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Application of Lasso for investigating QPO properties in black-hole binaries

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Quasi-periodic oscillations (QPOs) are sub-second variability observed during outbursts in black-hole low-mass X-ray binaries (BH LMXBs), and their origin is still under debate. The QPO analysis has basically relied on the fast Fourier transform (FFT) in which the frequency resolution degrades as the length of the light curve gets shorter. Here we applied the period analysis by the least absolute shrinkage and selection operator (Lasso) to the X-ray light curve of the BH LMXB named XTE J1550−564, in which type-C QPOs having the peak frequency of 5.7 Hz were detected, and compared the FFT and Lasso spectra. The FFT power spectrum becomes messy around the QPO peak frequency as the length of the light curve is shorter and shorter. On the other hand, we found a single and sharp peak in the Lasso power spectrum of the 1.4-s light curve. We repeated estimating the Lasso spectrum of each 1.4-s light curve by shifting the window throughout the 2-ks entire light curve, and obtained the half width at half maximum (HWHM) of the histogram of the resolved QPO frequencies. The HWHM was consistent with the HWHM of the Lorentzian in the FFT spectrum of the entire light curve. These results suggest that the Lasso technique is powerful for investigating the short-term variability of the QPO frequency and that the Lasso analysis result is consistent with the FFT analysis result.

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1. Introduction

Black-hole low-mass X-ray binaries (BH LMXBs) are close binary systems composed of a black hole as the primary star and a K or M-type dwarf as the companion star. The gas is transferred from the companion to the primary, which forms an accretion disk around the BH. LMXBs exhibit outbursts at multi-wavelengths, which represent sudden brightening of the accretion disk [1]. The source of the outbursts is the thermal-viscous instability triggered by partial ionization of hydrogen at the accretion disk [2].

During outbursts, BH LMXBs undergo dramatic state transitions in X-rays, basically between the low/hard state to the high/soft state. In the hard state, the accretion rate to the BH is low, and the optically-thick disk is truncated far from the BH. An optically-thin and geometrically-thick hot accretion flow present inside the truncated disk, which is called an advection-dominated accretion flow (ADAF). This plasma is the source of hard X-rays by inverse Compton scattering of photons from the optically-thick disk. To the contrary, the optically-thick disk extends to the innermost stable circular orbit (ISCO) in the soft state. The X-ray spectrum is dominated by multi-color black body emission in this state.

One of the most characteristic features in BH LMXBs is sub-second variability. Many systems show quasi-periodic oscillations (QPOs) during outbursts. QPOs are classified into several types, and type-C QPOs, a kind of low-frequency QPOs, are characterized by high-amplitude and a narrow peak of a Lorentzian in the power spectrum. The centroid frequency of type-C QPO is strongly correlated with spectral state [3]. Several models for type-C QPOs have been proposed, and it is widely accepted that Lense–Thirring precession is the source of type-C QPOs, in which the ADAF flow is dragged by the BH spin. However, some observations are negative to this model, and the physical mechanisms are unclear for other types of QPOs [4].

To approach the physical origin of QPOs, timing analyses are essential. Traditionally, QPO analyses have been performed using the fast Fourier transform (FFT). However, the FFT has limitations in tracking the short-time evolution of power spectra since it needs long-term light curves. Also, this method is not applicable to unevenly sampled light curves with seasonal gaps, which are normal for astronomical time-series data. We have to introduce another method if we obtain more information than the QPO frequency and its deviation from the long-term light curve, such as the time evolution of the QPO frequency, phase, amplitude, and light-curve profile, which would be helpful to test the proposed models.

Our purpose in this article is to investigate the short-term evolution of the QPO frequency. We chose XTE J1550−564 as a target object in this work. XTE J1550−564 is a BH LMXB with the BH mass of $9.10 \pm 0.61\,M_{\odot}$, and the orbital period of 1.55 days. The distance is $D = 4.38^{+0.58}_{-0.41}$ kpc, and the inclination angle is $i = 74.7^{\circ} \pm 3.8^{\circ}$ [5]. QPOs have been confirmed in this source during the 1998 outburst [6]. The Rossi X-ray Timing Explorer (RXTE) satellite obtained a dense light curve of this source, which is suitable for our purpose. In this article, we explain the limitations of the FFT analyses, introduce another technique, and demonstrate that the power spectrum obtained by our introduced method is consistent with the FFT spectrum. This article is structured as follows. Section 2 describes the data reduction and analysis methods. Section 3 shows our results, and we discuss the results in Section 4.

