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Thermal Effects in Ising Cosmology

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We consider the rapidly expanding phase of the universe and model it by de Sitter (dS) and a real scalar field. Using arguments form holography we propose that the deviation of the spectral index n_S of scalar fluctuations from unity may by controlled almost entirely by the critical exponent η of the $d = 3$ Ising model. We compute the thermal propagators of the scalar field in dS background and propose that non-trivial thermal effects as seen by an 'out' observer can be encoded by η which fixes completely a number of cosmological observables.

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1. Part 1: The spectral Index

1.1 The Calculation of n_S from dS/CFT

The universe is believed to have gone through a violent phase of expansion i.e. inflation, mainly due to the fact that it provides an adequate solution to the horizon problem. During the same epoch, the primordial fluctuations are believed to have directly impacted the cosmic microwave background (CMB) radiation which is strangely measured as isotropic. This means that the temperatures in two present spacelike points are nearly identical, which suggests that at some point in their past, these points where timelike. In order to describe these effects, we define the scalar power spectrum at super-Hubble scales:

$$\mathcal{P}_S = \frac{|\boldsymbol{k}|^3}{2\pi^2} \left|\phi^2_{|\boldsymbol{K}|}\right| \tag{1}$$

where $\phi_{|\boldsymbol{K}|}$ the mode of the quantum scalar field $\phi(x)$. Then, the scale invariance of the CMB is characterized by the value of the scalar spectral index

$$n_S = 1 + \frac{\partial \ln \mathcal{P}_S}{\partial \ln |\boldsymbol{k}|} \tag{2}$$

so that when $n_S = 1$, the spectrum is completely scale invariant.

Arguably, the most important result that we obtained from the Planck experiment [1] is the further restriction of the deviation of the scalar spectral index away from unity:

$$\delta n_S = 1 - n_S^{\text{exp}} \simeq 0.036 \tag{3}$$

while various theoretical endeavors have been made in order to describe the above deviation. Starting from the $d = 4$ Poincare patch of dS spacetime which spans from the cosmological time $\tau \in (-\infty, 0]$:

$$ds^2 = dt^2 - a^2(t)dx^2 = a^2(\tau)\left(d\tau^2 - dx^2\right), \qquad a(\tau) = \frac{1}{H^2\tau^2}, \qquad H = \frac{1}{a^2(\tau)}\frac{da(\tau)}{d\tau} \tag{4}$$

our goal is to anticipate the above value of δn_S by first considering a scalar field propagating on the bulk dS space and its dual, a Large N non-unitary CFT living on the boundary. In particular, we consider an observer that starts their journey at the start of inflation ($\tau_{\text{in}} \to -\infty$) and flows towards a time close to the horizon ($\tau_{\text{out}} \to 0$). On the boundary, we will argue that this can be seen as a RG flow from UV to the IR Fixed point (FP) where the $\tau_{\text{in}}(\tau_{\text{out}})$ is known to correspond to a UV FP on the dS boundary [3].

We propose that the said dual theory is the 3d Ising model which is described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_i \sigma\right)^2 - \lambda\sigma^4 \tag{5}$$

where σ is the corresponding boundary field and $i = 1, 2, 3$ the spatial indices. Our main motivation arises from the fact that one of the critical exponents of the model is $\eta = 2\gamma_\sigma$ with γ_σ being the anomalous dimension of the field, where various numerical computations [4] showed that $\eta \simeq 0.036$. Since, this value is similar to the measured deviation δn_S, we want to check whether a deeper connection is hidden and the equation

$$n_S = 1 - \eta \tag{6}$$

holds or if this is just a mere coincidence. After all, the anomalous dimensions arise when one lets a theory flow away from one of its fixed points, resulting to the breaking of its scale invariance. This naturally fits with our description that during inflation there is a corresponding RG flow of the boundary theory.

The dS/CFT correspondence provides the required link between the boundary anomalous dimensions and the bulk fluctuations. Specifically, a boundary operator $O_{|\boldsymbol{k}|}$ of dimension $\Delta_O = 3$, will be dual to the scalar curvature perturbations $\zeta_{|\boldsymbol{k}|} = z(\tau)\phi_{|\boldsymbol{k}|}$ of dimension $\Delta_\zeta = 0$ as longs as:

$$\langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle \sim \frac{1}{\langle \phi_{|\boldsymbol{k}|} \phi_{-|\boldsymbol{k}|} \rangle} \sim \frac{1}{\langle \zeta_{|\boldsymbol{k}|} \zeta_{-|\boldsymbol{k}|} \rangle} \tag{7}$$

where $z(\tau)$ is a factor determined by the (conformal time τ) time-dependent classical background while $\langle \phi_{|\boldsymbol{k}|} \phi_{-|\boldsymbol{k}|} \rangle \sim \langle \zeta_{|\boldsymbol{k}|} \zeta_{-|\boldsymbol{k}|} \rangle$ is a gauge-invariant result [5]. Then, by substituting the above equation into the definition of the spectral index, one receives:

