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Carroll gravity from the conformal approach

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We show how electric and magnetic Carroll gravity can be constructed using conformal methods. We first review the conformal approach to relativistic Einstein-Hilbert gravity. Next, we consider a set of independent and dependent gauge fields of the conformal Carroll algebra, paying special attention to the way intrinsic torsion tensor components appear in their transformation rules. We then couple a single electric/magnetic massless scalar to these conformal Carroll gauge fields and show how, upon gauge-fixing the dilatations, we obtain a non-conformal version of electric/magnetic Carroll gravity.

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1. Introduction

Carroll symmetries have been under intense study in recent years for a variety of reasons. First of all, since they are the symmetries of null surfaces, they play a central role in investigations of black hole horizons [1] and the asymptotic null boundary of flat spacetime. Moreover, a conformal extension of Carroll symmetries corresponds to the asymptotic BMS symmetries of flat spacetime that play a fundamental role in Carroll holography [2–4]. Carroll gravity theories have also been constructed; they occur in two versions that are called electric [5] and magnetic Carroll gravity [6].

In this work we will focus on a conformal extension of the Carroll symmetries. This will enable us to generalize the so-called conformal technique to the construction of actions for Carroll gravity without conformal symmetry. This conformal technique is based on the observation that there is a relation between a dynamical scalar field and gravity. Coupling a relativistic dynamical scalar to gauge fields of conformal gravity and setting the scalar equal to a constant value to gauge-fix the dilatations, one obtains the Einstein-Hilbert action of general relativity. It also works the other way around. Replacing the Vielbein by the product of the (compensating) scalar and a conformal Vielbein, such that this product is invariant under dilatations, and restricting to flat spacetime one re-obtains the Lagrangian of the dynamical scalar.

We will show how this program can be extended to construct Carroll gravity. An important difference with the relativistic case is that in the Carroll case there are two types of massless scalars called 'electric' and 'magnetic' scalars. Correspondingly, we will find that the conformal method relates these two scalar field theories to the electric [5] and magnetic [6] types of Carroll gravity.

An important difference between the pseudo-Riemannian geometry of general relativity and the Carroll geometry underlying Carroll gravity is that in the latter case there exists a special type of torsion tensor, called intrinsic torsion, that is independent of any spin-connection. While in general relativity non-zero torsion usually occurs only after coupling to matter, in the Carroll case these intrinsic torsion tensors can occur even in the absence of matter. Setting them to zero leads to constraints on the geometry and they can therefore not be ignored. In this work we will carefully include these intrinsic torsion tensors in the conformal method.

2. Einstein-Hilbert gravity from the conformal approach

In this section, we will review how the conformal approach can be used to obtain relativistic Einstein-Hilbert (EH) gravity. The first step of this method consists of constructing a set of suitable independent and dependent gauge fields of the relativistic conformal algebra. In a second step, these gauge fields are coupled to a massless, so-called compensating scalar, to render its action invariant under local Lorentz transformations, dilatations and special conformal transformations. The EH action is then obtained in a third step, which consists of gauge-fixing the dilatations and special conformal transformations (i.e., the conformal symmetries that are not part of the Poincaré algebra).

2.1 A minimal set of gauge fields of the conformal algebra

In applying the conformal method, one first writes down a set of independent and dependent conformal algebra gauge fields. To do this, one starts from gauge fields in the adjoint representation of the relativistic conformal algebra. The latter is spanned by the generators of translations $P_{\hat{A}}$, Lorentz transformations $M_{\hat{A}\hat{B}} = -M_{\hat{B}\hat{A}}$, special conformal transformations $K_{\hat{A}}$ and dilatations D and has the following non-zero commutation relations

$$\begin{aligned} \left[M_{\hat{A}\hat{B}}, M_{\hat{C}\hat{D}}\right] &= 4\,\eta_{[\hat{A}[\hat{C}} M_{\hat{D}]\hat{B}]}\,, & \left[P_{\hat{A}}, M_{\hat{B}\hat{C}}\right] &= 2\,\eta_{\hat{A}[\hat{B}} P_{\hat{C}]}\,, \\ \left[K_{\hat{A}}, M_{\hat{B}\hat{C}}\right] &= 2\,\eta_{\hat{A}[\hat{B}} K_{\hat{C}]}\,, & \left[P_{\hat{A}}, K_{\hat{B}}\right] &= 2\,\eta_{\hat{A}\hat{B}} D + 2\,M_{\hat{A}\hat{B}}\,, \\ \left[D, P_{\hat{A}}\right] &= P_{\hat{A}}\,, & \left[D, K_{\hat{A}}\right] &= -K_{\hat{A}}\,. \end{aligned} \tag{1}$$

Here, the indices $\hat{A}$, $\hat{B}$ take on the values $0, 1, \cdots, D-1$ [1] and $\eta_{\hat{A}\hat{B}} = \mathrm{diag}(-1, 1, \cdots, 1)$ is the Minkowski metric. In what follows, we will mostly focus on the Lorentz transformations, dilatations and special conformal transformations and collectively refer to these as "homogeneous conformal transformations".

The gauge fields in the adjoint representation of the algebra (1), that are associated to the generators $P_{\hat{A}}$, $M_{\hat{A}\hat{B}}$, $K_{\hat{A}}$ and D will be denoted by $E_\mu{}^{\hat{A}}$ (called the Vielbein), $\Omega_\mu{}^{\hat{A}\hat{B}}$ (called the spin-connection), $F_\mu{}^{\hat{A}}$ and B_μ respectively. Their infinitesimal gauge transformation rules under dilatations (with parameter Λ_D), Lorentz transformations (with parameter $\Lambda^{\hat{A}\hat{B}}$) and special conformal transformations (with parameter $\Lambda_K{}^{\hat{A}}$) are given by:

$$\begin{aligned} \delta_0 E_\mu{}^{\hat{A}} &= -\Lambda^{\hat{A}}{}_{\hat{B}}\, E_\mu{}^{\hat{B}} - \Lambda_D\, E_\mu{}^{\hat{A}}\,, \\ \delta_0 \Omega_\mu{}^{\hat{A}\hat{B}} &= \partial_\mu \Lambda^{\hat{A}\hat{B}} - 2\,\Lambda^{[\hat{A}|}{}_{\hat{C}}\, \Omega_\mu{}^{\hat{C}|\hat{B}]} - 4\,\Lambda_K{}^{[\hat{A}}\, E_\mu{}^{\hat{B}]}\,, \\ \delta_0 F_\mu{}^{\hat{A}} &= \partial_\mu \Lambda_K{}^{\hat{A}} - \Lambda_{K\hat{B}}\, \Omega_\mu{}^{\hat{B}\hat{A}} - \Lambda_K{}^{\hat{A}}\, B_\mu - \Lambda^{\hat{A}}{}_{\hat{B}}\, F_\mu{}^{\hat{B}} + \Lambda_D\, F_\mu{}^{\hat{A}}\,, \\ \delta_0 B_\mu &= \partial_\mu \Lambda_D + 2\,\Lambda_K{}^{\hat{A}}\, E_{\mu\hat{A}}\,. \end{aligned} \tag{2}$$

We use the subscript "0" on δ_0 to indicate that the final transformations that are used in the conformal approach are not the ones given in (2), but rather a modification thereof, as will be explained later. We will also often make use of the curvatures of translations $P^{\hat{A}}$ and Lorentz transformations $M^{\hat{A}\hat{B}}$ that are defined in the following way

$$\begin{aligned} R_{\mu\nu}(P^{\hat{A}}) &\equiv 2\,\partial_{[\mu} E_{\nu]}{}^{\hat{A}} + 2\,\Omega_{[\mu}{}^{\hat{A}\hat{B}}\, E_{\nu]\hat{B}} + 2\,B_{[\mu}\, E_{\nu]}{}^{\hat{A}}\,, \\ R_{\mu\nu}(M^{\hat{A}\hat{B}}) &\equiv 2\,\partial_{[\mu} \Omega_{\nu]}{}^{\hat{A}\hat{B}} + 2\,\Omega_{[\mu|}{}^{[\hat{A}|}{}_{\hat{C}}\, \Omega_{|\nu]}{}^{\hat{C}|\hat{B}]} + 8\,F_{[\mu}{}^{[\hat{A}}\, E_{\nu]}{}^{\hat{B}]}\,. \end{aligned} \tag{3}$$

These transform covariantly, i.e. without the derivative of a parameter, under (2).

The multiplet of independent gauge fields (2) is not minimal in the sense that it realizes the conformal algebra with more independent fields than needed for the construction of gravitational theories. One can however reduce the number of independent gauge fields by imposing the following two, so-called "conventional", constraints on the curvatures:

$$R_{\mu\nu}(P^{\hat{A}}) = T_{\mu\nu}{}^{\hat{A}}\,, \qquad \text{and} \qquad R_{\mu\hat{B}}(M^{\hat{A}\hat{B}}) = 0\,. \tag{4}$$

The tensor $T_{\mu\nu}{}^{\hat{A}}$ will be referred to as the torsion. [2] While the torsion is usually taken to be zero in discussions of the relativistic conformal approach, this will be too restrictive in the Carrollian

[1] We use a hatted index $\hat{A}$ here, since the unhatted index A will be used as a spatial index in the Carrollian case.

[2] It can be identified as the torsion of an affine connection $\Gamma_{\mu\nu}{}^\rho$ that is introduced via the Vielbein postulate

$$\partial_\mu E_\nu{}^{\hat{A}} + \Omega_\mu{}^{\hat{A}\hat{B}} E_{\nu\hat{B}} + B_\mu E_\nu{}^{\hat{A}} - \Gamma_{\mu\nu}{}^\rho E_\rho{}^{\hat{A}} = 0\,. \tag{5}$$

generalization. This is because in non-Lorentzian geometry part of the torsion is intrinsic and setting this intrinsic torsion to zero amounts to imposing constraints on the geometry. In order to anticipate the Carrollian case, we will for the moment keep the torsion $T_{\mu\nu}{}^{\hat{A}}$ arbitrary. When we discuss how EH gravity arises from the conformal approach in the next subsection, we will revert back to the case of zero torsion that is usually found in the literature.

The constraints (4) can be used to solve the spin-connection $\Omega_\mu{}^{\hat{A}\hat{B}}$ and the special conformal gauge field $F_\mu{}^{\hat{A}}$ in terms of the Vielbein $E_\mu{}^{\hat{A}}$, the dilatation gauge field B_μ and the torsion $T_{\mu\nu}{}^{\hat{A}}$ as follows:

$$\begin{aligned} \Omega_\mu{}^{\hat{A}\hat{B}}(E,B,T) &= \Omega_\mu{}^{\hat{A}\hat{B}}(E,T) + 2\, E_\mu{}^{[\hat{A}} E^{\hat{B}]\nu} B_\nu \,, \\ F_\mu{}^{\hat{A}}(E,B,T) &= -\frac{1}{2(D-2)} \left[R'_{\mu\hat{B}}(M^{\hat{A}\hat{B}}) - \frac{1}{2(D-1)}\, E_\mu{}^{\hat{A}}\, R'_{\hat{B}\hat{C}}(M^{\hat{B}\hat{C}}) \right] . \end{aligned} \tag{6}$$

Here, $\Omega_\mu{}^{\hat{A}\hat{B}}(E,T)$ is the usual torsionful spin-connection

$$\Omega_\mu{}^{\hat{A}\hat{B}}(E,T) = E^{[\hat{A}|\nu} \left(2\partial_{[\mu} E_{\nu]}{}^{|\hat{B}]} - T_{\mu\nu}{}^{|\hat{B}]} \right) - \frac{1}{2} E_{\mu\hat{C}}\, E^{\hat{A}\nu}\, E^{\hat{B}\rho} \left(2\partial_{[\nu} E_{\rho]}{}^{\hat{C}} - T_{\nu\rho}{}^{\hat{C}} \right) , \tag{7}$$

and $R'_{\mu\nu}(M^{\hat{A}\hat{B}})$ is the Lorentz transformation curvature with the $F_{[\mu}{}^{[\hat{A}} E_{\nu]}{}^{\hat{B}]}$ term deleted:

$$R'_{\mu\nu}(M^{\hat{A}\hat{B}}) = 2\,\partial_{[\mu}\Omega_{\nu]}{}^{\hat{A}\hat{B}}(E,B,T) + 2\,\Omega_{[\mu|}{}^{[\hat{A}|}{}_{\hat{C}}(E,B,T)\,\Omega_{|\nu]}{}^{\hat{C}|\hat{B}]}(E,B,T) \,. \tag{8}$$