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2. Observations and analysis methods

2.1 Data selection and reduction

We analyzed the RXTE/PCA data of ObsID: 30191-01-01-00 in this paper. The timing analyses of this data were performed by [7], and significant type-C QPOs were detected. We can compare our results with theirs. We determined to use the binned mode data because they have the highest time resolution.

We used HEAsoft version 6.35.1 for data reduction. We generated a good time interval (GTI) using `maketime` version 2.16 with a filter file generated by `xtefilt` version 1.8. We extracted light curves with 0.005-s bin in the 2–40 keV band using `saextract` version 4.4 and applied the barycentric correction with `faxbary` version 2.5. The total exposure time is 2-ks. A part of the extracted light curve is represented in the left panel of Figure 1. We also extracted the power spectrum of the entire light curve and confirmed the QPO feature (see the right panel of Figure 1).

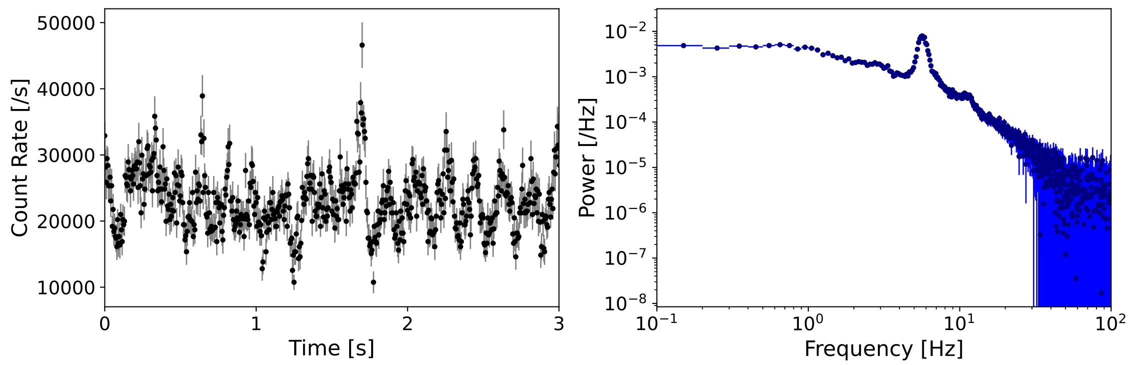

Figure 1: Observed light curve of XTE J1550−564 and its power spectrum. (Left) Part of the light curves of ObsID: 30191-01-01-00 of the RXTE/PCA data. (Right) Enlarged view of the binned power spectrum of the entire light curve. This shows a Lorentzian shape centered at the peak frequency of ~5.7 Hz.

2.2 Fast Fourier transform (FFT)

The FFT is a fast algorithm for numerically performing the discrete Fourier transform. Specifically, it is defined as

$$\min_{X} \|y - FX\|_2^2. \tag{1}$$

where F is the inverse Fourier transform matrix, X is the Fourier coefficients, and y is the time-series data, which is the observed light curve in this case. We can compute X under the condition $\dim(y)/2 \geq \dim(X)$ [8]. However, optical light curves usually have seasonal gaps and/or are unevenly sampled. The FFT cannot be directly applied to this kind of sparse time series data. In this paper, we used the `scipy.signal` package of Python for the FFT analyses. Also, the FFT power spectrum is greatly affected by window functions. For example, the power spectrum deviates from the true shape if the light curve has just a finite length.

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2.3 Least absolute shrinkage and selection operator (Lasso)

The least absolute shrinkage and selection operator (Lasso) is a technique of compressed sensing, which was proposed by [9]. The Lasso is defined as

$$\hat{X}_{\rm LAR} = \mathrm{argmin}_X \left(F(X) \equiv \frac{1}{2N_{\rm sample}} \|y - FX\|_2^2 + \lambda \|X\|_1 \right). \tag{2}$$

Here, $\lambda\|X\|_1$ stands for the ℓ_1-norm penalty term with the weighting coefficient λ (≥ 0), and $N_{\rm sample}$ denotes the number of data points in the light curve. If $\lambda = 0$, equation (2) becomes identical with the least-squares regression method. The Lasso technique was introduced in the period search of the astronomical light curve by [10] for the first time.

