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Halo-independent bounds on the WIMP-nucleon couplings of long-range interactions from direct detection and neutrino observations

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We discuss the bounds on WIMP-proton and WIMP-neutron couplings of spin-independent and spin-dependent long-range interactions via massless mediator. We update the bounds in the Standard Halo Model for direct detection and the neutrino signal from WIMP annihilation in the Sun, and set halo-independent bounds using the single-stream method.

In the case of a massless mediator the capture rate in the Sun diverges and is regularized by removing the contribution of WIMPs locked into orbits that extend beyond the Sun-Jupiter distance. I discuss the dependence of the SHM bounds on the Jupiter cut showing that it can be sizable for a WIMP-proton coupling of a spin-dependent long-range interaction and a WIMP mass exceeding 1 TeV.

I show that the halo-independent analysis shows that mostly the relaxation of the bounds compared to the SHM is of the same order of that for contact interactions, relatively moderate in the low and high WIMP mass regimes and large for intermediate WIMP mass range. However, in the case of a WIMP-proton coupling of a spin-dependent long-range interaction, the relaxation of the bounds becomes not reliable at large WIMP mass range due to the sensitivity of the SHM capture rate in the Sun to low incoming WIMP speeds. In contrast, the halo–independent bounds are robust against the details of the velocity distribution including the Jupiter cut and the local escape speed.

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1. Introduction

Weakly Interacting Massive particles (WIMPs) are still one of the popular candidates of Dark Matter (DM) which is believed to make up around 25% of the density of the Universe. Although there are many evidences and candidates of DM, they have not been observed yet. Therefore world–wide effort is under way to detect WIMPs in the use of three main strategies to detect DM. One of them is direct detection (DD), which indicates the interactions between DM and Standard Model (SM) particles. The other one is indirect detection measuring signals from DM annihilation to SM particles, i.e. neutrino telescope (NT), and the final one can be done through accelerators observing DM creation.

Analyzing signals from DD and NTs, two major uncertainties arise: the WIMP–nucleus interaction and the WIMP speed distribution. In general, most WIMP–searching experiments make specific assumptions on these two aspects. Considering the WIMP–nucleus interaction, standard spin–independent (SI) / –dependent (SD) interactions are the most common choices. On the other hand, in terms of the speed distribution of WIMPs, analytic estimations and numerical simulations of Galaxy formation implies that the Standard Halo Model (SHM) is represented by Maxwellian. In addition, only zero–th–order approximation of the speed distributions is provided by the isothermal model. However, only statistical average properties of galactic halos can be identified through numerical simulations of Galaxy formation, and by the growing number of observed dwarf galaxies it is implied that our halo is not perfectly thermalized. In order to avoid this kind of problem, we applied Halo–independent (HI) approaches.

Halo–independent method provides the most conservative bounds with the only one constraint:

$$\int_{u=0}^{\infty} f(u)du = 1, \tag{1}$$

for any arbitrary speed distribution $f(u)$. In this approach, DD experiments have a lower threshold on speed, $u_{th}^{\rm DD}$, below which the signal vanishes due to a recoil energy threshold E_R^{th}. Therefore, in order to cover the full speed range, DD experimental results need to be combined with signals of NTs from WIMPs captured in the Sun, which are favored for low or even vanishing WIMP speed. This complementarity between two kinds of experiments was used in Ref. [1] to develop a "single–stream method" with the only constraint Eq. (1).

The applicability of the single–stream method is only valid with a single coupling. Based on that, we have discussed bounds on c_i^p and c_i^n obtained from the halo–independent single–stream method in both cases of elastic and inelastic interactions in Refs. [2, 3]. Compared to those analysis, the WIMP–nucleon Hamiltonian of long–range interactions can enhance sensitivity on the specific velocity distribution. Therefore, we discuss the bounds on long–range couplings $\alpha_{\rm SI}^{p,n}$, $\alpha_{\rm SD}^{p,n}$ with massless mediator from the combination of signals from DD and NT using five DD experimental data and measurements of three NTs to calculate capture rate of WIMPs in the Sun.