$$n_S - 1 = 3 - \frac{1}{\langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle} \left(\frac{\partial}{\partial \ln |\boldsymbol{k}|} \langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle \right) \tag{8}$$

while the Callan-Symanzik equation for a general 2-point function in momentum space is

$$\left(\frac{\partial}{\partial \ln |\boldsymbol{k}|} - \beta_\lambda \frac{\partial}{\partial \lambda} + (3 - 2\Delta_O) \right) \langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle = 0 \tag{9}$$

where $\beta_\lambda = \mu \frac{\partial \lambda}{\partial \mu}$ the beta-function and the operator dimension $\Delta_O = [\Delta_O] + \Gamma_O$ can be written as a sum of the classical dimension plus the corresponding anomalous dimension:

$$\Gamma_O = -\mu \frac{\partial}{\partial \mu} \left(Z_\sigma^{-1} z_O \right) . \tag{10}$$

Here, Z_σ^{-1} and z_O are the wave renormalizations of the boundary field and operator correspondingly. Subsequently, we can combine eqs. (8) and (9) in order to obtain for $[\Delta_O] = 3$:

$$n_S = 1 - 2\Gamma_O - \beta_\lambda \frac{\partial}{\partial \lambda} \ln \langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle . \tag{11}$$

A similar result was also found in [6], [7], [8], nevertheless the slow-roll approximation was used along with parameter fixing in contrast to our work where we reach $\delta n_S = 0.036$ automatically.

In order to proceed, we need to recall that since $O_{|\boldsymbol{k}|}$ is coupled to $\zeta_{|\boldsymbol{k}|}$, its dimension is fixed: $\Delta_O = 3$ which results into $\Gamma_O = 0$. This encourages us to suppose that the trace of the stress energy tensor $\Theta = \delta^{ij} T_{ij}$ is the operator O in question. Then,

$$n_S = 1 - \beta_\lambda \frac{\partial}{\partial \lambda} \ln \langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle, \tag{12}$$

and since O should contain two fields (in order for it to be identified as Θ), we can express its anomalous dimension as

$$\Gamma_O = -\gamma_O + 2\gamma_\sigma \tag{13}$$

where $\gamma_O = \mu \frac{\partial}{\partial \mu} z_O$ the total anomalous dimension of the operator. Consequently, $\gamma_O = 2\gamma_\sigma \equiv \eta$ meaning that our hypothesis that η truly fixes the deviation of n_S, is true as long as the eigenvalue equation

$$\beta_\lambda \frac{\partial}{\partial \lambda} \langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle = 2\gamma_\sigma \langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle \tag{14}$$

is satisfied for $O = \Theta$. Furthermore, in the vicinity of the IR FP ($\lambda \simeq \lambda^*$) we can approximately write

$$\langle O_{|\boldsymbol{k}|} O_{-|\boldsymbol{k}|} \rangle = \frac{c_O}{|x|^{2d}} \tag{15}$$

so that the coupling c_O satisfies the eigenvalue equation

$$\beta_\lambda \partial_\lambda c_O = 2\gamma_\sigma c_O. \tag{16}$$

The leading order solution is:

$$c_O \simeq \left(\frac{16\pi^2 - 3\lambda}{\lambda} \right)^\eta \tag{17}$$

which vanishes at the FP, as it should. Following [6], the boundary RG flow can be understood from the bulk's perspective as the late time equation of motion

$$\frac{dH}{d\tau} = -\frac{1}{2a} \left(\frac{d\phi}{dt} \right)^2 \tag{18}$$

in $M_{\rm pl} = 1$ units where H the corresponding bulk Hubble constant. Then one can relate the time-dependence of H, away from the conformal limits, and the RG flow via the identification $\mu = aH$ and $\lambda = \phi$, with $a(\tau)$ the scale factor of the late time geometry. The latter identification is not sufficient however to reproduce our RG flow because it is not able to see wave function renormalization, therefore it must be generalized. A possible identification for that purpose is:

$$\lambda \simeq \phi + \ln(H\tau|)^{2\gamma_\sigma/\beta_\lambda} \tag{19}$$

which would indeed gives the eigenvalue eq. (14) near the FP. A similar identification was wound in [9] and while the above consideration does not serve as proof, it does provide a plausible argument.

1.2 Tensor Fluctuations

Although the use of the dS/CFT correspondence in order to interpret the bulk effects via the use of boundary arguments seems natural, it does come with unfortunate implications. In particular, the dual of dS is known to be a non-unitary CFT with negative central charge while the 2-point function of the gravitational waves $\gamma_{\mu\nu}$ is inversely proportinal to the central charge of the CFT. Hence, if P_T is the tensor power spectrum, [5] has shown that

$$P_T \sim \langle \gamma\gamma \rangle \sim \frac{1}{c_T^*} \sim -\frac{1}{R_{\rm dS}^2} \tag{20}$$

where c_T^* the central charge and $R_{\rm dS}$ the radius of dS. On the other hand, the interesting physics happens just outside the FP, where the scalar to tensor ration

$$r = \frac{P_S}{P_T} > 0 \tag{21}$$

should be positive and small. In order to deal with this seemingly inconsistency, we will show that the effective coupling that determines the tensor spectrum is positive away from the FP but at some point before it reaches the FP, it becomes negative.