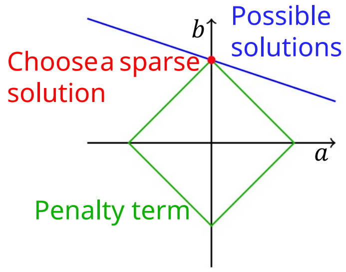

Figure 2: Arranged version of a figure in [8]. An example of the Lasso analysis for the problem with two variables.

Let us consider a linear regression with a straight line, for example. The image of the regression with the Lasso for this problem is depicted in Figure 2. The model equation is $y = ax + b$ in this case. If we have only one pair of observations (x, y), all pairs of variables (a, b) on the blue line are possible solutions. However, we can choose a sparse solution (the red point) when applying the ℓ_1-norm (the green diamond). Here, a is exactly zero.

The Lasso technique is applicable to sparse astronomical time series data, i.e., in the case that $\dim(y)/2 < \dim(X)$, which is a great advantage [10]. The time series data does not need to have evenly-spaced samples. The period search by the Lasso method is less affected by window functions, while the FFT spectrum becomes more diluted if the length of the time-series data becomes shorter. In this paper, we used the `LassoCV` method from Python's scikit-learn `linear_model` package and determined the regularization parameter λ by the cross-validation method.

3. Results

First, we tested whether the FFT method is suitable or not for investigating the short-term variability of the QPO frequency. We applied the FFT to six different light curves, each of which has a different observational length. The results are shown in Figure 3. By comparing with the power spectrum of the entire light curve, the spectral shape does not change very much when applying the FFT to a half of and a quarter of the entire light curve (see the right panel of Figure 1 and the top left and top middle panels of Figure 3). However, the shorter the light curve becomes, the lower the frequency resolution is, and the lower the S/N is. In the lower panels of Figure 3, each

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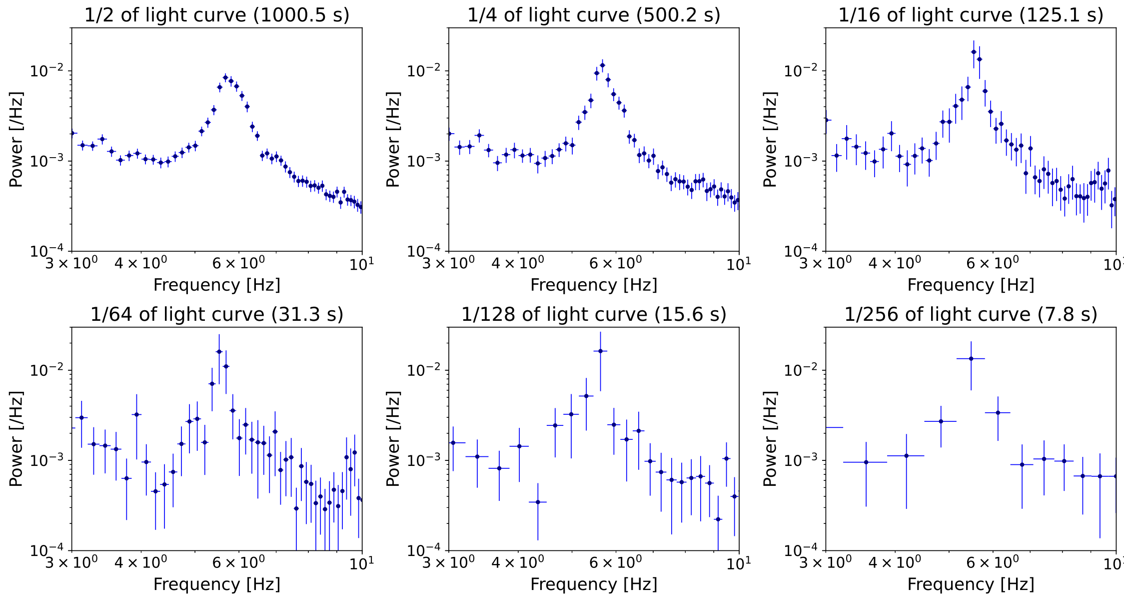

Figure 3: Binned power spectra of six different light curves of XTE J1550−564. From the upper left to the lower right, the observation length of the light curve is shorter and shorter. The error bars represent 1σ errors.