2. WIMP–nucleus scattering in the limit of a massless mediator based on the non–relativistic effective theory

Since WIMP is a kind of cold Dark Matter, the scattering process is non–relativistic (NR), where the WIMP–nucleon interaction can be parameterized with an effective Hamiltonian $\mathcal{H}$ with spin of WIMPs up to 1/2 as like Refs. [4, 5]. In particular, with momentum transfer $\vec{q}$, $\mathcal{H}$ of long–range interactions is written as:

$$\mathcal{H} = \sum_{\tau=0,1} \left(\frac{\alpha^{\tau}_{\rm SI}}{q^2 + M_0^2} 1_{\chi} 1_N t^{\tau} + \frac{\alpha^{\tau}_{\rm SD}}{q^2 + M_0^2} \vec{S}_{\chi} \cdot \vec{S}_N t^{\tau} \right), \tag{2}$$

with M_0 the mass of the mediator, which is considered as 0 in this work. In this equation, $1_{\chi(N)}$ and $\vec{S}_{\chi(N)}$ are identity operators and spins of WIMP (nucleon) while q is the transferred momentum. $t^0 = 1$, $t^1 = \sigma^3$ represent the 2×2 identity and third Pauli matrix in isospin base, while α^0_j and α^1_j (with j=SI, SD) are the isoscalar and isovector coupling constants related to couplings to protons (α^p_j) and neutrons (α^n_j) in the relation of $\alpha^0_j = \alpha^p_j + \alpha^n_j$ and $\alpha^1_j = \alpha^p_j - \alpha^n_j$. Based on this Hamiltonian, the WIMP–nucleus scattering amplitude is [4, 5]:

$$\begin{aligned} &\frac{1}{2j_{\chi}+1} \frac{1}{2j_T+1} |\mathcal{M}_T(q^2)|^2 = \\ &= \frac{4\pi}{2j_T+1} \sum_{\tau,\tau'=0,1} \left\{ \frac{\alpha^{\tau}_{\rm SI}\alpha^{\tau'}_{\rm SI}}{q^4} W^{\tau\tau'}_{TM}(q) + \frac{j_{\chi}(j_{\chi}+1)}{12} \frac{\alpha^{\tau}_{\rm SD}\alpha^{\tau'}_{\rm SD}}{q^4} \left[W^{\tau\tau'}_{T\Sigma'}(q) + W^{\tau\tau'}_{T\Sigma''}(q) \right] \right\}, \end{aligned} \tag{3}$$

where j_{χ} and j_T are spins of the WIMP and the target nucleus respectively, while the $W^{\tau\tau'}_{Tk}$'s (with $k = M, \Sigma', \Sigma''$) denote the nuclear response functions (nuclear form factors) defined in Refs. [4–6], where M represents standard SI interaction and a combination of Σ' and Σ'' indicates standard SD interaction. From this scattering amplitude, the differential cross section for the WIMP–nucleus scattering can be calculated with fixed recoil energy ($E_R = q^2/2m_T$) as:

$$\frac{d\sigma_T}{dE_R}(q^2) = \frac{2m_T}{4\pi w^2} \left[\frac{1}{2j_{\chi}+1} \frac{1}{2j_T+1} |\mathcal{M}_T(q^2)|^2 \right], \tag{4}$$

where m_T is the mass of the target nuclei and w is the incoming WIMP speed. This differential cross section is applied to both DD and capture in the Sun identically.

