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On the threshold behaviour of heavy top production

Torbjörn Sjöstrand[a,*]

[a] *Department of Physics, Lund University, Box 118, SE-221 00 Lund, Sweden*

E-mail: torbjorn.sjostrand@fysik.lu.se

The observation of an excess of $\mathrm{t\bar{t}}$ production in the threshold region, by CMS and ATLAS, has been interpreted as a toponium contribution, i.e. from below-threshold $\mathrm{t\bar{t}}$ virtual states. The news here is the nontrivial experimental extraction of such a signal, not its existence as such. Indeed, already 35+ years ago an NRQCD Green's function approach was used to model the above- and below-threshold production of $\mathrm{t\bar{t}}$ pairs in $\mathrm{pp/p\bar{p}}$ collisions. The relevant cross section equations from that study are now (re-)implemented in the Pythia 8 event generator. While the above-threshold part is straightforward, the physical interpretation and modelling of below-threshold events is nontrivial, and a final prescription is cross-checked against two simpler ones. Cross sections and some event properties are presented.

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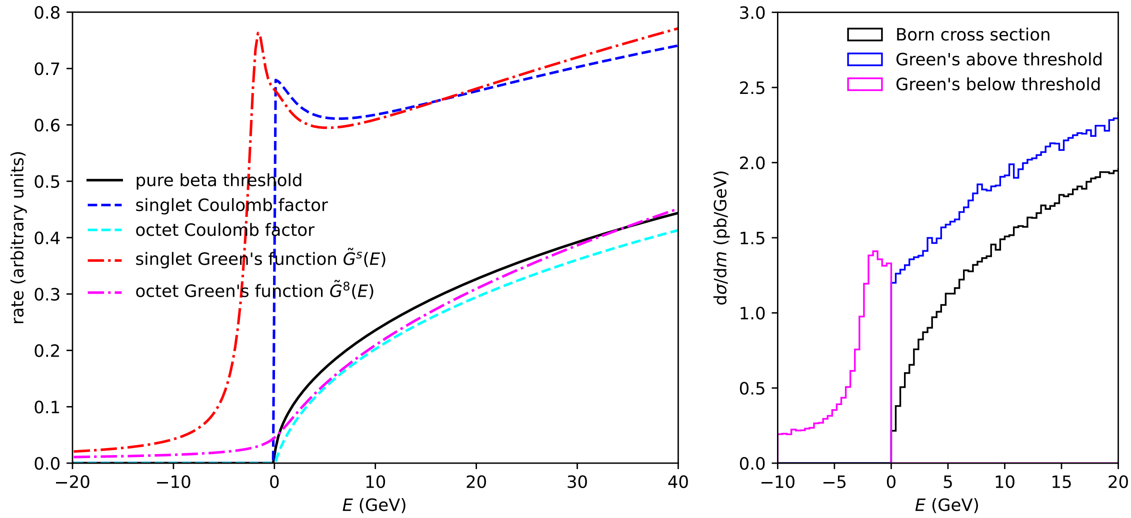


(a) *(b)*

Figure 1: The threshold behavior. *(a)* Analytical expressions for singlet and octet cases, $E = m_{\mathrm{t\bar{t}}} - 2m_{\mathrm{t}}$. *(b)* Cross section in a four-mass scenario simulation for the expected singlet/octet mix, $E = \hat{m} - m_{\mathrm{t}1} - m_{\mathrm{t}2}$.

This writeup is one of four dedicated to the study of the $\mathrm{t\bar{t}}$ threshold region, and notably the contribution from below-threshold $\mathrm{t\bar{t}}$ virtual states. The experimental background, i.e. the recent CMS [1] and ATLAS [2] observation of a cross section excess, is covered by Baptiste Ravina. Two examples of recent theoretical studies are presented by Benjamin Fuks [3] and Sven-Olaf Moch [4]. Here we revive what may be the first study of the heavy-top threshold region in $\mathrm{pp/p\bar{p}}$ collisions, from 1989/90 [5, 6], as a follow-up of the earlier $\mathrm{e^+e^-}$ study by Fadin and Khoze [7]. See also [8].

Relative to the Born-level $\mathrm{t\bar{t}}$ production cross section, a first extension is to consider the Coulomb effects of gluon exchange between the outgoing t and $\mathrm{\bar{t}}$. Multiple gluon exchanges can be resummed to an all-orders expression. This leads to an enhanced cross section when the $\mathrm{t\bar{t}}$ pair is in a colour singlet state, and a reduced one when in a colour octet state. Based on relative colour factors we will assume that $\mathrm{gg \to t\bar{t}}$ is 2/7 singlet and 5/7 octet, while $\mathrm{q\bar{q} \to t\bar{t}}$ is all octet. Higher-order corrections, e.g. from the emission of a gluon, could change these numbers.