Let us call such a coupling the "C-function" $C(e)$ that depends on $e = |x_1 - x_2|$. Since the 2 point function of the boundary stress energy tensor $\boldsymbol{T}_{\mu\nu}$ couples to $\langle\gamma\gamma\rangle$ inversely, we define the C-function in d dimensions as

$$\langle \boldsymbol{T}_{\mu\nu}\boldsymbol{T}_{\rho\sigma}\rangle = \frac{A_{\mu\nu\rho\sigma}}{e^{2d}} \sim \frac{C(e)}{e^{2d}}F_{\mu\nu\rho\sigma} \tag{22}$$

with $F_{\mu\nu\rho\sigma}$ a tensor structure independent of e. We then proceed to decompose the stress energy tensor

$$\boldsymbol{T}_{\mu\nu} = T_{\mu\nu} + \Theta\frac{\delta_{\mu\nu}}{d} \tag{23}$$

into a traceless $T_{\mu\nu}$ and trace Θ part, so that the left-hand side of (22) can be written as a sum of the "trace" $\langle\Theta\Theta\rangle$, "mixed" $\langle\Theta T_{\mu\nu}\rangle$ and "traceless" $\langle T_{\mu\nu}T_{\rho\sigma}\rangle$ two point functions. Furthermore, the most general form of the tensor structure on the right hand side is given in [10]:

$$\begin{aligned}\langle \boldsymbol{T}_{\mu\nu}\boldsymbol{T}_{\rho\sigma}\rangle &= \frac{c_1}{e^{2d+4}}e_\mu e_\nu e_\rho e_\sigma + \frac{c_2}{e^{2d+2}}(e_\mu e_\nu \delta_{\rho\sigma} + e_\rho e_\sigma \delta_{\mu\nu})\\ &+ \frac{c_3}{2d+2}(e_\mu e_\rho \delta_{\nu\sigma} + e_\nu e_\rho \delta_{\mu\sigma} + e_\mu e_\sigma \delta_{\nu\rho} + e_\nu e_\sigma \delta_{\mu\rho})\\ &+ \frac{c_4}{e^{2d}}\delta_{\mu\nu}\delta_{\rho\sigma} + \frac{c_5}{e^{2d}}(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma})\end{aligned} \tag{24}$$

while its form on the FP is given in [11]. Even with the conservation equation

$$\partial^\mu\langle \boldsymbol{T}_{\mu\nu}\boldsymbol{T}_{\rho\sigma}\rangle = 0 \tag{25}$$

it is not enough to lead to an obvious preference for a particular C-function, unlike the $d = 2$ case. In our case however, we have an extra constraint coming from the eigenvalue eq. (16) which in coordinate space for the individual couplings $c_i, i = 1, 2.., 5$ can be written as:

$$\frac{d}{de}c_i = -\eta c_i \tag{26}$$

whose general solution is

$$c_i(e) = c_i^*\left[\frac{e}{e_*}\right]^{-\eta} \tag{27}$$

where e_* and c_i^* are the values of e and the individual couplings on the FP. If we define the most general form of the C-function as a linear combination of the five couplings and their values on the FP, we find

$$C = \left(a\left[\frac{e}{e_*}\right]^{-\eta} + b\right)c_T^* \tag{28}$$

where the coefficients a, b are restricted by the conditions that the C-function on the boundary is equal to the central charge and that it is a monotonically decreasing function:

$$C^* = c_T^* \Rightarrow a + b = 1 \tag{29a}$$

$$\frac{d}{de}C < 0 \Rightarrow a < 0. \tag{29b}$$

Now, starting at some point away from the FP where the C-function is positive, we can use eqs. (29) in order to find:

$$C > 0 \Rightarrow a < a_c \equiv \frac{1}{1 - \left[\frac{e}{e_*}\right]^{-\eta}} \tag{30}$$

and as we approach the IR FP ($e \to e_*$) the denominator tends to zero which means that the critical value $a_c \to -\infty$. Consequently, for any initial value of a, at some point along the RG flow, a_c will become lesser that a due to its running, resulting into a negative C-function. The above picture is summarized in fig. 1.

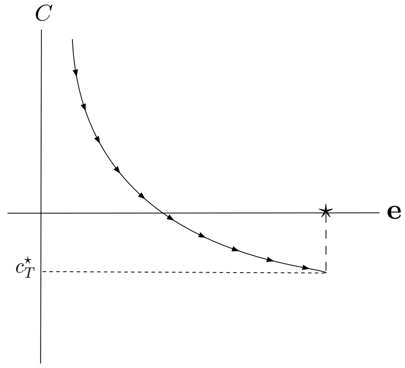

Figure 1: The graph of a general C-function depicting its flow from positive towards negative values as it approaches the IR FP.

