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Effects of perturbation for transition operators of double-β decay on nuclear matrix element

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We show that the effective axial-vector current couplings ($g_A^{\rm eff}$) for the neutrinoless double-β ($0\nu\beta\beta$) and the two-neutrino double-β ($2\nu\beta\beta$) decays are close to each other in the decay instance of ^{136}Xe→^{136}Ba. This is a remarkable finding because $g_A^{\rm eff}$ for the $0\nu\beta\beta$ decay was unknown at all. This is shown by our calculation with the transition operator perturbed by the residual nucleon-nucleon interaction. The nuclear wave functions are supplied by the quasiparticle random-phase approximation with phenomenological interactions. The sum of the correction terms of the nuclear matrix element due to the perturbation is half the leading term in absolute value and negative. The influence of the perturbation on the half-life for the $2\nu\beta\beta$ decay is also shown to be significantly large. The more perturbation, the longer the half-life.

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The neutrinoless double-β ($0\nu\beta\beta$) decay is a predicted nuclear decay caused by the weak interaction [1]. If this is found, it implies the existence of the Majorana neutrino; this is identical with its antiparticle. This new neutrino induces physics beyond the standard model at least because the lepton number conservation is broken. The leptogenesis also needs this neutrino [2]. The decay probability predicted theoretically has a large uncertainty, and its origin is in the nuclear matrix element (NME), e.g., [3]. This transition matrix element between the initial and final nuclear states significantly depends on the method to calculate the matrix element. The goal of our study is to solve this uncertainty problem of the $0\nu\beta\beta$ NME. Usually, the effective axial-vector current coupling ($g_A^{\rm eff}$) is used for the NME of the weak decay, however, that for the $0\nu\beta\beta$ NME is unknown at all. This is the major cause of the uncertainty problem. Our approach to this problem is to obtain $g_A^{\rm eff}$ by simulating the exact half-life by the perturbed calculation [4]. This approach is applicable for both the $0\nu\beta\beta$ and the two-neutrino double-β ($2\nu\beta\beta$) decays. We compare the $g_A^{\rm eff}$ for the two decay modes and clarify whether the $g_A^{\rm eff}$ for the $0\nu\beta\beta$ NME ($g_{A,0\nu}^{\rm eff}$) is quite different from $g_A^{\rm eff}$ of the $2\nu\beta\beta$ NME ($g_{A,2\nu}^{\rm eff}$).

We introduce the perturbed initial and final states $|\mathcal{I}\rangle$ and $|\widetilde{\mathcal{F}}\rangle$, respectively, as

$$
\begin{aligned}
|\mathcal{I}\rangle &= |I^{(0)}\rangle + |I^{(1)}\rangle + |I^{(2)}\rangle, \\
|\widetilde{\mathcal{F}}\rangle &= |\widetilde{F}^{(0)}\rangle + |\widetilde{F}^{(1)}\rangle + |\widetilde{F}^{(2)}\rangle,
\end{aligned} \tag{1}
$$

where the tilde of the final state indicates that the leptons are included, and the suffix corresponds to the order of the perturbation. Each of the perturbed terms can be obtained by the Rayleigh-Schrödinger perturbation theory; the perturbation interaction is specified below. Our starting point is the transition matrix element $\langle\widetilde{\mathcal{F}}|H_W|\mathcal{I}\rangle$, where H_W denotes the weak interaction. By inserting Eq. (1) into this equation and taking the second-order terms, it is obtained that

$$
\langle\widetilde{F}^{(0)}|H_W|I^{(2)}\rangle + \langle\widetilde{F}^{(1)}|H_W|I^{(1)}\rangle + \langle\widetilde{F}^{(2)}|H_W|I^{(0)}\rangle. \tag{2}
$$