The Coulomb expressions do not take into account the finite-top-width effects, nor the below-threshold states. Instead NRQCD Green's function expressions can be derived for the behaviour in the threshold region. These contain both continuum terms and an infinite (but rapidly convergent) sum over the virtual states, (pseudo)bound for the singlet case.

Newly encoded old formulae were compared with the 1990 results, for 140 and 200 GeV top masses, and agreement was found. In Fig. 1*a* results are shown for a current $m_{\mathrm{t}} = 172.5$ GeV value. Of note is that the Green's functions diverge above the threshold region. In our numerical studies, therefore a smooth transition to the more reliable Coulomb expression is done between 10 and 20 GeV above the threshold. In a full event generation setup, one may instead prefer to transition to an NLO description [9]. Correspondingly, the below-threshold contribution is damped to zero between −10 and −20 GeV.

The question now arises how to apply these equations in practice. For the above-threshold part the basic prescription is to generate a $\mathrm{t\bar{t}}$ pair according to an overall weight

$$\mathrm{BW(t)} \times \mathrm{BW(\bar{t})} \times \mathrm{PDF}(x_1) \times \mathrm{PDF}(x_2) \times \mathrm{ME} \times \mathrm{PS} \times (\tilde{G}(E)/\beta_{\mathrm{t}}) ,$$

where a standard accept/reject step is added to generate events with unit weight. Here BW denotes the Breit-Wigner mass distributions, PDF the parton distributions of the incoming gluons or quarks, ME the (squared) matrix elements for $\mathrm{gg} \to \mathrm{t\bar{t}}$ and $\mathrm{q\bar{q}} \to \mathrm{t\bar{t}}$ respectively, and PS the phase space of the production process. Note that the PS expression contains a factor

$$\beta_{\mathrm{t}} = \sqrt{\left(1 - \frac{m_{\mathrm{t1}}^2}{\hat{s}} - \frac{m_{\mathrm{t2}}^2}{\hat{s}}\right) - 4\,\frac{m_{\mathrm{t1}}^2}{\hat{s}}\,\frac{m_{\mathrm{t2}}^2}{\hat{s}}} ,$$

where m_{t1} and m_{t2} are the two BW-selected masses, while m_{t} is reserved for the on-shell value, and $\hat{s} = \hat{m}^2$ is the squared invariant mass of the $\mathrm{t\bar{t}}$ system. In the final factor, that takes us beyond the Born expression, the β_{t} is replaced by the rescaled Green's expression $\tilde{G}(E) = (4\pi/m_{\mathrm{t}}^2)\,\mathrm{Im}\,G(E)$, where $E = \hat{m} - m_{\mathrm{t1}} - m_{\mathrm{t2}}$. Note that the BW factors smear the threshold $E = 0$ away from being at a fixed $2m_{\mathrm{t}} = 345$ GeV to become an event-by-event number.

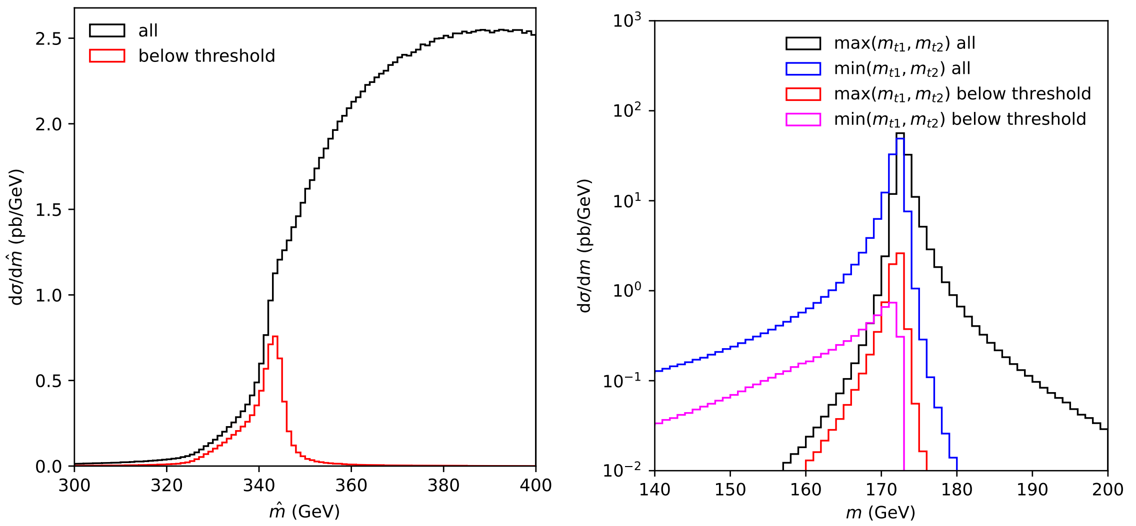
This approach does not work for $E < 0$, where formally phase space is vanishing. But a below-threshold state is subject to the weak decays of the t and $\mathrm{\bar{t}}$. From the final state a theorist's detector would reconstruct some m'_{t1} and m'_{t2}, where $m'_{\mathrm{t1}} + m'_{\mathrm{t2}} < m_{\mathrm{t1}} + m_{\mathrm{t2}}$ so that $E' = \hat{m} - m'_{\mathrm{t1}} - m'_{\mathrm{t2}} > 0$. Therefore the $E < 0$ states are never detectable as such, but show up as a deformation of the BW distributions, lowering the average mass. We will strive towards such an implementation, via two simpler models that will act as sanity checks.