From now on, it will be understood that the spin-connection $\Omega_\mu{}^{\hat{A}\hat{B}}$ and special conformal gauge field $F_\mu{}^{\hat{A}}$ are given by the dependent expressions $\Omega_\mu{}^{\hat{A}\hat{B}}(E,B,T)$ and $F_\mu{}^{\hat{A}}(E,B,T)$ of (6), whenever they appear in covariant derivatives or curvatures.

Note that the transformations of the dependent spin-connection and special conformal gauge field under homogeneous conformal transformations follow from varying their explicit expressions (6) under the transformation rules (2) of the independent fields $E_\mu{}^{\hat{A}}$ and B_μ.[3] The result of this typically leads to the transformation rules (2) of the dependent fields, modified with extra terms. These extra terms arise in particular when the conventional constraints (4) are not left invariant by (2) and their role is to ensure invariance of these constraints under the thus modified transformation rules. Let us denote the infinitesimal action of these modified homogeneous conformal transformations by δ. One then finds that the homogeneous conformal transformations of the independent and dependent gauge fields are, after imposing (4), given by:

$$\begin{aligned} \delta E_\mu{}^{\hat{A}} &= -\Lambda^{\hat{A}}{}_{\hat{B}}\, E_\mu{}^{\hat{B}} - \Lambda_D\, E_\mu{}^{\hat{A}} \,, \qquad\qquad \delta B_\mu = \partial_\mu \Lambda_D + 2\,\Lambda_K{}^{\hat{A}}\, E_{\mu\hat{A}} \,, \\ \delta\Omega_\mu{}^{\hat{A}\hat{B}}(E,B,T) &= \partial_\mu \Lambda^{\hat{A}\hat{B}} - 2\,\Lambda^{[\hat{A}|}{}_{\hat{C}}\, \Omega_\mu{}^{\hat{C}|\hat{B}]}(E,B,T) - 4\,\Lambda_K{}^{[\hat{A}}\, E_\mu{}^{\hat{B}]} \,, \\ \delta F_\mu{}^{\hat{A}}(E,B,T) &= \partial_\mu \Lambda_K{}^{\hat{A}} - \Lambda_{K\hat{B}}\, \Omega_\mu{}^{\hat{B}\hat{A}}(E,B,T) - \Lambda_K{}^{\hat{A}}\, B_\mu - \Lambda^{\hat{A}}{}_{\hat{B}}\, F_\mu{}^{\hat{B}}(E,B,T) \\ &\quad + \Lambda_D\, F_\mu{}^{\hat{A}}(E,B,T) + \frac{2}{D-2} \Lambda_K{}^{\hat{C}} \left[\delta^{[\hat{A}}_{\hat{C}} T_{\mu\hat{B}}{}^{\hat{B}]} - \frac{1}{2(D-1)} E_\mu{}^{\hat{A}} T_{\hat{C}\hat{B}}{}^{\hat{B}} \right] . \end{aligned} \tag{9}$$

The inclusion of the term involving B_μ in this Vielbein postulate ensures that the affine connection $\Gamma_{\mu\nu}{}^\rho$ is dilatation invariant; it is however not metric-compatible. The geometric structure introduced thus far is that of a Cartan-Weyl space-time, i.e., a space-time endowed with a metric $g_{\mu\nu}$ and a connection $\Gamma_{\mu\nu}{}^\rho$ that is not metric-compatible, but for which a trace-free part $Q_{\mu\nu\rho} - \frac{1}{D} Q_{\mu\sigma}{}^\sigma g_{\nu\rho}$ of the non-metricity tensor $Q_{\mu\nu\rho} \equiv -\nabla_\mu g_{\nu\rho}$ vanishes. The trace $-(1/D) Q_{\mu\nu}{}^\nu$ is called the Weyl connection and is in this case given by $-2B_\mu$.

[3] This requires that one also assigns a homogeneous conformal transformation rule $\delta T_{\mu\nu}{}^{\hat{A}}$ for the torsion tensor. In what follows, we will assume that $T_{\mu\nu}{}^\rho = T_{\mu\nu}{}^{\hat{A}} E_{\hat{A}}{}^\rho$ is invariant under homogeneous conformal transformations.

In case the torsion $T_{\mu\nu}{}^{\hat{A}}$ is zero, both conventional constraints (4) are invariant under (2). The homogeneous conformal transformation rules of the dependent spin-connection and special conformal gauge field are then not modified with respect to (2), as is seen explicitly by setting $T_{\mu\nu}{}^{\hat{A}}$ equal to zero in (9). Finally, note that the independent gauge field B_μ transforms with a shift under special conformal transformations and is thus a Stueckelberg field that can be set to zero by fixing the special conformal transformations. The only independent gauge field left is then the Vielbein $E_\mu{}^{\hat{A}}$, so that one indeed ends up with the minimal number of fields needed for a gravitational theory.