Since the perturbation interaction is arbitrary, one can set it to $H_W + :V:$, where $:V:$ is the normal-ordered residual nucleon-nucleon interaction. The ordering is defined referring to the Hartree-Fock-Bogoliubov ground state of the initial or final state. The choice depends on the structure of the concrete correction term. By taking the terms with two H_W and one $:V:$, the basic equation is obtained of the first-order correction to the NME of the $\beta\beta$ decay. This procedure yields two identical terms. In one, first, the system is perturbed by H_W, and then the transition occurs by H_W. In the other, the two steps are exchanged, but this is a double counting. Thus, the redundant factor of two is removed. The explicit equation can be found in Ref. [4]. This non-trivial derivation is necessary because the intermediate states of the $\beta\beta$ decay are virtual states. In other words, H_W^2 is not the transition operator of the $\beta\beta$ decay. This derivation method is an extension of the one to derive the leading-order $\beta\beta$ NME [5]. The phase space factor is the same as that for the leading-order decay probability.

We use three correction terms for the $0\nu\beta\beta$ NME, which are linear with respect to $:V:$. One of them is obtained from

$$
\begin{aligned}
M_{0\nu}^{(JVJ)} &= \frac{4\pi R}{g_A^2}\int d^3\boldsymbol{x}\, d^3\boldsymbol{y}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{1}{|\boldsymbol{q}|}\exp[i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})](-)\sum_{BB'}\langle F|\mathcal{J}_\mu(\boldsymbol{x})|B'\rangle \\
&\quad\times\frac{\langle B'|:(-V):|B\rangle}{E_{B'}+|\boldsymbol{q}|-\frac{1}{2}(E_I+E_F)}\frac{\langle B|(-\mathcal{J}^\mu(\boldsymbol{y}))|I\rangle}{E_B+|\boldsymbol{q}|-\frac{1}{2}(E_I+E_F)}.
\end{aligned} \tag{3}
$$

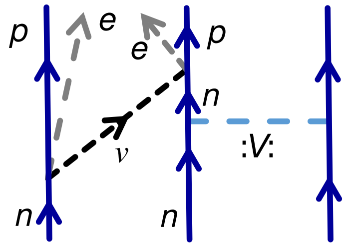

Figure 1: Diagram of two-body current correction term for $0\nu\beta\beta$ NME. The solid lines express the proton (p) or the neutron (n) states. The horizontal dashed line shows the perturbation interaction, and the tilted dashed line (closed) is the neutrino propagator. The tilted dashed line (open) indicates the electron. The two gathering points of the nucleon, the neutrino, and the electron are the vertexes of the weak interaction.

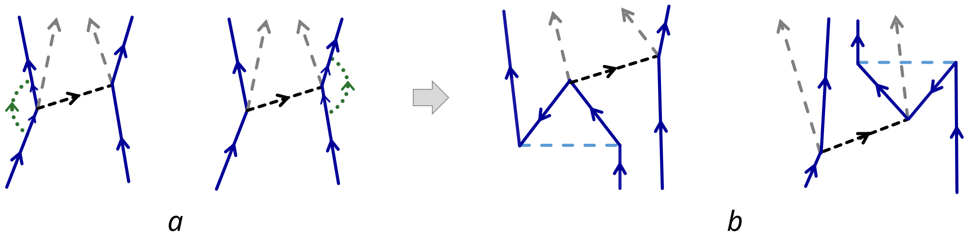

Figure 2: Diagrams of vertex corrections. The dotted line of the left diagrams (a) indicates the pion propagator. For the other parts, see the caption of Fig. 1.