2.1 Direct detection rate

In a DD experiment, the number of expected events is given by:

$$R_{\rm DD} \;=\; M\tau_{\rm exp} \left(\frac{\rho_{\chi}}{m_{\chi}} \right) \sum_T N_T \int du\, f(u)\, u \int_{E_{R,th}}^{E_R^{\rm max}} dE_R\, \zeta_{exp} \frac{d\sigma_T}{dE_R}, \tag{5}$$

where ζ_{exp} includes experimental features such as energy resolution and the efficiency while $M\tau_{\rm exp}$ and N_T indicates exposure and the number of targets per unit mass in the detector. The function $f(u)$ represents the normalized WIMP–incoming speed distribution and ρ_{χ} is the local density of WIMP.

Due to the energy threshold of detectors, DD experiments requires larger speed than the value which corresponds to the momentum threshold. Therefore the integration over the energy in Eq. (5) for a given u does not diverge in the case of long–range interaction via a massless mediator (Eq. (2)).

2.2 Capture rate in the Sun

WIMPs with low speed can be captured in the Sun gravitationally, where they annihilate and deposit a neutrino flux observed by NTs. From the same differential cross section Eq. (4), capture rate is:

$$C_\odot = \left(\frac{\rho_\chi}{m_\chi}\right) \int du\, f(u)\, \frac{1}{u} \int_0^{R_\odot} dr\, 4\pi r^2\, w^2 \times \sum_T \rho_T(r)\, \Theta(u^C_{\max,T} - u) \int_{E^C_{\min}}^{E^C_{\max}} dE\, \frac{d\sigma_T}{dE_R}\,. \tag{6}$$

Here $w = \sqrt{u^2 + v_{\rm esc}^2(r)}$ is the boosted incoming WIMP speed at a distance r from the center of the Sun, where $v_{\rm esc}(r)$ is the local escape speed at r while $\rho_T(r)$ is the number density of target nuclei in the Sun.

When $u \to 0$, the capture rate based on the effective Hamiltonian (Eq. (2)) diverges because minimum energy of the integration range of Eq. (6) vanishes. Physically, this implies that a WIMP is captured and locked into a bound orbit, with the gravitational disturbances putting upper cut r_0 on the size of the bounded orbit. The capture rate is sensitive to r_0 and diverges for $r_0 \to \infty$ because the cross section may diverge at small q. Considering r_0, a WIMP with initial position r requires a minimum speed $v_e(r)^2 = v_{esc}(r)^2 - v_{esc}(r_0)^2$ to reach r_0. In this case, there is a minimum energy which a WIMP with speed u needs to lose:

$$E^{\rm C}_{\min} \to \frac{1}{2} m_\chi (u^2 + v_{esc}(r_0)^2) = \frac{1}{2} m_\chi u^2 + E^{\rm C}_{\rm cut}\,, \tag{7}$$

in order to be locked into a bound orbit with aphelion r_0. In this way $E^{\rm C}_{\min}$ does not vanish and the integral of Eq. (6) gives a finite result. During our analysis, we assume r_0 as the distance between the Sun and the Jupiter, corresponding to:

$$v_{\rm esc}(r_0) = v_{\rm cut} \simeq 18.5\,{\rm km/s}\,. \tag{8}$$

3. Halo–independent bounds with the single–stream method

To obtain halo–independent bounds, we calculate the number of expected events in a DD experiment (Eq. (5)) or the WIMP capture rate in the Sun (Eq. (6)) as:

$$R = \int_0^{u_{\max}} du\, f(u)\, H(u), \tag{9}$$

with $H(u)$ for DD,

$$H(u) = H_{\rm DD}(u) = M\tau_{\rm exp} \left(\frac{\rho_\chi}{m_\chi}\right) u \sum_T N_T \int_{E_{R,th}}^{E_R^{\max}} dE\, \zeta_T\, \frac{d\sigma_T}{dE_R}\,, \tag{10}$$

and for capture,

$$H(u) = H_C(u) = \left(\frac{\rho_\chi}{m_\chi}\right) \frac{1}{u} \int_0^{R_\odot} dr\, 4\pi r^2 w^2 \times \sum_T \rho_T(r)\, \Theta(E^{\rm C}_{\rm max} - E^{\rm C}_{\rm min}) \int_{E^{\rm C}_{\rm min}}^{E^{\rm C}_{\rm max}} dE\, \frac{d\sigma_T}{dE_R}\,. \tag{11}$$