The mirror solution is very simple: mirror the $E < 0$ contribution, so that events with $E > 0$ get a total weight $(\tilde{G}(E) + \tilde{G}(-E))/\beta_{\mathrm{t}}$. This is a robust Monte Carlo procedure that works for a poor experimental $\mathrm{t\bar{t}}$ mass resolution. The drop of the PDFs with x gives too low an integrated cross section, however, and the BW distribution is not deformed.

In the mass shift solution two separate runs are needed, one with $\tilde{G}(E)/\beta_{\mathrm{t}}$ and another with $\tilde{G}(-E)/\beta_{\mathrm{t}}$, where the latter is with a shifted $m_{\mathrm{t}} = 172.5 \to 168.7$ GeV. Thereby $\langle E \rangle = +3.82 \to -3.90$ GeV for the $\tilde{G}(-E)$ part relative to the $m_{\mathrm{t}} = 172.5$ GeV threshold, so that average PDF weights work out, but some kinematics distributions are still off.

The preferred four-mass hybrid solution is more tricky. In it the m_{t1} and m_{t2} masses are first picked. Phase space is then sampled in the extended region $\hat{m} > m_{\mathrm{t1}} + m_{\mathrm{t2}} - 20$ GeV, i.e. as far below threshold as $\tilde{G}(E)$ is non-vanishing. If the chosen $\hat{m}$ gives an $E > 0$ then the normal $\tilde{G}(E)/\beta_{\mathrm{t}}$ weight is applied. If not, two new masses $m'_{\mathrm{t1}} < m_{\mathrm{t1}}$ and $m'_{\mathrm{t2}} < m_{\mathrm{t2}}$ are selected according to the normal Breit-Wigners, repeatedly until $E' = \hat{m} - m'_{\mathrm{t1}} - m'_{\mathrm{t2}} > 0$. Then a hybrid weight $\tilde{G}(E)/\beta'_{\mathrm{t}}$ is applied, i.e. with the original $E < 0$ but the new β'_{t} based on m'_{t1} and m'_{t2}. Overall this gives a good rendering of the desired $\tilde{G}(E)$, Fig. 1*b*, although a small discontinuity at $E = 0$ indicates that the match is not perfect. The $\langle E \rangle = -4.24$ GeV for the $E < 0$ part lines up well with the previous numbers.

The integrated cross section for the $E < 0$ part is 5.70, 6.27 and 6.76 pb, respectively, for the three scenarios above. The first number clearly is too low owing to the too high $\langle x \rangle$ values. The difference between the last two could partly come from a further fine-tuning of the $\hat{m}$ spectrum, but also partly from the above-mentioned small discontinuity at $E = 0$. An intermediate value of 6.5 pb could be compared with the experimental signals $8.8^{+1.2}_{-1.4}$ pb for CMS and 9.0 ± 1.3 pb for ATLAS. It is important to remember, however, that a realistic comparison would need to consider



(a) *(b)*

Figure 2: Example of event properties for the $\mathrm{t\bar{t}}$ pair mass range $300 < \hat{m} < 400$ GeV. *(a)* Cross section $\mathrm{d}\sigma/\mathrm{d}\hat{m}$. *(b)* Cross section for the heavier and lighter of the t and $\bar{\mathrm{t}}$, respectively.

the theoretical and experimental handling of the whole threshold region. On the former side, the choice of PDF, of α_s, of transition away from the Green's function near $E = 0$, and so on, also matter. And, most interestingly, whether higher-order effects could affect the colour singlet/octet composition assumed here, which likely would act in favour of enhancing the colour-singlet fraction [6, 8]. In the all-octet extreme the 6.76 pb would drop to 2.40 pb, and in the all-singlet one rise to 21.5 pb, with large changes also for $E > 0$.