Here, $\mathcal{J}_\mu(\boldsymbol{x})$ denotes the nucleon current density. The g_A is the axial-vector current coupling, and it is either $g_A^{\rm eff}$ or $g_A^{\rm bare} = 1.267$ [6] depending on the calculation. The latter is the value of g_A for a nucleon in the vacuum. $|B\rangle$ ($|B'\rangle$) is the intermediate nuclear state. $|F\rangle$ is the nuclear part of $|\widetilde{\mathcal{F}}^{(0)}\rangle$, and $|I\rangle = |I^{(0)}\rangle$. The energies of these states are denoted by E_B, $E_{B'}$, E_F and E_I, respectively. The neutrino momentum is denoted by $\boldsymbol{q}$. R is the nuclear root-mean-square radius. Equation (3) includes more terms than that necessary for our correction term, which represents the correction of the transition operator and is expressed by the connected diagram of Fig. 1. That is our two-body current correction (2bc). We use two more correction terms called the vertex correction, which correspond to the diagrams b of Fig. 2.

Now, we explain our new $g_A^{\rm eff}$. Usually, $g_A^{\rm eff}$ is defined by the reproduction of the experimental half-life [7]. This definition can be written in the form of a linear equation with respect to the NME components, and for the $0\nu\beta\beta$ decay, it is

$$(g_{A,0\nu}^{\rm eff})^2 M_{0\nu}^{\rm GT(0)} - g_V^2 M_{0\nu}^{\rm F(0)} = (g_A^{\rm bare})^2 M_{0\nu}^{\rm GT(exp)} - g_V^2 M_{0\nu}^{\rm F(exp)}, \tag{4}$$

where $M_{0\nu}^{\rm GT(0)}$ and $M_{0\nu}^{\rm F(0)}$ are the leading-order Gamow-Teller (GT) and Fermi (F) NME for the $0\nu\beta\beta$ decay. $M_{0\nu}^{\rm GT(exp)}$ and $M_{0\nu}^{\rm F(exp)}$ are those included in the experimental half-life implicitly; these are unknown. We assume that this virtual experimental value includes $g_A^{\rm bare}$ implicitly. The exact value of $g_{A,0\nu}^{\rm eff}$ is unknown. The vector coupling constant g_V is always equal to one in our study. We replace the experimental NME components by the corresponding perturbed ones;

$$(g_{A,0\nu}^{\rm eff}(\mathrm{ld;pt}))^2 M_{0\nu}^{\rm GT(0)} - g_V^2 M_{0\nu}^{\rm F(0)} = (g_A^{\rm bare})^2 M_{0\nu}^{\rm GT} - g_V^2 M_{0\nu}^{\rm F}. \tag{5}$$

Table 1: Calculated GT and Fermi NMEs of the $0\nu\beta\beta$ and the $2\nu\beta\beta$ decay modes of ^{136}Xe with the Skyrme interaction SkM* [9]. Leading, vc, and 2bc stand for the leading order, the vertex correction, and the two-body current terms, respectively. The sum of them is called the perturbed NME. The sign is chosen so that the GT leading term is positive. These results were taken from Ref. [4].

	$0\nu\beta\beta$ NME		$2\nu\beta\beta$ NME	
	GT	Fermi	GT	Fermi
Leading	3.095	−0.467	0.102	−0.002
vc	1.332	−0.984	0.055	−0.033
2bc	−2.731	1.758	−0.192	0.030
Perturbed (sum)	1.696	0.307	−0.035	−0.005

$M^{\mathrm{GT}}_{0\nu}$ is the sum of $M^{\mathrm{GT}(0)}_{0\nu}$ and the GT correction terms and is called the perturbed term. The GT correction term is the sum of the three components diagrammatically shown by Figs. 1 and 2; for the explicit equations, see Ref. [4]. $M^{\mathrm{F}}_{0\nu}$ is defined analogously for the Fermi term. The $g^{\mathrm{eff}}_{A,0\nu}(\mathrm{ld};\mathrm{pt})$ is a simulation of the exact $g^{\mathrm{eff}}_{A,0\nu}$. The first argument, ld, indicates that this $g^{\mathrm{eff}}_{A,0\nu}$ is used with the leading-order NME components, and the second argument, pt, implies that the perturbed NME components and g^{bare}_{A} are referred to in the definition of $g^{\mathrm{eff}}_{A,0\nu}$. The analogous definition is applied for $g^{\mathrm{eff}}_{A,2\nu}(\mathrm{ld};\mathrm{pt})$ for the $2\nu\beta\beta$ decay.