Considering single WIMP–nucleon coupling α at a time, event rate from both D and NT is less than its experimental maximum value $R_{\rm max}$:

$$R = R(\alpha^2) = \int_0^{u_{\rm max}} du\, f(u) H(\alpha^2, u) \leq R_{\rm max}\,. \tag{12}$$

Due to the factorization, $H(\alpha^2, u) = \alpha^2 H(\alpha = 1, u)$, Eq. (12) can be re–expressed as:

$$R(\alpha^2) = \int_0^{u_{\rm max}} du\, f(u) \frac{\alpha^2}{{\alpha^2}_{\rm max}(u)} H({\alpha^2}_{\rm max}(u), u) = \int_0^{u_{\rm max}} du\, f(u) \frac{\alpha^2}{{\alpha^2}_{\rm max}(u)} R_{\rm max} \leq R_{\rm max}\,, \tag{13}$$

where $\alpha_{\rm max}(u)$ represents an upper limit on the the coupling at single speed stream u. And experimental maximum is:

$$H({\alpha^2}_{\rm max}(u), u) = {\alpha^2}_{\rm max}(u) H(\alpha = 1, u) = R_{\rm max}\,. \tag{14}$$

Using Eqs. (13 and 14) one can obtain the upper bound over whole streams of an arbitrary WIMP speed distribution $f(u)$ as:

$$\alpha^2 \leq \left[\int_0^{u_{\rm max}} du \frac{f(u)}{{\alpha^2}_{\rm max}(u)}\right]^{-1}. \tag{15}$$

In order for this calculation to be possible, experimental sensitivity should be able to cover the full WIMP speed range. Since DD experiments have a lower threshold $u^{\rm DD}_{\rm min}$ while NT experiments have maximum value $u^{\rm C}_{\rm max}$, the combination of both DD and NT is needed to cover the full range of WIMP speeds. During our analysis, we realized that the HI bounds can be calculated in two situations.

When the $\alpha_{\rm max}$ of NT and the DD intersect at $\tilde{u}$, we call that value of $\alpha_{\rm max}$ as $\tilde{\alpha}$ and use to find HI bounds. In the case where intersection happens, **Case I**, the HI bound on the coupling is:

$$\alpha^2 \leq 2\tilde{\alpha}^2 = \alpha^2_{\rm HI}. \tag{16}$$

On the other hand, in **Case II** where $\tilde{\alpha}^2 < (\alpha^{\rm DD})^2_{\rm max}(u)$ even though intersection happens, the HI bound is described as:

$$\alpha^2 \leq (\alpha^{\rm DD})^2{}_{\rm max}(u_{\rm max}) + \tilde{\alpha}^2 = \alpha^2_{\rm HI}. \tag{17}$$

This situation occurs because with increasing u, due to the suppression of the nuclear form factor at large momentum transfer q or finite experimental energy bin, DD experiments loose its sensitivity. HI bounds obtained through **Case II** becomes sensitive to the choice of $u_{\rm max}$. In order to investigate this effect, we repeat calculations with a larger value of $u_{\rm max}$ = 8000 km/s.

We calculate HI upper bounds at each m_χ following either Eq. (16) or Eq. (17) combining one DD experiment with one NT data. As we analyze more than one DD and NT, such calculation is repeated for each combination of DD and NT and we take the most constraining value.