Examples of other event properties are shown in Fig. 2, with BW smearing fully taken into account. The $E < 0$ contribution dominates for low $\hat{m}$, as should be expected. But the much larger $E > 0$ event rate means that its BW fluctuations still give a significant contribution at low $\hat{m}$. Of note is also that the sharp peak structure of Fig. 1 is smeared out and lost in the overall picture. For the individual t and $\bar{\mathrm{t}}$ masses, $m_{\mathrm{t}1}$ and $m_{\mathrm{t}2}$ above threshold and $m'_{\mathrm{t}1}$ and $m'_{\mathrm{t}2}$ below it, obviously the latter two have suppressed high-mass tails. But the heavier of $m'_{\mathrm{t}1}$ and $m'_{\mathrm{t}2}$ stays close to on-shell, while the lighter alone can be forced well off-shell, a familiar outcome when Breit-Wigners are involved.

In the 1989 study the below-threshold contribution was only integrated numerically, i.e. no full event generation was attempted. Therefore only the Coulomb expressions were presented in the public code. When Pythia 8 [10] was written the experimentalists had turned to NLO codes for the $\mathrm{t\bar{t}}$, and so only the simple Born cross sections were carried over. In the recent 8.316 release the scenarios studied here are made available, with a set of free parameters, such as the singlet/octet composition. Also do recall that Pythia does generate complete final states starting from the production process in focus here, with further free parameters.

There is one important catch: the top quarks are assumed unpolarized and uncorrelated at production, and only the intermediate W spin is taken into account in the $\mathrm{t} \to \mathrm{bW}^+ \to \mathrm{b}\ell^+\nu_\ell$ (or $\to \mathrm{bq\bar{q}}'$) decay chain. In the threshold region a pseudoscalar spin state should dominate, which thus is missed. There is room for improvement: a user hook in the code allows you to repeatedly generate new decay angle configurations, assuming you have relevant matrix element code for an accept/reject step. Alternatively some external program could be delegated to handle the decays.

In summary, this note described the re-implementation of NRQCD expressions for the $t\bar{t}$ production in the threshold region, including the contribution from states below the threshold. In spite of its 35+ years age, it may still serve as a useful reference. A new feature is the generation of below-threshold events, which appears to work reasonably well. The code is made available in Pythia 8.316.

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References

[1] CMS collaboration, *Observation of a pseudoscalar excess at the top quark pair production threshold*, Rept. Prog. Phys. **88** (2025) 087801 [2503.22382].

[2] ATLAS collaboration, *Observation of a cross-section enhancement near the ttbar production threshold in root-s = 13 TeV pp collisions with the ATLAS detector*, ATLAS-CONF-2025-008.

[3] B. Fuks, K. Hagiwara, K. Ma and Y.-J. Zheng, *Simulating toponium formation signals at the LHC*, Eur. Phys. J. C **85** (2025) 157 [2411.18962].

[4] M.V. Garzelli, G. Limatola, S.O. Moch, M. Steinhauser and O. Zenaiev, *Updated predictions for toponium production at the LHC*, Phys. Lett. B **866** (2025) 139532 [2412.16685].

[5] V.S. Fadin, V.A. Khoze and T. Sjostrand, *ON THE THRESHOLD BEHAVIOR OF HEAVY TOP PRODUCTION*, in *24th Rencontres de Moriond: New Results in Hadronic Interactions*, pp. 19–32, 1989.

[6] V.S. Fadin, V.A. Khoze and T. Sjostrand, *On the Threshold Behavior of Heavy Top Production*, Z. Phys. C **48** (1990) 613.

[7] V.S. Fadin and V.A. Khoze, *Threshold Behavior of Heavy Top Production in e+ e- Collisions*, *JETP Lett.* **46** (1987) 525.

[8] V.S. Fadin and V.A. Khoze, *Production of a pair of $t\bar{t}$ quarks near threshold*, *Sov. J. Nucl. Phys.* **53** (1991) 692.

[9] P. Nason, S. Dawson and R.K. Ellis, *The Total Cross-Section for the Production of Heavy Quarks in Hadronic Collisions*, Nucl. Phys. B **303** (1988) 607.

[10] C. Bierlich et al., *A comprehensive guide to the physics and usage of PYTHIA 8.3*, SciPost Phys. Codeb. **2022** (2022) 8 [2203.11601].