 * 10^3 \quad (140)$$

$$\text{and thus } \ln(\frac{m_\mu}{\text{“string”}}) = \frac{\Delta a_\mu}{a_\mu}/2 * \frac{\text{“monopole energy scale”}^2}{m_\mu^2} \quad (141)$$

$$\frac{214(19)(37) * 10^{-8}}{2 * (1.920 * 10^{-3})^2} = 0.290(9\%)(17\%)$$

$$\text{(taking “monopole energy scale” as exact)} \quad (142)$$

$$\text{this means } \frac{m_\mu}{“string''} = \exp(0.290) = 1.34 \quad (143)$$

$$\text{thus “string”} = 0.105658GeV/1.34 \quad (144)$$

$$= 0.0790GeV \quad (145)$$

$$\text{to compare with “string”} = \frac{2.18}{e} * 0.16GeV \quad (146)$$

$$= 0.128GeV \quad (147)$$

It deviates by only a factor 1.6. The to the anomalies in the anomalous magnetic moments fitted “string energy scale” would be 0.10 GeV.

So this choice of making the $<(am)^2>$ in the logarithm the "string energy scale", while the occurance in the overall factor be "monopole energy scale" fits wonderfully the small anomalous to the anomalous magnetic moments! It should be stressed that these "energy scales" are gettable from our fit of the scales to the straight line, so even if only some of the energy scales, we dream about, are truly physical we can get this, using the straight line fit, just found wonderful numbers for the deviations from the anomalous magnetic moments from the Standard Model values. It must though be admitted that we so to speak predict the negative deviation for the electron anomalous magnetic moment and thus predict that it must be the Cs experiment value of the fine structure constant, that is the right one. However, for the muon anomalous magnetic moment the " string energy scale" and the muon mass are so close, that the sign of the logarithm of these two numbers becomes uncertain, and therefore we do not predict the sign of the muon anomaous magnetic moment deviations; but we can certainly fit it to be positive, as it is.

5. Conclusion

We have put forward a list of energy scales, all in principle determined from some measurements although heavily having to be treated by some theory to be determined, and found that they all fall on a straight line in a certain plot. This plot has on one axis the power of the lattice length a, i.e. n in a^n, which is relevant for the energy scale in question. On the other axis we have simply the logarithm of the energy of the energy scale.

Under the speculation of there existing a fluctuating lattice - meaning a say Wilson lattice, but with the important feature, that it is somewhere very tight and somewhere with very big masks, and even extended somewhere in some directions and somewhere in other directions - we (only to order of magnitude, but still some fitting) have fitted these different energy scales by arguing, that they result from averaging the different powers of the link length, which are relevant for the different scales. The point is that with the extremeliy broad distribution of link sizes, which we assume - a Log Normal distribution (sometimes called a Galton or a Gibrath distribution) with a large width, σ - the energy extracted from the average of different powers of the link length a can differ by appreciable factors. In fact the energy scale extracted from the average of the nth power, $<a^n>$, is for dimensional reasons $\sqrt[n]{<a^n>}^{-1}$, and one calculate in the Log Normal distribution trivially, that such energy scales become of the form

$$\sqrt[n]{<a^n>}^{-1} = a_0^{-1}\exp(-\frac{\sigma * n}{2}) \tag{148}$$

where there are only the **two** parameters, a_0 and σ, fitting order of magnitudewise the whole series of about 9 different energy scales, all related to some experimental data, although often in a very theory dependent way. These theories are not very trustable, rather a kind of phenomenological models. Nevertheless it is not so bad again:

The "Fermion tip scale" can in principle be considered an extrapolation of the fermion masses in the Standard Model put up after their number after mass counted from the highest mass. The extrapolation then back to number zero gives as the extrapolated mass what we call the "fermion tip scale"; it turns out to be a parabolic extrapolation and thus not so accruate (for the minmum point,which is "fermion tip scale"), but in principle it is defined.

The energy scale, at which the running fine structure constants are closest to be in the ratios as if we had SU(5) unification, is also order of magnitude-wise making sense, and of course the Planck scale is order of magnitude-wise well-defined. Similarly, if we think of hadrons being approximately described by a string theory - as historically was the starting point for string theory[73] - the order of magnitude for, say, the string tension is pretty well defined.

So at least the energy scale values of four of our nine scales are not so ill defined.

If you believe, that the doubtfull dimuon resonance with mass $27GeV$ really is a monopole for some Standard Model gauge group $S(U(2) \times U(3))$ or a bound state of some such monopoles, of course this mass $27GeV$ points out a well defined energy scale, so if you counted that, we would have even 5 numerically sensible energy scales.

Even having five energy scales with reasonably well-defined energy numbers, when plotted with the logarithm of the energy number versus the supposed relevant power n of the lattice length, fall on a stright line is quite a remarkable coincidence unless it has some explanation. Of course the fluctuating lattice is the sort of explanation which is called for.

It is, however, quite intriguing, since while the approximate SU(5) unification scale and the Planck scale are easily imagined to come from some fundamental lattice or other type of cut off, the hadron string scale is basically known to be given from the running of the QCD strong coupling α_S and thus should not be given a priori from mysterious phantasy lattice (fluctuating or not). Similarly, even if the domain wall of Froggatt‘s and mine dark matter model existed, then it should presumably be understandable as e.g. QCD vacuum having several phases appearing, when considering the vacuum as function of the quark masses (“ Columbia Plot”[76, 77]). But that would again be given by QCD and seemingly have little to do with a fundamental lattice, even if there were such one.