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4. Analysis

First we consider the sensitivity on the specific behavior of velocity distribution when $u \to 0$ especially for the capture in the Sun. To justify ourselves adopting r_0 as distance from the Sun to the Jupiter, we varied the $v_{\rm cut}$ by multiplying or dividing by a factor of two. As shown as blue band in Fig. (1), this effect does not affect much on the HI bounds combining DD and NT because such sensitivity only lies at small u range while HI bounds are obtained at rather higher value of speed. As expressed above Eqs. (16 and 17), HI bounds might be sensitive to the value of u_{max} so that we repeated this calculation with larger value. This situation is represented in Fig. (1) as dotted lines leading to conversion from **Case I** to **Case II** especially at large m_χ region in case of $\alpha^{p,n}_{SI}$ and α^n_{SD}, while this happens at small m_χ region for α^p_{SD}.

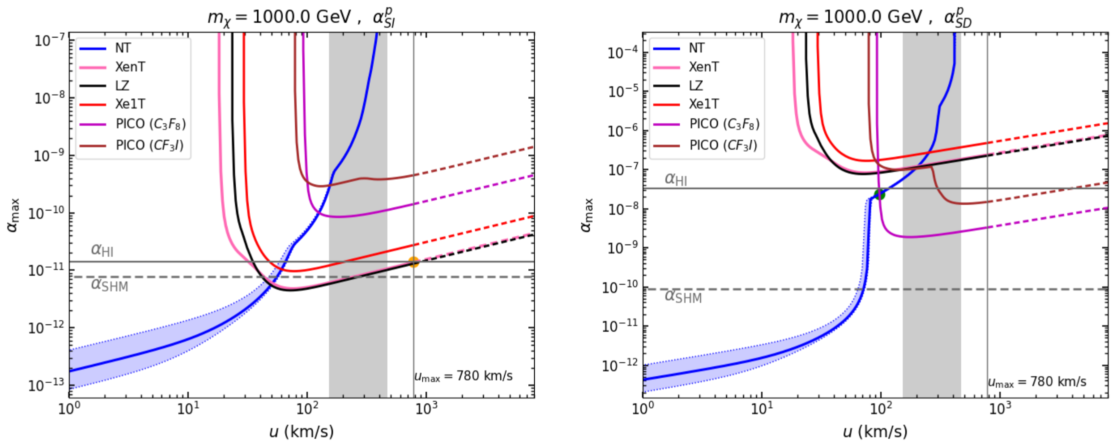

Figure 1: Examples of $\alpha_{\rm max}$ as a function of the WIMP speed u for different DD and NT experiments at m_χ = 1 TeV. The gray shaded region represents the range of u for which $\int f_{\rm MB}(u)du \simeq 0.8$.

Our results are shown in Fig. (2), where the black solid line represents HI limit for each couplings using $u_{\rm max}$=780 km/s. As mentioned above, we repeat the calculation with larger value of $u_{\rm max}$=8000 km/s indicated by red dotted line. Even with such unrealistic value, this effect is rather mild (only by a factor $\lesssim$ 2-3). When $u_{\rm max}$ becomes large, the situation is described by **Case II**, where α_{max} is released. This happens due to the suppression of the nuclear form factor at large momentum transfer or finite experimental energy bin of detectors. As shown in Fig. (2), HI bounds have similar shape except α^p_{SD}, where the most contributing targets for DD and NT are hydrogen and fluorine respectively. $u^C_{{\rm max},{}^1H}$ drops below $u^{DD}_{th,{}^{19}F}$ and the complementarity between them is lost. The next–leading target is ^{14}N, whose contribution is more than 6 orders of magnitude smaller. This loss of sensitivity explains this steep increase.