2.2 Einstein-Hilbert gravity from the compensating mechanism

We will now show how the conformal approach can be used to obtain the EH Lagrangian of general relativity

$$\mathcal{L}_{\text{EH}} = \frac{1}{2\kappa^2} E R\,, \tag{10}$$

with κ^2 the gravitational coupling constant, in a two-step process. In a first step, we will couple a massless compensating scalar to the independent and dependent gauge fields of the conformal algebra that were discussed in the previous subsection. The resulting action of this scalar is invariant under the homogeneous conformal transformations and, manifestly, under diffeomorphisms. EH gravity is recovered in a second step, by gauge-fixing the dilatations and special conformal transformations. In this section, we will set the torsion $T_{\mu\nu}{}^{\hat{A}}$ equal to zero and we will denote the explicit solutions (6) for the dependent spin-connection and special conformal gauge field in this zero torsion case by $\Omega_\mu{}^{\hat{A}\hat{B}}(E,B)$ and $F_\mu{}^{\hat{A}}(E,B)$.

As starting point, we consider the Lagrangian of a free, massless scalar field ϕ in flat spacetime:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\,\partial_{\hat{A}}\phi\partial^{\hat{A}}\phi\,, \tag{11}$$

Note that the sign in front of this Lagrangian is the opposite of the usual, physical one. This will guarantee that the EH action comes out with the correct sign. We then assume that the transformation rule of the scalar field ϕ under local homogeneous conformal transformations is given by a dilatation with scaling weight w:

$$\delta\phi = w\,\Lambda_D\,\phi\,, \tag{12}$$

and we replace the ordinary derivatives in (11) by covariant ones, by coupling to the gauge fields of the conformal algebra, discussed in the previous subsection. The first order derivative of ϕ that transforms covariantly (i.e., without a derivative of a parameter) under homogeneous conformal transformations is defined by:

$$D_{\hat{A}}\phi \equiv E_{\hat{A}}{}^{\mu}\left(\partial_\mu - w\,B_\mu\right)\phi\,. \tag{13}$$

Replacing the derivatives in (11) by covariant ones we end up with the following Ansatz for the Lagrangian of a scalar coupled to gauge fields of the conformal algebra:

$$\mathcal{L}_{\text{Ansatz}} = \frac{E}{2}\,D_{\hat{A}}\phi\,D^{\hat{A}}\phi\,, \tag{14}$$

where $E = \det(E_\mu{}^{\hat{A}})$. As it stands, this Lagrangian is however not invariant under special conformal transformations. In particular, one finds that its variation under special conformal transformations is, after partial integration, given by:

$$\delta_K \mathcal{L}_{\text{Ansatz}} = w\, E\, E_{\hat{A}}{}^{\mu} \left(\delta_K F_\mu{}^{\hat{A}}(E,B)\right) \phi^2 + w\,(2w+2-D)\, E\, \Lambda_K{}^{\hat{A}}\, B_{\hat{A}}\, \phi^2 \,, \tag{15}$$

where

$$\delta_K F_\mu{}^{\hat{A}}(E,B) = \partial_\mu \Lambda_K{}^{\hat{A}} - \Lambda_{K\hat{B}}\, \Omega_\mu{}^{\hat{B}\hat{A}}(E,B) - \Lambda_K{}^{\hat{A}}\, B_\mu \,, \tag{16}$$

is an infinitesimal special conformal transformation of $F_\mu{}^{\hat{A}}(E,B)$ (see eq. (9) with $T_{\mu\nu}{}^{\hat{A}} = 0$). One then sees that the Ansatz Lagrangian (14) can be made invariant under special conformal transformations by adding a term $-w\, E\, F_{\hat{A}}{}^{\hat{A}}(E,B)\, \phi^2$ and by choosing the dilatation weight w as

$$w = \frac{D-2}{2} \,. \tag{17}$$

This choice of w also ensures invariance under dilatations, so that the Lagrangian

$$\mathcal{L}_{\text{conf}} = \mathcal{L}_{\text{Ansatz}} - w\, E\, F_{\hat{A}}{}^{\hat{A}}(E,B)\, \phi^2 = \frac{E}{2}\left[D_{\hat{A}}\phi D^{\hat{A}}\phi - 2\, w\, F_{\hat{A}}{}^{\hat{A}}(E,B)\, \phi^2\right] . \tag{18}$$

is fully invariant under homogeneous conformal transformations.

The Lagrangian (18) is gauge equivalent to the EH Lagrangian. Indeed, gauge-fixing the dilatations and special conformal transformations by setting:

$$\phi = \frac{2}{\kappa}\sqrt{\frac{D-1}{D-2}}\,, \qquad\qquad B_\mu = 0\,, \tag{19}$$

with κ the gravitational coupling constant, and using the explicit expressions (for $T_{\mu\nu}{}^{\hat{A}} = 0$) (6), (7), (8), leads to the EH Lagrangian:

$$\mathcal{L}_{\text{conf}} \xrightarrow{\text{apply gauge-fixing (19)}} \mathcal{L}_{\text{EH}} = \frac{1}{2\kappa^2} E\, R\,. \tag{20}$$

This finishes our review of the conformal compensating technique in the relativistic case.

3. A conformal approach to Carroll gravity

Here, we will show how the conformal compensating technique reviewed in the previous section can be adapted to construct two different Carroll gravity theories. The existence of these two theories is due to the fact that there exist two massless Carroll scalar field theories that are called 'electric' and 'magnetic' in the literature [7–11]. In subsection 3.1, we will first discuss the construction of a multiplet of independent and dependent gauge fields of the conformal Carroll algebra, an Inönü-Wigner contraction of the relativistic conformal algebra. In subsection 3.2, we will use this multiplet to construct conformally coupled versions of electric and magnetic Carroll scalars and adopt gauge-fixing conditions, giving rise to two different Carroll gravity theories.

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3.1 Independent and dependent gauge fields of the conformal Carroll Algebra

The conformal Carroll algebra is a particular Inönü-Wigner contraction of the relativistic conformal algebra (1). It consists of the generators of time translations H, spatial translations P_A, spatial rotations J_{AB}, Carroll boosts J_{0A}, a singlet special conformal transformation K, a vector special conformal transformation K_A and a dilatation D. Here, the index $A = 1, \cdots, D-1$ is used to denote a flat spatial index. As in the relativistic case, we will collectively refer to the transformations associated to J_{AB}, J_{0A}, K/K_A and D as homogeneous conformal Carroll transformations. The commutation relations of the conformal Carroll algebra are obtained by suitably re-instating the speed of light c in the generators of the relativistic conformal algebra (1) and taking the limit $c \to 0$. We refrain from giving these commutation relations explicitly here and instead refer to [12–14].