The remaining 4 energy scales are, however, only estimable by using some phenomenologigal theory fitting the related data and comming out with some number from that fitting: E.g. you can theoretically fit neutrino oscillations by many models, and these different models can make the typical or average see-saw neutrino have by orders of magnitude deviating values, so that a see-saw-neutrino-mass scale is uncertain with a few orders of magnitude. Similarly the inflation time Hubble-Lemaitre constant is quite model dependent, and the domain wall tension in our dark matter model is very badly determined, even if our dark matter model should be true.

The least trustable of the scales is the “scalars scale”, which is only my own speculation: I propose that there should be an energy scale, for which there are several scalars having their mass, and if they have expectaion values in vacuum also the scale of these expectation values. Nevertheless even this worst among the nine scales is formally involved with an experimental connection in as far as some typical scales for ratios of standard model Fermions can be used to fix it. In fact since the fermions in the Satandard Model deviate from each other by big factors of the order of 100 or more, explaining this the effect of some fermion masses needing breaking of more approximately conserved charges of some type yet to be discovered than other fermions in Standard Model, is a natural idea. Now such only weakly broken symmetries could appear, when fermions at the see-saw scale of mass have symmetries broken by scalar vacuum expectation values of size given by the “scalars scale”. This scale is namely by our straight line fit lower than the “see-saw scale” by a factor say 251. The ratio of the “see-saw scale” to the “scalars scale” should thus be a typical ratio of Standard Model Fermions, and thus in principle some experimental connection is there. In fact in[1] I looked up for some more than once occuring ratio values of pairs of Standard model

Fermions and found:

$$\frac{m_t}{m_b} \approx \frac{m_b}{m_s} \approx \frac{m_s}{m_u} \approx 43 \quad (149)$$

$$\text{or if Yukawa coupling} = \sqrt{4\pi}, \frac{\text{“}see - saw''}{\text{“}scalars''} \approx 43 * 3.54 = 152. \quad (150)$$

5.1 The anomalous magnetic moments

All this was already in the Universe article[1], but a quite new section, also not present at the time of the conference, was the attempt to estimate the deviation of the anomalous magnetic moments of electron and muon from Standard Model values, which our fluctuating lattice would give to these anomalous magnetic moments. Our estimation here is, however, very preliminary in the sense, that we only take a “representative integral” for the finite integral supposedly providing the lowest order loop integral for the anomalous magnetic moment. But the result for fitting the deviations of the anomalous magnitic moments may not depend so much on the exact “representative integral”, since our calculation turned out to be mainly a logarithmically divergent term cut off in the high energy end by the cut off loop-momentum identified (by our assumption) with scale of the “string”, which is modulo order unity the scale of “hadronic strings”. We could namely then hope, that many different barely finite integrals going into the anomalous magnetic moment calculation would by being modified by some factor loop momentum squared q would also go logartihmically divergent in a very similar way, so that the difference in the relative correction would be small. The over all scale/strength of the cut off effect is by our calculation and assumptions, however, the scale we call in the table the “monopole scale”. That it is such a rather simple logarithmic divergence cut off makes it likely that the various types of integrals for which we should have calculated the correction to get the true correction to the full anomalous magnetic moments, would likely all be corrected by the same factor. If so, then we should already have performed the correct calculation.

Our model predicts that the correction to the electron anomalous magnetic moment should be negative in the sense of diminishing experimental value compared to the theoretical anomalous magnetic moment. Thus our model has only a chanse of being right, if we use the value of the fine structure constant determined by use of caesium Cs, while the one using rubidium leads to no significant deviation of theoretical and experimental magnetic moments for the electron. However, the sign of the muon anomalous magnetic moment is due to an accident not comming out of our model, but the order of magnitude of the deviations from theory of both the anomalous magnetic moments - electron and muon - come out surprisingly well. The coincidense, which makes the muon anomalous magnetic moment have the unpredictable sign , is that, what we call the hadron string scale, which is given as 0.16 GeV in our table, is very close to the muon mass, which is 0.105658 GeV. It is namely the logarithm of the ratio of these two numbers corrected by some order unity factor that determines the sign of the muon anomalous magnetic moment contribtuion from our cut off.

So the our model has the small deviations of theory from experiment for the anomalous magnetic moments as a great success, although we must admit that it is an a bit uncertain discussion why one should use for the overall magnitude of the deviations the “monopole scale” in stead of e.g. also the “string” scale. But if one does not use the monopole scale one gets completely wrong magnitude for the deviations.

We think this calculation of the anomalous magnetic moment corrections in our model deserves more precise calculations.

If our speculation that one of our energy scales is one of monopole masses - probably some monopoles for the gluon fields and presumably what was found at the mass 27 GeV decaying into two muons were not a genuine monopole, but rather a "hadron like" bound state of a couple of monopoles, so that no net monopole charge would prevents its decay into two muons. But one could still ask, if such monopoles could have an important role for confinement? One could ask:

Could it be that confinement is not appearing for truly continuous gluon fields, but that it is only possible to have confinement, if there effectively are some monopoles, like on a lattice as used in lattice calculations, or some physically existing gluon-monopoles, even if they should be as heavy as of the order of 27 GeV?

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