To understand an effect of HI bounds comparing to that from SHM, we introduce relaxation factor:

$$r_f^2 = \frac{\alpha^2_{\rm HI}}{(\alpha^{\rm exp}_{\rm SHM})^2} = \alpha^2_{\rm HI}\int_0^{u_{\rm max}} du \frac{f_M(u)}{(\alpha^{\rm exp})^2_{\rm max}(u)} = \alpha^2_{\rm HI}\left\langle \frac{1}{(\alpha^{\rm exp})^2_{\rm max}}\right\rangle \simeq \alpha^2_{\rm HI}\left\langle \frac{1}{(\alpha^{\rm exp})^2_{\rm max}}\right\rangle_{\rm bulk}. \quad (18)$$

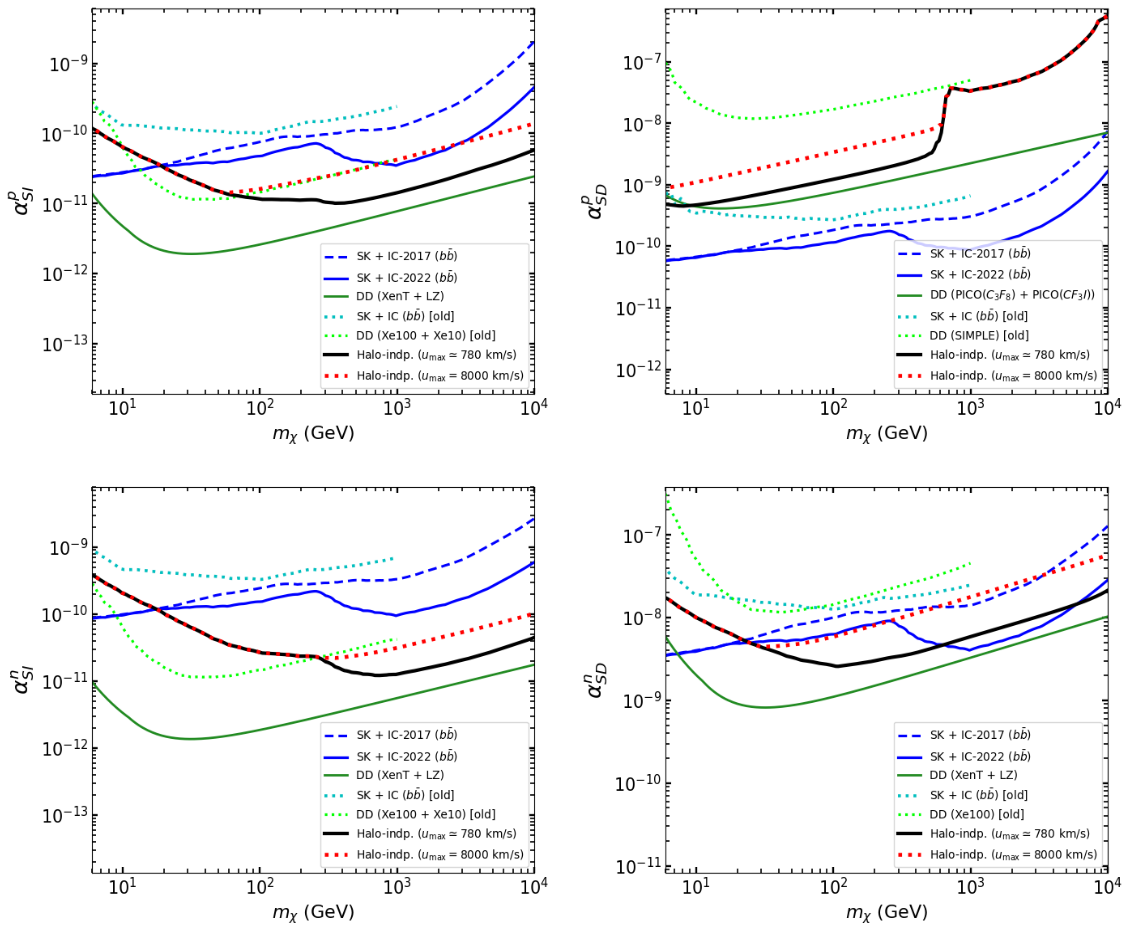

Figure 2: $\alpha^{p,n}_{SI,SD}$ as a function of the WIMP mass m_χ. HI bounds are represented in black solid lines or red dotted lines where bounds assuming SHM are plotted separatedly with different DD and NTs.