The exact computation of the C-function requires a simultaneous analysis of the boundary and the bulk. For the former, one needs to calculate the finite parts of $\langle \boldsymbol{T}_{\mu\nu} \boldsymbol{T}_{\rho\sigma} \rangle$ via the ϵ-expansion or any other renormalization process, while for the latter, a better understanding of the time-dependent background is needed in order to compute the spectra $\mathcal{P}_S$ and $\mathcal{P}_T$ exactly.

A second implication arising from dS/CFT, comes from the fact that the dual of dS is ought to be a non-unitary large N theory while the Ising model is unitary and of $N = 1$. Fortunately, this is not prohibitory since the value of η is pretty much N-independent (see [12]) while we can think of the Ising model being dual to an AdS system which is permitted since its dual is a unitary CFT. Then, we can perform an analytic continuation from AdS to dS which does not affect the value of η and its effect on n_S. Finally, we believe that this argument justifies the use of the Ising critical exponent.

2. Part 2: The Thermal Effects

2.1 Propagators and Temperature

In this section, we will turn our attention towards the bulk which we will model by a 4d dS background described by the metric (4) written in conformal time τ and the action

$$\mathcal{S} = \frac{1}{2} \int d^3x d\tau \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - (m^2 + \xi \mathcal{R}) \phi^2 \right] \tag{31}$$

of a free massive scalar field ϕ where $\mathcal{R}$ is the scalar curvature in dS. The corresponding equation of motion (eom) is:

$$\Phi''_{|\boldsymbol{k}|} + \omega^2_{|\boldsymbol{k}|} \Phi''_{|\boldsymbol{k}|} = 0, \qquad \Phi_{|\boldsymbol{k}|} = \frac{\phi_{|\boldsymbol{k}|}}{a} \tag{32}$$

with

$$\omega^2_{|\boldsymbol{k}|} = |\boldsymbol{k}|^2 + m^2_{\text{dS}}, \qquad m^2_{\text{dS}} = \frac{1}{\tau^2} \left(\frac{m^2}{H^2} + 12\xi - \frac{d^2 - 1}{4} \right) \tag{33}$$

the angular frequency. The general solution the eom (32) is a linear combination of the Bessel functions of classical order:

$$\nu_{cl} = \frac{3}{2}\sqrt{1 - \frac{4m^2}{9H^2} - \frac{48\xi}{9}} \tag{34}$$

with $\nu_{cl} = \frac{3}{2}$ corresponding to the scale invariant case as we will soon see.

We already know that dS is characterized by the temperature:

$$T_{dS} = \frac{H}{2\pi}, \tag{35}$$

hence we suspect that the corresponding thermal effects have left an imprint on the CMB. In order to regulate these effects, we will proceed to calculate the thermal propagators of our theory using the two real-time formalisms: the Schwinger-Keldysh (SK) contour and thermo-field dynamics (TFD). Actually, both formalism require us to double the degrees of freedom instead of trading the time coordinate for temperature and interestingly enough, we will show that their results turn out to be equivalent.

In the context of the SK path integral, the doubled degrees of freedom is related to a forward (+) and a backward (−) branch in conformal time evolution. The field propagator $\mathcal{D}$ in such a basis takes on a 2×2 matrix structure:

$$\mathcal{D}(\tau_1 - \tau_2) = \begin{pmatrix} \mathcal{D}_{++} & \mathcal{D}_{+-} \\ \mathcal{D}_{-+} & \mathcal{D}_{--} \end{pmatrix} = \begin{pmatrix} \langle \mathcal{T}\Phi(\tau_1)\Phi(\tau_2)\rangle & \langle \tilde{\Phi}(\tau_1)\Phi(\tau_2)\rangle \\ \langle \Phi(\tau_1)\tilde{\Phi}(\tau_2)\rangle & \langle \tilde{\mathcal{T}}\tilde{\Phi}(\tau_1)\tilde{\Phi}(\tau_2)\rangle \end{pmatrix} \tag{36}$$

with Φ ($\tilde{\Phi}$) the field living on the forward (backward) part of the contour and $T(\tilde{T})$ denoting the time (anti-time) ordering.

The SK contour consists of three individual parts (see fig. 2), namely the forward and backward branches C_+, C_- which span from $\tau_{in} \longrightarrow \tau_{out}$ and reverse. In addition, a third "thermal" branch C_3 is needed, which spans in an imaginary direction $(\tau_{in}, \tau_{in} - i\frac{\beta}{2})$ with no inflation, in order to insert temperature $\beta = \frac{1}{T}$ into the system. We also introduce the propagators:

$$\mathcal{D}_{33}(\tau_1 - \tau_2) = \langle \mathcal{T}\Phi_3(\tau_1)\Phi_3(\tau_2)\rangle \tag{37a}$$

$$\mathcal{D}_{3+}(\tau_1 - \tau_2) = \langle \Phi(\tau_1)\Phi_3(\tau_2)\rangle \tag{37b}$$

$$\mathcal{D}_{3-}(\tau_1 - \tau_2) = \langle \tilde{\Phi}(\tau_1)\Phi_3(\tau_2)\rangle, \tag{37c}$$

where Φ_3 is the field defined in the thermal branch. Next, we require that τ_{out}, τ_{in} are junction points of the propagators and their derivatives following [13] while we also identify τ_{in} with $\tau_{in} - i\frac{\beta}{2}$ so that the KMS condition:

$$\langle \Phi(\tau_1)\Phi(\tau_2)\rangle = \langle \Phi(\tau_2)\Phi_3(\tau_1 - i\frac{\beta}{2})\rangle \tag{38}$$

is satisfied. This ensures that our thermal propagator is correct.