It is the aim of this article to compare $g^{\mathrm{eff}}_{A,0\nu}(\mathrm{ld};\mathrm{pt})$ and $g^{\mathrm{eff}}_{A,2\nu}(\mathrm{ld};\mathrm{pt})$. How different are they? This is a non-trivial question because the momentum of the neutrino is quite different between the $0\nu\beta\beta$ and $2\nu\beta\beta$ decays. This question is also deeply related to the uncertainty problem of the $0\nu\beta\beta$ NME. The calculation was performed for $^{136}\mathrm{Xe} \to {}^{136}\mathrm{Ba}$. This decay instance was chosen because the QRPA is a good approximation for these nuclei [8]. The nuclear wave functions are supplied by the quasiparticle random-phase approximation (QRPA) with the Skyrme, the Coulomb, and the contact pairing (isoscalar and isovector) interactions. The perturbation interaction $:V:$ is the proton-neutron components of the Skyrme interaction.

Table 1 shows the $0\nu\beta\beta$ and the $2\nu\beta\beta$ NME, and each of them shows the GT and the Fermi components. The GT component of the $0\nu\beta\beta$ NME is reduced by half due to the correction terms, in which the 2bc term has the major contribution. The Fermi component shows a similar tendency but for the different sign. In addition, the effect of the correction terms is so large that the sign of the Fermi component changes. The GT component of the $2\nu\beta\beta$ NME shows a tendency similar to that of the $0\nu\beta\beta$ NME but with the sign change. The Fermi component of the $2\nu\beta\beta$ NME is special. This term vanishes if the isospin symmetry is exact. It is seen that both the leading and the perturbed terms are very small. It is also seen that the two correction terms cancel each other.

Table 2 shows four calculated half-lives for the $2\nu\beta\beta$ decay of ^{136}Xe, and the experimental value is also shown. The differences in the calculations are the used g_A (g^{bare}_{A} or $g^{\mathrm{eff}}_{A,2\nu}(\mathrm{ld};\mathrm{pt})$) and the used NME components (the leading or the perturbed one). In the fourth row of the calculations is an over-perturbed half-life in terms of our method; $g^{\mathrm{eff}}_{A,2\nu}(\mathrm{ld};\mathrm{pt})$ is intended to be used with the leading NME components, but we used the perturbed ones to see what half-life is obtained. It is

Table 2: Calculated and experimental half-lives of the $2\nu\beta\beta$ decay. The used components of the NME are shown in Table 1. The definition of $g^{\rm eff}_{A,2\nu}(\rm ld; pt)$ is the same as Eq. (5) but for the $2\nu\beta\beta$ NME. We used $g_A^{\rm bear}$ = 1.267 [6] and $g^{\rm eff}_{A,2\nu}(\rm ld; pt)$ = 0.696. The experimental half-life value was taken from Ref. [10].

Used GT and Fermi components	Used g_A	Half-life (10^{21} y)
Leading	$g_A^{\rm bear}$	0.03
Perturbed	$g_A^{\rm bear}$	0.26
Leading	$g^{\rm eff}_{A,2\nu}(\rm ld; pt)$	0.34
Perturbed	$g^{\rm eff}_{A,2\nu}(\rm ld; pt)$	4.77
Experimental		2.18

Table 3: Obtained $g_A^{\rm eff}$ by different methods, which are defined by the second through the fourth columns. The second column shows the decay mode, and the third column specifies the GT and Fermi NMEs used with the $g_A^{\rm eff}$. The half-life reproduced by the $g_A^{\rm eff}$ is specified in the fourth column; $T^{0\nu}_{1/2}(M^{\rm GT}_{0\nu}, M^{\rm F}_{0\nu}, g_A^{\rm bare})$ is the half-life calculated with those NME components and the g_A shown in the arguments. The same equation of the half-life but for the $2\nu\beta\beta$ decay is used for the $g_A^{\rm eff}$ in the second data row. For the explicit equations, see Ref. [4]. $T^{2\nu(\rm exp)}_{1/2}$ denotes the experimental half-life, and the value is shown in Table 2. The fifth column shows the values of the $g_A^{\rm eff}$ for SkM*. These results were taken from Ref. [4].