To compare the effect of HI approach, we provide relaxation factors of both cases of contact interactions and long–range interactions in Fig. 3. Through relaxation factors, the weakening of bounds from halo models different from SHM can be evaluated. Relaxation factors of long–range interactions have similar behavior to those of contact interactions because the speed interval which determines bounds is far from u close to zero. An exception of general pattern of relaxation factors is α^p_{SD} which is driven by capture. α^p_{SD} obtained based on SHM is driven by capture, which is quite sensitive to specific behavior of $f(u)$ when $u \to 0$. This implies that relaxation factor of α^p_{SD} becomes not reliable for $m_\chi > 1$ TeV. Therefore, we conclude that with the exception of $\alpha^p_{\rm SD}$ at large WIMP mass ($m_\chi \gtrsim 1$ TeV), the impact of HI approach on the bounds on couplings of long–range interactions is in the similar order to that of contact interaction cases.

5. Conclusions

In this work we discuss about HI bounds on SI and SD long–range interactions with massless mediator comparing to those obtained from SHM. In order to cover full WIMP incoming speed

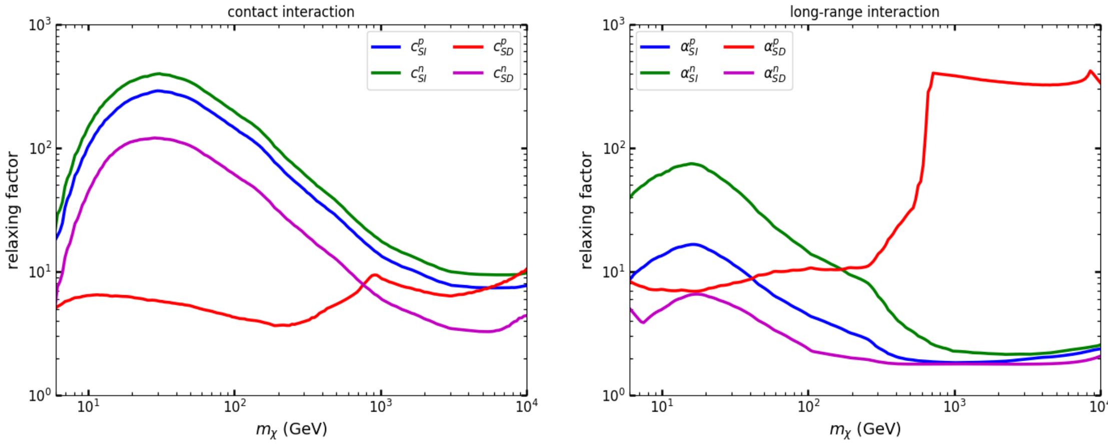

Figure 3: Relaxation factors defined in Eq. (18). Left–hand plot: contact interaction; right–hand plot: long–range interaction via massless propagator.

range, we combine DD experiments and capture in the Sun fixing the WIMP annihilation channel to $b\bar{b}$. We put an upper cut on r_0 to the Sun–Jupiter distance to avoid possible divergence of the energy integral in the calculation of capture rate. During this analysis, we find that the effect of choice of r_0 and $u_{\max}$ is rather mild reaching less than a factor of 2 or 3. Moreover, we analyze the sensitivity of the couplings on a WIMP speed distributions comparing HI bounds to bounds from SHM based on the single–stream method [1]. Through relaxation factors, we realize that with the exception of $\alpha^p_{\rm SD}$ at large m_χ, the effect of HI approach is in the similar order of that of contact interactions. On the other hand for $\alpha^p_{\rm SD}$ the relaxation factor becomes not reliable for $m_\chi \gtrsim 1$ TeV, because in this case the SHM bound is driven by capture in the Sun and it is very sensitive to the specific behavior of speed distribution. For further details, we refer to original paper of this proceeding Ref. [7].

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