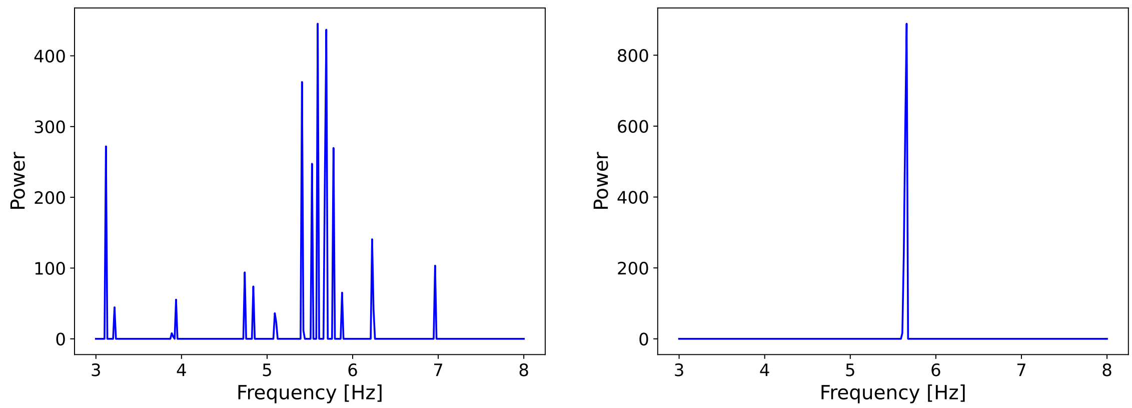

Figure 4: Lasso spectra of a part of the light curve of XTE J1550−564. (Left) The power spectrum of a 14-s light curve corresponding to ~80 cycles of QPOs. (Right) The power spectrum of a 1.4-s light curve corresponding to ~8 cycles of QPOs.

data point is generated by binning of ten points. As a result, we could not distinguish whether the spectral shape is a wide Lorentzian or is diluted by the window function.

We next applied the Lasso to the light curves having the lengths of 14 s and 1.4 s, which include ~80 cycles and ~8 cycles of QPOs, respectively. We found a single sharp peak in the Lasso spectrum of the latter light curve, while the Lasso spectrum of the former light curve has many small peaks around 5.7 Hz (see Figure 4).

We modeled the FFT spectrum of the entire light curve by the following equation. The result

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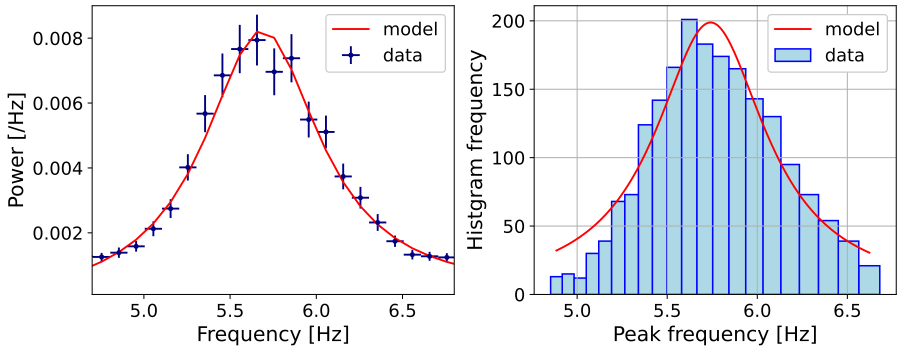

Figure 5: Comparison between our FFT and Lasso analysis results. (Left) The binned FFT spectrum of the entire light curve (blue points) and the modeling result (the red line). (Right) The histogram of peak frequencies in the Lasso spectra for many short-term light curves.