In analogy to the relativistic case, we then introduce gauge fields τ_μ, $e_\mu{}^A$, $\omega_\mu{}^{AB}$, $\omega_\mu{}^{0A}$, f_μ, $g_\mu{}^A$ and b_μ (associated to H, P_A, J_{AB}, J_{0A}, K, K_A and D, respectively) in the adjoint representation of the conformal Carroll algebra. Their transformation rules under J_{AB}, J_{0A}, K/K_A and D, with respective gauge parameters λ^{AB}, λ^{0A}, $\lambda_K/\lambda_K{}^A$ and λ_D, are given by:

$$\begin{aligned}
\delta_0\tau_\mu &= -\lambda^{0A}e_{\mu A} - \lambda_D\tau_\mu\,, \qquad \delta_0 e_\mu{}^A = -\lambda^A{}_B e_\mu{}^B - \lambda_D e_\mu{}^A\,,\\
\delta_0\omega_\mu{}^{AB} &= \partial_\mu\lambda^{AB} - 2\lambda^{[A|}{}_C\omega_\mu{}^{C|B]} - 4\lambda_K{}^{[A}e_\mu{}^{B]}\,,\\
\delta_0\omega_\mu{}^{0A} &= \partial_\mu\lambda^{0A} + \omega_\mu{}^A{}_B\lambda^{0B} - \lambda^A{}_B\omega_\mu{}^{0B} + 2\lambda_K{}^A\tau_\mu - 2\lambda_K e_\mu{}^A\,,\\
\delta_0 f_\mu &= \partial_\mu\lambda_K - \lambda_K b_\mu - \lambda^{0A}g_{\mu A} + \lambda_{KA}\omega_\mu{}^{0A} + \lambda_D f_\mu\,,\\
\delta_0 g_\mu{}^A &= \partial_\mu\lambda_K{}^A + \omega_\mu{}^A{}_B\lambda_K{}^B - \lambda_K{}^A b_\mu - \lambda^A{}_B g_\mu{}^B + \lambda_D g_\mu{}^A\,,\\
\delta_0 b_\mu &= \partial_\mu\lambda_D + 2\lambda_K{}^A e_{\mu A}\,.
\end{aligned} \tag{21}$$

As in the relativistic case, the subscript "0" on δ_0 indicates that these are not the final conformal Carroll transformation rules that we will use to construct Carroll gravity theories.

In what follows, we will need the following curvatures:

$$R_{\mu\nu}(H) = 2\,\partial_{[\mu}\tau_{\nu]} + 2\,\omega_{[\mu}{}^{0A}\,e_{\nu]A} + 2\,b_{[\mu}\,\tau_{\nu]}\,, \tag{22a}$$
$$R_{\mu\nu}(P^A) = 2\,\partial_{[\mu}e_{\nu]}{}^A + 2\,\omega_{[\mu}{}^{AB}\,e_{\nu]B} + 2\,b_{[\mu}\,e_{\nu]}{}^A\,, \tag{22b}$$
$$R_{\mu\nu}(J^{AB}) = 2\,\partial_{[\mu}\omega_{\nu]}{}^{AB} + 2\,\omega_{[\mu|}{}^{[A|}{}_C\,\omega_{|\nu]}{}^{C|B]} + 8\,g_{[\mu}{}^{[A}\,e_{\nu]}{}^{B]}\,, \tag{22c}$$
$$R_{\mu\nu}(J^{0A}) = 2\,\partial_{[\mu}\omega_{\nu]}{}^{0A} + 2\,\omega_{[\mu|}{}^A{}_B\,\omega_{|\nu]}{}^{0B} + 4\,f_{[\mu}\,e_{\nu]}{}^A - 4g_{[\mu}{}^A\,\tau_{\nu]}\,, \tag{22d}$$

that transform covariantly under the homogeneous conformal Carroll transformations (21). We will also often need the dual Vielbeine $(\tau^\mu, e_A{}^\mu)$ that are defined by the following duality relations:

$$\tau^\mu\tau_\mu = 1\,, \quad \tau^\mu e_\mu{}^A = 0\,, \quad \tau_\mu e_A{}^\mu = 0\,, \quad e_\mu{}^A e_B{}^\mu = \delta^A_B\,, \quad e_\mu{}^A e_A{}^\nu = \delta^\mu_\nu - \tau^\mu\tau_\nu\,. \tag{23}$$

These dual Vielbeine will be used to convert a curved index μ into flat indices 0 and A, respectively. For instance, given a one-form X_μ and a two-form $X_{\mu\nu}$ we define the different projections to flat time and spatial components as follows:

$$X_0 \equiv \tau^\mu X_\mu\,, \qquad X_A \equiv e_A{}^\mu X_\mu\,, \qquad X_{0A} \equiv \tau^\mu e_A{}^\nu X_{\mu\nu}\,, \qquad X_{AB} \equiv e_A{}^\mu e_B{}^\nu X_{\mu\nu}\,. \tag{24}$$

Similar to what happened in the relativistic case, the multiplet of conformal Carroll algebra gauge fields (21) contains more independent fields than are necessary in a gravitational theory.