As a result, corrections of thermal nature are introduced in the propagator (36) which we can compute by making two assumptions. Firstly, we set $\tau_{in} \longrightarrow -\infty$ where we assume the Bunch Davies vacuum i.e. the mode functions $\Phi_{|\boldsymbol{k}|}$ are expressed in terms of Hankel functions and that their classical order is $\nu_{cl} = \frac{3}{2}$. Then, the in-in SK thermal propagator is:

$$\mathcal{D}_{\frac{\beta}{2}} = \mathcal{D} + n_B\left(\frac{\beta}{2}\right)\left(\mathcal{D}_{++} + \mathcal{D}^*_{++}\right)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{39}$$

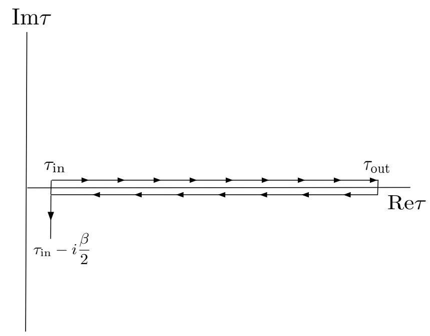

Figure 2: The SK contour consisting of the forward C_+, backward C_- and thermal C_3 branches.

with

$$n_B(\beta) = \frac{e^{-\beta\omega_{|\boldsymbol{k}|}}}{1 - e^{-\beta\omega_{|\boldsymbol{k}|}}} \tag{40}$$

the Bose-Einstein number density written as a function of the inverse of the temperature of the system β.

On the other hand, in the TFD approach the doubled Hilbert space is seen as the tensor product of the Hilbert spaces $\mathcal{H}, \tilde{\mathcal{H}}$ of positive and negative momenta correspondingly. The fields living in these Hilbert spaces are Φ and $\tilde{\Phi}$ which we Fourier expand:

$$\Phi(\tau, \boldsymbol{x}) = \int d^3\boldsymbol{k} \left[\alpha^-_{\boldsymbol{k}} u^*_{|\boldsymbol{k}|}(\tau) + \alpha^+_{-\boldsymbol{k}} u_{|\boldsymbol{k}|}(\tau)\right] e^{i\boldsymbol{k}\boldsymbol{x}} \tag{41a}$$

$$\tilde{\Phi}(\tau, \boldsymbol{x}) = \int d^3\boldsymbol{k} \left[\tilde{\alpha}^+_{\boldsymbol{k}} u^*_{|\boldsymbol{k}|}(\tau) + \tilde{\alpha}^-_{-\boldsymbol{k}} u_{|\boldsymbol{k}|}(\tau)\right] e^{i\boldsymbol{k}\boldsymbol{x}} \tag{41b}$$

where $\alpha^\pm_{\boldsymbol{k}}, \tilde{\alpha}^\pm_{\boldsymbol{k}}$ the corresponding ladder operators of $\mathcal{H}, \tilde{\mathcal{H}}$. These operators satisfy the commutation relations

$$[\alpha^-_{\boldsymbol{k}}, \alpha^+_{\boldsymbol{q}}] = \delta^{(3)}(\boldsymbol{k} - \boldsymbol{q}), \qquad [\alpha^-_{\boldsymbol{k}}, \alpha^-_{\boldsymbol{q}}] = [\alpha^+_{\boldsymbol{k}}, \alpha^+_{\boldsymbol{q}}] = 0, \qquad [\alpha^\pm_{\boldsymbol{k}}, \tilde{\alpha}^\pm_{\boldsymbol{q}}] = 0. \tag{42a}$$

Then main point of the TFD structure is that the doubling of the degrees of freedom enable us to define a pure thermal vacuum $|0, \beta\rangle$ by:

$$\alpha^-_{\boldsymbol{k}}(\beta)|0, \beta\rangle = 0 \tag{43}$$

where $\alpha^\pm_{\boldsymbol{k}}(\beta), \tilde{\alpha}^\pm_{\boldsymbol{k}}(\beta)$ are the thermal analogues of the ladder operators given by the rotation

$$\begin{pmatrix} \alpha^-_{\boldsymbol{k}}(\beta) \\ \tilde{\alpha}^+_{\boldsymbol{k}}(\beta) \end{pmatrix} = U(\beta; \boldsymbol{k}) \begin{pmatrix} \alpha^-_{\boldsymbol{k}} \\ \tilde{\alpha}^+_{\boldsymbol{k}} \end{pmatrix} . \tag{44}$$