is shown in the left panel of Figure 5.

$$ax + b + C\frac{\Gamma^2}{(X-\mu)^2 + \Gamma^2}, \tag{3}$$

where, μ is the peak frequency of QPOs, and Γ is the half width at half maximum (HWHM) of a Lorentzian. The derived peak frequency and HWHM are 5.69(2) Hz and 0.41(3), respectively. In addition, we repeated applying the Lasso method to each 1.4-s light curve by shifting the window by 0.7 s throughout the entire light curve, and extracted the frequency at which the power is maximized. The right panel of Figure 5 shows the histogram of the sum of the derived peak frequencies. We modeled this histogram by equation (3), and extracted the peak frequency to be 5.74(1) Hz and the HWHM to be 0.38(3). The HWHM of the FFT spectrum is consistent with that of the histogram of peak frequencies in the Lasso spectra within the 1σ error, while the peak frequency is not.

4. Discussion

Our results suggest that the Lasso method enables detection of the QPO frequency even from the light curve having a very short observational length, though it is difficult to track the short-term variability of the QPO frequency by the FFT method (see Figures 3 and 2). This would be because the Lasso spectrum is less affected by window functions. It was demonstrated by [11] that the Lasso technique is useful for tracking the short-term variations of periodic signals. Our result would reflect this advantage of the Lasso method.

On the other hand, the Lasso spectrum is slightly different from the FFT spectrum (see Figure 5). The same analysis was performed by [7], and they obtained consistency between the FFT and Lasso analysis results. We explored the difference between their and our analyses, and found that we provided the evenly-spaced vector of λ not in the linear scale but in the logarithmic scale when determining the λ value by using the cross-validation technique. We corrected to give the λ vector having evenly-spaced samples in the linear scale and in the period space, and retried the same analysis. As a result, we obtained the modified histogram of peak frequencies, which is consistent with the FFT spectrum within 1σ errors. Figure 6 shows the difference in the selected λ value

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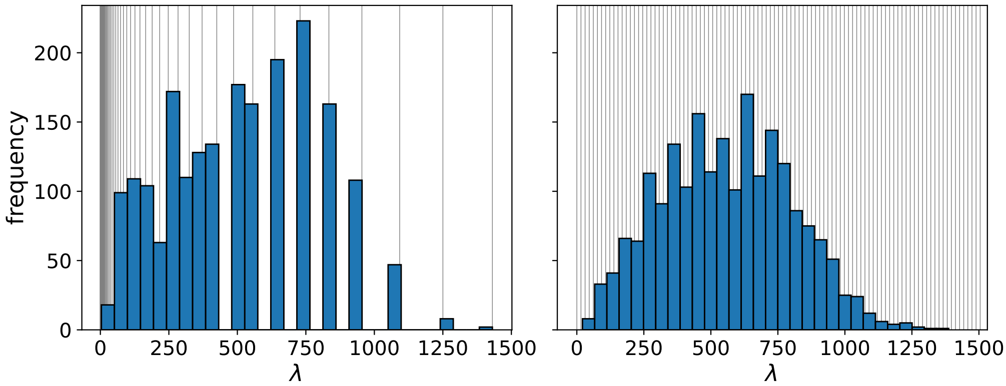

Figure 6: Histogram of λ determined by the cross-validation technique. Black vertical lines represent the input λ vector. (Left) The case for giving evenly-spaced vectors of λ in the logarithmic scale and in the frequency space. (Right) The case for giving evenly-spaced vectors of λ in the linear scale and in the period space.

between our old and new analyses. The Lasso analysis result is consistent with the FFT analysis result.

The HWHM of the Lorentzian in the FFT spectrum of the entire light curve is consistent with the deviation of the sum of QPO frequencies derived from many short-term light curves by the Lasso method. Also, the number of detected frequencies decreases with the shorter light curve, and only a single frequency appears in the Lasso power spectrum of 1.4-s light curves. This suggests that the QPO signal has a kind of lifetime of ~8 cycles and the QPO frequency changes with timescales of a couple of seconds. We will try the same analysis on other ObsIDs in the future to confirm whether this interpretation is correct or not.

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