Specification of $g_A^{\rm eff}$	Decay	GT and Fermi NMEs	Half-life reproduced	$g_A^{\rm eff}$
$g^{\rm eff}_{A,0\nu}$(ld;pt)	$0\nu\beta\beta$	Leading	$T^{0\nu}_{1/2}(M^{\rm GT}_{0\nu}, M^{\rm F}_{0\nu}, g_A^{\rm bare})$	0.796
$g^{\rm eff}_{A,2\nu}$(ld;pt)	$2\nu\beta\beta$	Leading	$T^{2\nu}_{1/2}(M^{\rm GT}_{2\nu}, M^{\rm F}_{2\nu}, g_A^{\rm bare})$	0.696
$g^{\rm eff}_{A,2\nu}$(ld;exp)	$2\nu\beta\beta$	Leading	$T^{2\nu(\rm exp)}_{1/2}$	0.422
$g^{\rm eff}_{A,2\nu}$(pt;exp)	$2\nu\beta\beta$	Perturbed	$T^{2\nu(\rm exp)}_{1/2}$	0.806

seen that the effect of the perturbation is quite significant; the result shows the differences of order of magnitude. Our method still underestimates the half-life. The over-perturbed half-life is larger than the experimental value within the same order.

Table 3 shows four $g_A^{\rm eff}$ obtained in our study. One is for the $0\nu\beta\beta$ NME, and the others are for the $2\nu\beta\beta$ decay. In the latter, two phenomenological $g_A^{\rm eff}$ are included. The most important result is the $g_A^{\rm eff}$s for the $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs are close to each other;

$$g^{\rm eff}_{A,0\nu}(\rm ld; pd) \simeq g^{\rm eff}_{A,2\nu}(\rm ld; pt). \tag{6}$$

At the leading order, the two $g_A^{\rm eff}$ coincide;

$$g^{\rm eff}_{A,0\nu}(\rm ld; ld) = g^{\rm eff}_{A,2\nu}(\rm ld; ld) = g_A^{\rm bare}. \tag{7}$$

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This is seen by replacing the perturbed NME components on the right-hand side of Eq. (5) by the leading NME components. Thus, the effect of the perturbation on the closeness relation between $g^{\rm eff}_{A,0\nu}$ and $g^{\rm eff}_{A,2\nu}$ is weak. The $g^{\rm eff}_{A,0\nu}$ is no longer unknown at all.

In summary, we calculated the NME for the $0\nu\beta\beta$ and the $2\nu\beta\beta$ decays of ^{136}Xe using the transition operator perturbed by the residual nucleon-nucleon interaction. This method is applied for the two decay modes on the same footing. The correction terms for the NME consist of the vertex correction and the two-body current correction terms. Their sum is negative to the leading term, and the absolute value is nearly 50% (the $0\nu\beta\beta$ GT component). Due to this variation, the calculated half-life increases by an order of magnitude compared to that of the leading-order calculation. We have found that the perturbed $g^{\rm eff}_A$s for the two decay modes are close to each other. The possibility is inferred that the phenomenological $g^{\rm eff}_{A,2\nu}$ for the $2\nu\beta\beta$ NME can be used for the $0\nu\beta\beta$ NME. We are now in the crucial stage to solve the uncertainty problem of the $0\nu\beta\beta$ NME.

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