Here U is a 2×2 matrix of the form:

$$U(\beta; \boldsymbol{k}) = \begin{pmatrix} \cosh\theta_{|\boldsymbol{k}|}(\beta) & -\sinh\theta_{|\boldsymbol{k}|}(\beta) \\ -\sinh\theta_{|\boldsymbol{k}|}(\beta) & \cosh\theta_{|\boldsymbol{k}|}(\beta) \end{pmatrix} \tag{45}$$

where

$$\sinh\theta_{|\boldsymbol{k}|}(\beta) = \sqrt{n_B(\beta)}, \qquad \cosh\theta_{|\boldsymbol{k}|}(\beta) = \sqrt{1 + \sinh\theta_{|\boldsymbol{k}|}(\beta)} = \frac{1}{\sqrt{1 - e^{-\beta\omega_{|\boldsymbol{k}|}}}} \tag{46}$$

the TFD parameters. Since the above rotation is mathematically similar to a Bogolyubov transformation (BT) in a curved background, we can think of TFD as its generalization which essentially enables us to calculate the thermal corrections arising due to the change of vacuum states. Consequently, the thermal propagator in this formalism is equal to:

$$\begin{aligned} \mathcal{D}_{\beta'} &= U^{-1}(\beta'; |\boldsymbol{k}|)\, \mathcal{D}\, U^{-1}(\beta'; |\boldsymbol{k}|)^T \\ &= \mathcal{D} + \left(\sinh^2\theta_{|\boldsymbol{k}|}(\beta') + \sinh\theta_{|\boldsymbol{k}|}(\beta')\cosh\theta_{|\boldsymbol{k}|}(\beta')\right)\left(\mathcal{D}_{++} + \mathcal{D}_{++}^*\right)\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \end{aligned} \tag{47}$$

where $\mathcal{D}$ is the zero-temperature propagator given in (36). One immediately notices that the thermal corrections found in both eqs. (39) and (47) have an analogous structure, though the latter contains an extra term along $\sinh\theta_{|\boldsymbol{k}|}(\beta')\cosh\theta_{|\boldsymbol{k}|}(\beta')$. Fortunately, the following trivial identity holds:

$$\frac{e^{-\beta\omega_{|\boldsymbol{k}|}}}{1 - e^{-\beta\omega_{|\boldsymbol{k}|}}} + \frac{e^{-\frac{\beta}{2}\omega_{|\boldsymbol{k}|}}}{1 - e^{-\beta\omega_{|\boldsymbol{k}|}}} = \frac{e^{-\frac{\beta}{2}\omega_{|\boldsymbol{k}|}}}{1 - e^{-\frac{\beta}{2}\omega_{|\boldsymbol{k}|}}} \tag{48}$$

which shows that the propagators (39) and (47) completely match as long as $\beta' = \beta$.

2.2 The line of constant physics and other observables

Now that we have managed to describe the thermal corrections of the scalar propagator via two equivalent processes, we can use either one to determine several important observables. At equal spacetime points and at the time of horizon exit, defined as $\tau H = 1$, we concentrate on horizon exiting modes that satisfy $|\boldsymbol{k}\tau| \leq 1$ and the thermal scalar power spectrum is defined as:

$$P_{S,\beta}\mathbf{1} = \mathcal{D}_\beta \mathbf{1}\Big|_{\tau_1=\tau_2}. \tag{49}$$

In addition, we define the parameter

$$\kappa \equiv \omega_{|\boldsymbol{k}|}|\tau|\Big|_{|\boldsymbol{k}\tau|=1} = \sqrt{-1 + \frac{m^2}{H^2} + 12\xi} \tag{50}$$

where $\omega_{|\boldsymbol{k}|}$ given by eq. (33). This parameter can be thought of as an analog to the order of the Hankel function $\nu_{\rm cl}$ with the special case $\nu_{\rm cl} = \frac{3}{2}$ obtained by $\frac{m^2}{H^2} + 12\xi = 0$, that gives a scale invariant CMB. This corresponds to $\kappa = i$.

As soon as one moves away from $\tau_{\rm in}$, due to the time-dependent dS background, the angular frequency is shown [14] to transform like:

$$\omega_{|\boldsymbol{k}|} \longrightarrow \Omega_{|\boldsymbol{k}|} = \omega_{|\boldsymbol{k}|}\left(\cosh^2\theta_{|\boldsymbol{k}|}(\beta) + \sinh^2\theta_{|\boldsymbol{k}|}(\beta)\right) \tag{51}$$

and subsequently, (50) generalizes into:

$$\kappa \longrightarrow \Lambda = \kappa\left(1 + 2\frac{e^{-2\kappa x}}{1 - e^{-2\kappa x}}\right) = \kappa\coth(x\kappa), \qquad x = \frac{\pi H}{2\pi T} \tag{52}$$

where x is the redefined temperature in order to keep our formulas as simple as possible. The corrections introduced in (51) can be interpreted as an appearance of a thermal mass which shifts the order away from $\frac{3}{2}$:

$$\nu_{\rm cl} \longrightarrow \nu = \nu_{\rm cl} + \nu_q \tag{53}$$

which in return shifts the scaling dimension of the bulk scalar field:

$$\Delta_- = \frac{d}{2} - \nu = \Delta_{\text{cl},-} - \nu_q. \tag{54}$$

Note that the dual partner will have:

$$\Delta_+ = \frac{d}{2} + \nu \tag{55}$$

while we are interested in the case where $(\Delta_+, \Delta_-)_{\text{cl}} = (0, 3)$. Furthermore, the emergence of the thermal mass can be seen as well from the eom:

$$\frac{d^2\phi}{d\tau^2} + 2aH\frac{d\phi}{d\tau} + \left(\frac{m^2}{H^2} + \xi\frac{\mathcal{R}}{H^2}\right)a^2H^2\phi = 0, \qquad \frac{dH}{d\tau} = -\frac{1}{2a}\left(\frac{d\phi}{d\tau}\right)^2 \tag{56}$$

since there seems to be a thermal backreaction that will result in the deformation of H at late times.

Then, we use (49) and define the thermal scalar spectral index:

$$n_{S,\beta} = 1 + \frac{d\ln\left(|\boldsymbol{k}|^3\mathcal{P}_{S,\beta}\right)}{d\ln|\boldsymbol{k}|} \tag{57}$$

so that its thermal deviation is:

$$\delta n_S = n_{S,\beta} - 1 = -\frac{2x}{\Lambda}\left[\frac{e^{-x\Lambda}}{1 - e^{-2x\Lambda}}\right]. \tag{58}$$

Starting with the initial condition $\kappa = i$ there are two interesting cases, namely zero temperature ($T = 0 \Rightarrow x \to \infty$) and dS-temperature ($T = T_{\text{dS}} \Rightarrow x = \pi$) which result into $\delta n_S = 0$, indicating that scale invariance is restored in zero and dS temperatures. This agrees with our earlier assumption that the journey of an observer starting from τ_{in} with $T = 0$ towards τ_{out} which is characterized by T_{dS} corresponds on the boundary to a flow from a UV towards an IR FP. In the intermediate times where $T \leq T_{\text{dS}}$ ($x \geq \pi$) we see that $\delta n_S \neq 0$ becomes a one parameter expression of Λ. We can fix this freedom by equating the value of δn_S with the experimental result (3) and let Λ flow along the $x : \infty \to \pi$ trajectory. As a result, there is a line of constant physics (LCP), showcased in fig. 3, which presents the different vales of Λ given by diffetent values of x that satisfy $\delta n_S = -0.036$.

	x
$\to 0$	$\to \infty$
10^{-6}	$3.5 \cdot 10^7$
0.01	1600
0.5	14.8
$\to 1.5117$	$\to \pi$

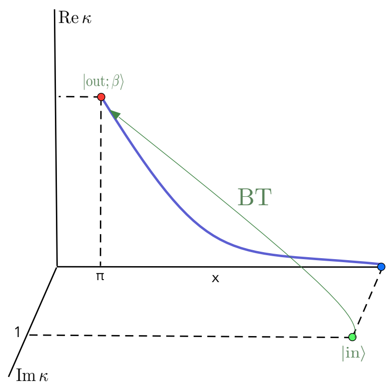

Figure 3: Left: A few points of the nearly conformal LCP defined by $n_S = -\eta$. Right: The Bogolyubov Transformation (BT) $|\text{in}\rangle \to |\text{out}; \beta\rangle$ and the LCP, on the complex plane.

Since our system has no free parameters, we can additionally compute several other observables which are also determined by $P_{S,\beta}$. For example, the running of the spectral index is:

$$n^{(1)}_{S,\beta} = \frac{dn_{S,\beta}}{d\ln|\boldsymbol{k}|} = \delta n_S \left[2 - \frac{1}{\Lambda^2} - \frac{x}{\Lambda}\left(1 + \frac{2e^{-2x\Lambda}}{1-e^{-2x\Lambda}}\right)\right] \tag{59}$$

which obtains the value when close to dS temperature $x \simeq \pi, \Lambda \simeq 1.5117$:

$$n^{(1)}_{S,\beta} = 0.0186\,, \tag{60}$$

while its value as measured via the Planck experiment [1] is:

$$n^{(1)}_{S,\mathrm{exp}} = 0.013 \pm 0.012\,. \tag{61}$$

Finally, the universal contribution to the non-Gaussiniaty parameter [15] can be expressed in terms of the number of e-folds $N = \int_{t_i}^{t_f} dtH$ and its derivatives in the in-vacuum, as [5]:

$$f_{NL} = \frac{5}{6}\frac{N_{\rho\rho}}{N_\rho^2} \tag{62}$$

with $N_\rho = \frac{\partial N}{\partial \rho}$, $N_{\rho\rho} = \frac{\partial^2 N}{\partial \rho^2}$ and $\rho = P_{S,\beta}$. It is computed to be:

$$f_{NL} = -\frac{5\left[x(-1+\Lambda^2)^2\left(1 + x\Lambda\cot(\frac{x\Lambda}{2})\right) + 2\Lambda^3\sinh(x\Lambda)\right]}{6\Lambda^2\left[x(-1+\Lambda^2) + \Lambda\sinh(x\Lambda)\right]}\,. \tag{63}$$

and for $x \simeq \pi$ and $=_\pi \simeq 1.5117$ this gives

$$f_{NL} = -1.7138. \tag{64}$$

while the experimental data fro a certain analysis [2] show:

$$f_{NL,\mathrm{exp}} = -1.7 \pm 5.2\,. \tag{65}$$

3. Conclusion

We considered a thermal scalar in dS background in the Bunch Davies vacuum. Starting from τ_{in}, time evolution placed us in the interior of the finite temperature phase diagram which ends at τ_{out} characterized by the dS temperature. We argued that the above can be understood as an RG flow of the boundary theory which seems to be in the same universality class as the 3d Ising model. Through holography, we presented how the critical exponent η could be the parameter that characterized the breaking of scale invariants which is observed in the CMB.

In addition, we used thermal field theory and managed to describe the thermal effects that appear in the scalar propagator and calculated the thermal power spectrum. This led to the calculation of the thermal deviation of n_S which in return gave us clues for the existence of a LCP. Lastly, we computed additional cosmological observables $\left(n^{(1)}_{S,\beta}, f_{NL}\right)$ which where well within current experimental bonds.

Finally, the emergence of a thermal mass in the equations of motion indicates towards a thermal backreaction that further alters the spacetime geometry through thermal corrections of the stress energy tensor. Hence, in order to calculate the new metric, one needs to first obtain the thermal $T_{\mu\nu}$, plug it in the Einstein equations and afterwards solve the differential equations. This of course is no easy feat, nevertheless if done correctly, it should answer the question of how the dS symmetry breaks down as soon as an observer moves away from their initial time $\tau_{\rm in}$ and in what way is it restored as soon as the system is heated up to dS temperature.

References

[1] Planck Collaboration, Y. Akrami et al., *Planck 2018 results. X. Constraints on inflation*, Astron. Astrophys. 641 (2020), arXiv:1807.06211 [astro-ph.CO].

[2] Planck Collaboration, Y. Akrami et al., *Planck 2018 results. IX. Constraints on primordial non-Gaussianity*, Astron. Astrophys. 641 (2020), arXiv:1905.05697 [astro-ph.CO].

[3] I. Antoniadis, P. O. Mazur and E. Mottola, *Conformal Invariance, Dark Energy, and CMB Non- Gaussianity*, JCAP 09 (2012) 024, arXiv:1103.4164 [gr-qc].

[4] M. Campostrini, A. Pelissetto, P. Rossi and Ettore Vicari, *25th-order high-temperature expansion results for three-dimensional Ising-like systems on the simple cubic lattice*, Phys. Rev. E 65 (2002) 066127, arXiv:cond-mat/0201180 [cond-mat].

[5] J. M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, JHEP 05 (2003) 013, arXiv:astro-ph/0210603 [astro-ph].

[6] F. Larsen, J. P. van der Schaar and R. G. Leigh, *De Sitter holography and the cosmic microwave background*, JHEP 04 (2002) 047, arXiv:hep-th/0202127 [hep-th].

[7] F. Larsen and R. McNees, *Inflation and de Sitter holography*, JHEP 07 (2003) 051, arXiv:hep-th/0307026 [hep-th].

[8] J. P. van der Schaar, *Inflationary perturbations from deformed CFT*, JHEP 01 (2004) 070, arXiv:hep-th/0307271 [hep-th].

[9] S. Fichet, *On holography in general background and the boundary effective action from AdS to dS*, JHEP 07 (2022) 113, arXiv:2112.00746 [hep-th].

[10] D. Karateev, *Two-point Functions and Bootstrap Applications in Quantum Field Theories*, arXiv:2012.08538 [hep-th].

[11] H. Osborn and A. Petkou, *Implications of Conformal Invariance in Field Theories for General Dimensions*, Annals Phys. 231 (1994) 311-362, arXiv:hep-th/9307010 [hep-th].

[12] J. Henriksson, *The critical O(N) CFT: Methods and conformal data*, Phys. Rept. 1002 (2023) 1-72 arXiv:2201.09520 [hep-th].

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[13] G. Semenoff and N. Weiss, *Feynman Rules for Finite Temperature Greens Functions in an Expanding Universe*, Phys. Rev. D31 (1985), 689.

[14] B. Garbrecht, T. Prokopec and M. G. Schmidt, *Particle number in kinetic theory*, Eur. Phys. J. C38 (2004) 135-143, arXiv:hep-th/0211219 [hep-th].

[15] P. Creminelli and M. Zaldarriaga, *Single field consistency relation for the 3-point function*, JCAP 10 (2004) 006, arXiv:astro-ph/0407059 [astro-ph].