In order to reduce the number of independent fields, we thus need to impose suitable curvature constraints that allow one to solve for some fields in terms of the remaining ones. By inspecting the list of curvatures given in (22), one sees that a minimal set of independent fields can be obtained in analogy to the relativistic case, by constraining the curvatures $R_{\mu\nu}(H)$ and $R_{\mu\nu}(P^A)$ of time and spatial translations, as well as the components $R_{\mu B}(J^{AB})$ and $R_{\mu A}(J^{0A})$ of the curvatures of spatial rotations and Carroll boosts. Here, we choose to constrain these curvatures as follows:

$$R_{\mu\nu}(H) = 0\,, \qquad R_{\mu\nu}(P^A) = 2\tau_{[\mu}e_{\nu]B}\, T_0{}^{\{B,A\}}\,, \tag{25a}$$

$$R_{\mu B}(J^{AB}) = 0\,, \qquad R_{\mu A}(J^{0A}) = 0\,, \tag{25b}$$

where $\{AB\}$ denotes the symmetric traceless part of AB and $T_0{}^{\{A,B\}}$ is an arbitrary symmetric traceless SO$(D-1)$–tensor. Note that we could have opted to equate $R_{\mu\nu}(H)$ and $R_{\mu\nu}(P^A)$ to arbitrary torsion tensors $T_{\mu\nu}$ and $T_{\mu\nu}{}^A$ respectively. In (25) we have not done this; instead we have put as many Carrollian torsion components equal to zero as possible, to closely mimic the reasoning that led to general relativity. Note however that we should not require that all torsion components vanish. The reason is that some components of $R_{\mu\nu}(P^A)$ do not contain (spin-connection or dilatation) gauge field components that appear algebraically and can be solved for. In our case, these components correspond to the following $\frac{1}{2}D(D-1)-1$ components of the spatial translation curvature:

$$R_0{}^{\{A}(P^{B\}})\,. \tag{26}$$

The corresponding torsion components [4]

$$T_0{}^{\{A,B\}} \equiv \tau^\mu e^{\{A|\nu} T_{\mu\nu}{}^{|B\}}\,, \tag{27}$$

define the intrinsic torsion. [5] They should not be set to zero, since doing so implies imposing the following geometric constraints

$$R_0{}^{\{A}(P^{B\}}) = 2\tau^\mu e^{\{A|\nu}\partial_{[\mu}e_{\nu]}{}^{|B\}} = 0\,, \tag{28}$$

and is therefore too restrictive.

The constraints (25) can be solved to express the following gauge field components

$$\omega_\mu{}^{AB}\,, \qquad \omega_0{}^{0A}\,, \qquad \omega^{[A|,0|B]}\,, \qquad b_0\,, \qquad g_\mu{}^A\,, \qquad f_\mu\,, \tag{29}$$

as fields that depend on the remaining independent field components

$$\tau_\mu\,, \qquad e_\mu{}^A\,, \qquad b_A\,, \qquad \omega^{(A|,0|B)}\,. \tag{30}$$

We will refrain from giving the explicit expressions for the dependent fields (29) here. From now on, we will however always assume that $\omega_\mu{}^{AB}$, $\omega_0{}^{0A}$, $\omega^{[A|,0|B]}$, b_0, $g_\mu{}^A$ and f_μ stand for their

[4]The comma in $T_0{}^{\{A,B\}}$ indicates that the two indices to the left of the comma are flat projections of the two curved indices of $T_{\mu\nu}{}^A$.

[5]Note that we use the term 'intrinsic torsion' in a more general sense than is typically done in the mathematical literature, where this term is reserved for the torsion tensor components that do not contain spin-connection components. This is appropriate for space-times with local Carroll symmetries. Since here however, we are dealing with the conformal Carroll algebra, we will for simplicity define the word intrinsic torsion tensor as a torsion tensor that does not contain spin-connection as well as dilatation gauge fields.

explicit expressions. Two differences with the relativistic case are worth pointing out. First, in the Carroll case it is not possible to solve for all spin-connection components; in particular the boost spin-connection components $\omega^{(A|,0|B)}$ remain independent. These independent boost connection components are related to the presence of intrinsic torsion, see e.g., [15]. Secondly, only the spatial components b_A of the dilatation gauge field are independent, while b_0 is dependent. It turns out that the dependent spin-connection fields depend only on b_A and not on the dependent b_0.

As in the relativistic case, varying the explicit expressions of the dependent fields (29) under the homogeneous Carroll transformations (21) of the independent fields (30), one finds that the resulting transformations are not necessarily given by the rules (21), but can contain extra terms. We denote these extra terms by Δ. They appear for instance in the transformations of the dependent spin-connection components $\omega_C{}^{AB}$ and $\omega_0{}^{0A}$. Indeed, the homogeneous conformal transformation rules $\delta\omega_C{}^{AB}$ and $\delta\omega_0{}^{0A}$ of their explicit expressions are found to be given by the rules $\delta_0\omega_C{}^{AB}$ and $\delta_0\omega_0{}^{0A}$ of (21), supplemented with extra terms $\Delta\omega_C{}^{AB}$ and $\Delta\omega_0{}^{0A}$, given by:

$$\Delta\omega_{C,AB} = \lambda^0{}_B T_{0\{A,C\}} - \lambda^0{}_A T_{0\{B,C\}}\,, \qquad \Delta\omega_0{}^{0A} = \lambda^0{}_B T_0{}^{\{A,B\}}\,. \tag{31}$$

The homogeneous conformal transformation rules of the dependent gauge fields $g_\mu{}^A$ and f_μ likewise acquire extra terms. We will refrain from giving the modifications in the transformation rules of all components of $g_\mu{}^A$ and f_μ, since they will not be needed. It will however be important that the combination $g_A{}^A + f_0$ does not receive any extra Δ-contributions in its homogeneous conformal transformation rules:

$$\Delta g_A{}^A + \Delta f_0 = 0\,. \tag{32}$$

3.2 Carroll Gravity from a Compensating Mechanism

Here, we will show how a particular electric and a magnetic Carroll gravity theory can be constructed using the conformal approach. To do this, we will first couple a massless electric and magnetic Carroll scalar field to the independent and dependent gauge fields of the conformal Carroll algebra that were constructed in the previous subsection. A special electric and a magnetic Carroll gravity theory are then obtained by gauge-fixing all superfluous conformal symmetries.

The electric case. Let us first discuss the electric case. As starting point we consider the following Lagrangian for a massless electric Carroll scalar field:

$$\mathcal{L}_{\text{electric scalar}} = -\frac{1}{2}\phi\partial_t\partial_t\phi\,. \tag{33}$$

Similar to what we did in the relativistic case, we will first make the corresponding action invariant under local homogeneous conformal Carroll transformations, by coupling the scalar ϕ to conformal Carroll gauge fields. To this effect, we assume that the scalar field ϕ only transforms under local homogeneous conformal Carroll transformations via a dilatation with weight w:

$$\delta\phi = w\,\lambda_D\,\phi\,. \tag{34}$$

We then replace the time derivative $\partial_t\phi$ of ϕ by a covariant derivative $D_0\phi$:

$$\partial_t\phi \to D_0\phi \equiv \tau^\mu\left(\partial_\mu - w\,b_\mu\right)\phi\,. \tag{35}$$

Since $D_0\phi$ transforms as

$$\delta\left(D_0\phi\right) = (w+1)\,\lambda_D\, D_0\phi\,, \tag{36}$$

under homogeneous conformal Carroll transformations, the second covariant time derivative $D_0D_0\phi$ of ϕ is given by

$$D_0D_0\phi \equiv \tau^\mu\left[\partial_\mu\left(D_0\phi\right) - (w+1)\,b_\mu\,D_0\phi\right]\,. \tag{37}$$

With the help of this covariant generalization of the second time derivative, we can propose the following Lagrangian that describes the coupling of ϕ to conformal Carroll gauge fields:

$$\mathcal{L}_{\text{electric coupling}} = -\frac{1}{2}e\phi D_0D_0\phi\,. \tag{38}$$

Here, $e = \det(\tau_\mu, e_\mu{}^A)$ which transforms under dilatations as

$$\delta e = -D\lambda_D e\,. \tag{39}$$

The proposed Lagrangian (38) is invariant under dilatations provided we take the scaling weight w to be

$$w = \frac{D-2}{2}\,. \tag{40}$$

Having constructed a Lagrangian that is invariant under homogeneous conformal Carroll transformations, we can gauge-fix the dilatations by imposing the gauge condition $\phi = 1$. This leads to the following Lagrangian:

$$\mathcal{L}_{\text{electric Carroll}} = \frac{w^2}{2}e\,b_0^2\,. \tag{41}$$

The explicit expression for the dependent dilatation gauge field component b_0 reads [6]

$$b_0 = -\frac{2}{D-1}\tau^\mu e_A{}^\nu\partial_{[\mu}e_{\nu]}{}^A \equiv -\frac{1}{D-1}\mathcal{T}_{0A}{}^A\,. \tag{42}$$

Using this in (41), we find a Lagrangian describing a particular form of electric Carroll gravity:

$$\mathcal{L}_{\text{electric Carroll}} = \frac{w^2}{2(D-1)^2}\,e\,\mathcal{T}_{0A}{}^A\mathcal{T}_{0B}{}^B\,. \tag{43}$$

Note that this is not the most general electric Carroll gravity theory, occurring in the literature [5]. In our notation, the Lagrangian of this general electric theory is given by

$$\mathcal{L} \sim e\left[T_0{}^{\{A,B\}}T_{0\{A,B\}} - \left(\frac{D-2}{D-1}\right)\mathcal{T}_{0A}{}^A\mathcal{T}_{0B}{}^B\right]\,. \tag{44}$$

Comparing with (43), we see that the conformal approach only reproduces the second term in this Lagrangian. The first term

$$\mathcal{L}_{\text{conformal Carroll}} \sim e\,T_0{}^{\{A,B\}}T_{0\{A,B\}}\,, \tag{45}$$

[6] We use here a calligraphic notation to indicate that $\mathcal{T}_{0A}{}^A$ is an intrinsic torsion tensor with respect to the Carroll algebra but not with respect to the conformal Carroll algebra.

transforms homogeneously under local dilatations without the need for a dilatation gauge field. As a consequence, this "conformal electric Carroll gravity" Lagrangian cannot be obtained by starting from a dynamical matter Lagrangian in the conformal approach.

The magnetic case. In this case, we wish to make the following Lagrangian [8, 16] of a massless magnetic Carroll scalar

$$\mathcal{L}_{\text{magnetic scalar}} = \pi\partial_t\phi - \frac{1}{2}\partial^A\phi\partial_A\phi\,, \tag{46}$$

invariant under homogeneous conformal Carroll transformations. Here, π is an independent Lagrange multiplier field. The Lagrangian (46) is, for any $\alpha \in \mathbb{R}$, invariant under the constant transformation

$$\delta\pi = \alpha\lambda_K\phi\,, \tag{47}$$

as well as under the constant Carroll boost transformations

$$\delta\pi = \lambda^{0A}\partial_A\phi\,, \qquad\qquad \delta\partial_A\phi = \lambda^0{}_A\partial_t\phi\,. \tag{48}$$

As before, we start from the assumption that the scalar field ϕ transforms under local homogeneous conformal Carroll transformations with a dilatation with scaling weight w. Like in the electric case, we replace the time derivative $\partial_t\phi$ of ϕ by the covariant derivative $D_0\phi$ defined in eq. (35). Similarly, we generalize the spatial derivative $\partial_A\phi$ of ϕ to the following covariant derivative:

$$D_A\phi \equiv e_A{}^\mu\left(\partial_\mu - w\,b_\mu\right)\phi\,. \tag{49}$$

Under the homogeneous conformal Carroll transformations this covariant derivative is found to transform as follows:

$$\delta\left(D_A\phi\right) = -\lambda_A{}^B\,D_B\phi + \lambda^0{}_A\,D_0\phi + (w+1)\,\lambda_D\,D_A\phi - 2\,w\,\lambda_{K\,A}\,\phi\,. \tag{50}$$

We now propose the following Ansatz Lagrangian to describe the coupling of a massless magnetic Carroll scalar to conformal Carroll gauge fields:

$$\mathcal{L}_{\text{Ansatz}} = e\pi D_0\phi - \frac{1}{2}eD^A\phi D_A\phi\,. \tag{51}$$

This Ansatz Lagrangian is manifestly invariant under spatial rotations. By taking

$$\delta\pi = \lambda^{0A}D_A\phi + \tfrac{1}{2}D\lambda_D\pi \qquad \text{and} \qquad w = \tfrac{1}{2}(D-2)\,, \tag{52}$$

it also becomes invariant under dilatations and Carroll boosts.

The Ansatz Lagrangian (51) is however not invariant under the vector special conformal transformations. After partial integration, one finds that its variation under K_A transformations is given by

$$\delta\mathcal{L}_{\text{Ansatz}} = -we\,\omega_{0,}{}^{0A}\lambda_{K\,A}\phi^2 - we\,e_A{}^\mu\left(D_\mu\lambda_K{}^A\right)\phi^2\,, \tag{53}$$

$$\text{with} \qquad e_A{}^\mu\left(D_\mu\lambda_K{}^A\right) \equiv e_A{}^\mu\left(\partial_\mu\lambda_K{}^A + \omega_\mu{}^A{}_B\lambda_K{}^B - \lambda_K{}^A b_\mu\right)\,.$$

Both terms can be canceled by adding a term involving the combination ${g_A}^A + f_0$ of the gauge fields of the scalar and vector special conformal transformation. Crucially, due to (32), the addition of this term does not break the boost symmetry. This gives rise to the following modified Lagrangian for the coupling of a massless magnetic Carroll scalar to conformal Carroll gauge fields:

$$\mathcal{L}_{\text{magnetic coupling}} = e\pi D_0\phi - \frac{e}{2}D^A\phi D_A\phi + we{g_A}^A\phi^2 + we f_0\phi^2 \,. \tag{54}$$

This Lagrangian is invariant under spatial rotations, boosts, dilatations and the vector special conformal transformations. However, we still need to verify that it is also invariant under the scalar special conformal transformations K. Allowing a K-variation $\delta_K\pi$ of the Lagrange multiplier π, we find

$$\delta_K\mathcal{L}_{\text{magnetic coupling}} = e\delta_K\pi D_0\phi + we\left(D_0\lambda_K\right)\phi^2 \,, \tag{55}$$

$$\text{with} \qquad D_0\lambda_K \equiv \tau^\mu\left(\partial_\mu\lambda_K - \lambda_K b_\mu\right) \,.$$

After partially integrating the second term, one then finds that (54) can be rendered invariant under K-transformations, provided one takes

$$\delta_K\pi = (D-2)\,\lambda_K\phi \,. \tag{56}$$

This local K-transformation reduces to the global K-transformation (47) for $\alpha = D-2$.

We now fix the dilatations by imposing $\phi = 1$ and the vector special conformal transformations by setting $b_A = 0$. After using the explicit expressions for the dependent gauge field components ${g_A}^A$ and f_0, one then finds that (54) reduces to the following Lagrangian for magnetic Carroll gravity:

$$\mathcal{L}_{\text{magnetic Carroll}} = \frac{1}{2}\frac{D-2}{D-1}e\left\{\pi\,{\mathcal{T}_{0A,}}^A - \frac{1}{4}\left[R'_{AB}(J^{AB})(e,\tau) + 2R'_{0A}(J^{0A})(e,\tau)\right]\right\} \,, \tag{57}$$

where

$$R'_{\mu\nu}(J^{AB}) = 2\,\partial_{[\mu}{\omega_{\nu]}}^{AB} + 2\,{\omega_{[\mu|}}^{[A|}{}_C\,{\omega_{|\nu]}}^{C|B]} \,, \tag{58}$$

$$R'_{\mu\nu}(J^{0A}) = 2\,\partial_{[\mu}{\omega_{\nu]}}^{0A} + 2\,{\omega_{[\mu|}}^A{}_B\,{\omega_{|\nu]}}^{0B}\,\tau_{\nu]} \,, \tag{59}$$

are curvature tensors with respect to the (non-conformal) Carroll algebra.

At first sight, the Lagrangian (57) differs from the result for magnetic Carroll gravity given in [6] due to the presence of the first term. However, as pointed out in [15], the curvature terms contain the independent spin-connection components $\omega^{(A,0B)}$ that act as Lagrange multipliers that impose the geometric constraints:

$$\mathcal{T}_0{}^{(A,B)} \equiv 2\tau^\mu e^{(A|\nu|}\partial_{[\mu}{e_{\nu]}}^{B)} = 0 \,. \tag{60}$$

In particular, this implies that the first term of (57) can be absorbed in a redefinition of the Lagrange multiplier field ${\omega_{A,}}^{0A}$. This is supported by the fact that the Lagrangian (57) is invariant under the K-transformation:

$$\delta\pi = (D-2)\,\lambda_K \,, \qquad\qquad \delta{\omega_{A,}}^{0A} = -2\lambda_K \,, \tag{61}$$

that can be gauge-fixed by setting $\pi = 0$. Doing this, one ends up with the following Lagrangian for magnetic Carroll gravity [6]:

$$\mathcal{L}_{\text{magnetic Carroll}} = -\frac{1}{8}\frac{D-2}{D-1}e\,\left[R'_{AB}(J^{AB})(e,\tau) + 2R'_{0A}(J^{0A})(e,\tau)\right]\,. \tag{62}$$

Alternatively, one could opt to gauge-fix the K-transformations by imposing $\omega_A{}^{0A} = 0$. The Lagrange multiplier field π would then behave as a special boost spin-connection component, as a result of compensating transformations.

4. Outlook

The results of this work can be used as a starting point to obtain non-trivial couplings of Carroll gravity to matter. This can be achieved by replacing the single scalar we have used here by a function of N scalars, of which only one is used to gauge-fix the dilatations. One may also further generalize this work by considering conformal Carroll algebras with an-isotropic dilatations that were studied in [13]. Such algebras exhibit different scaling weights for the longitudinal and transverse Vierbeine. This suggests a realization of conformal Carroll symmetries in terms of scalar field theories with different numbers of time and spatial derivatives. Yet another possible generalization concerns foliated conformal Carroll geometries corresponding to an extended object with p spatial directions. This involves a more general decomposition of the relativistic flat index into $p+1$ longitudinal and $D-p-1$ transverse directions. Finally, the conformal technique could also be applied to other non-Lorentzian algebras than the Carroll algebra.

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