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Developments in collinear and TMD heavy boson probability density functions

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Parton probability density functions are derived via a DGLAP-type equation involving QCD and electroweak terms. The Parton-Branching (PB) method is employed in order to derive the solution and the evolution is obtained at leading order (LO) for electroweak bosons and next-to-leading order (NLO) for QCD partons.
This novel solution is performed through an initial fit of QCD parton density functions to HERA deep inelastic scattering (DIS) data, along a perturbative generation of electroweak boson densities. Finally, the obtained densities in both collinear (PDF) and transverse-momentum dependent (TMD) form are provided in a parametrised form for phenomenological studies and its implementation in standard HEP libraries.

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1. Introduction

Theoretical studies in high-energy physics and especially in hadronic processes, heavily depend on the treatment of parton densities due to their perturbative contribution from the evolution equations and non-perturbative term at the hadronic mass scale. Nowadays, both QCD parton and photon densities are already determined, but heavy boson densities are not yet available in parametrised form, nor they are implemented in HEP libraries such as LHApdf or TMDlib. Therefore, the current work derives a complete set of parton densities for QCD partons and electroweak (EW) bosons: γ, Z^0 and $W^\pm$, from a solution of the extended DGLAP equation where Parton Branching (PB) method [1] includes both the QCD partons and the EW bosons contributions.

2. Evolution equations for QCD partons and electroweak bosons

The evolution of parton densities is described by the DGLAP equation, where the splitting function can be decomposed in terms of resolvable and non-resolvable contributions (in the notation of Ref. [1]). In the PB method, this decomposition is rewritten using Sudakov form factors, which replace the plus-prescription and allow a probabilistic interpretation, as discussed in Ref. [1]. In the current approach, any initial photon contribution is neglected since the focus is the contribution at large scales and the consistent treatment of photon and heavy boson densities. The determination of effective $W^\pm$ densities has been already extensively discussed and more recently revisited in Ref. [2].

The effective coupling α_{eff} for photons is $\alpha_{eff} = \alpha_{em}$, whereas for processes where the Z^0 boson or the $W^\pm$ boson are involved, it is obtained from the total production $q_i\bar{q}_j$-cross section, and is given by: $\mathrm{Z} : \alpha_{eff} = \frac{\alpha_{em}}{4\sin^2\theta_{\mathrm{W}}\cos^2\theta_{\mathrm{W}}}(V_f^2 + A_f^2)$ $\quad \mathrm{W} : \alpha_{eff} = \frac{\alpha_{em}|V_{qq}|^2}{4\sin^2\theta_{\mathrm{W}}}$, respectively with θ_W being the Weinberg angle and V_f, A_f being the vector and axial couplings of the Z-boson to the fermions, and V_{qq} being the CKM element.

In the massless limit, the splitting functions for EW topologies at LO are matched to the corresponding ones of QCD partons modulo the coupling and different colour factors, and the self-coupling, which in the case of the photon, does not exist. An average is performed over all transverse polarization states, and any longitudinal contribution from the EW bosons is neglected, since this polarisation information is only needed for dedicated validation studies (see talk slides). A full account, including different polarization states, is given in Ref. [2].

The momentum weighted density for photons, Z- and W-bosons are:

$$\mu^2 \frac{\partial x f_\gamma(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{em}}{2\pi}\int_x^1 \frac{dz}{z}\left(1+(1-z)^2\right)\sum_{u,d} e_i^2 \left[\frac{x}{z}F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right] \tag{1}$$

$$\mu^2 \frac{\partial x f_{\mathrm{Z}}(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{em}}{8\pi\sin^2\theta_{\mathrm{W}}\cos^2\theta_{\mathrm{W}}}\int_x^1 \frac{dz}{z}\left(1+(1-z)^2\right)\sum_{u,d}\left(V_i^2 + A_i^2\right)\left[\frac{x}{z}F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right] \tag{2}$$

$$\mu^2 \frac{\partial x f_{\mathrm{W}^-}(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{em}}{8\pi\sin^2\theta_{\mathrm{W}}}|V_{qq}|^2\int_x^1 \frac{dz}{z}\left(1+(1-z)^2\right)\frac{x}{z}\left[D\left(\frac{x}{z},\mu^2\right) + \bar{U}\left(\frac{x}{z},\mu^2\right)\right] \tag{3}$$

$$\mu^2 \frac{\partial x f_{\mathrm{W}^+}(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{em}}{8\pi\sin^2\theta_{\mathrm{W}}}|V_{qq}|^2\int_x^1 \frac{dz}{z}\left(1+(1-z)^2\right)\frac{x}{z}\left[U\left(\frac{x}{z},\mu^2\right) + \bar{D}\left(\frac{x}{z},\mu^2\right)\right] \tag{4}$$

where the quark densities $f_i(x, Q^2)$ are combined as follows:

$$xF_u = xU = xu + xc, x\bar{F}_u = x\bar{U} = x\bar{u} + x\bar{c}, \quad xF_d = xD = xd + xs, x\bar{F}_d = x\bar{D} = x\bar{d} + x\bar{s} \tag{5}$$

Additionally, the evolution equation for collinear densities includes transverse momenta of the partons (and bosons). Besides, the transverse momenta of the emitted partons can be calculated, as detailed in Ref. [1], where the evolution scale is paired to a physical scale.

2.1 Mass treatment for Heavy Bosons

The three mass-treatment scenarios (referred to as MassCutSchemes), are summarised as follows:

- MassCutScheme=0: In this scenario, the Zero Mass Variable Flavour Number Scheme (ZMVFN), the heavy boson mass M_V acts as a cut-off, similarly to the Effective Vector Boson Approximation (EWA) approach presented in [3]: $\alpha_{eff} \equiv \alpha_{eff} \cdot \Theta\left(q^2 - M_V^2\right)$
- MassCutScheme=1: This scheme is considered the default scenario and it is based on the introduction of an additional term. Here, measurements of charged-current cross sections in lepton-proton collisions result in a non-zero cross section in the energy range $Q^2 < M_{\mathrm{W}}^2$. This encourages the inclusion of a suppression factor (similar to the treatment in the DIS cross section) [2]: $\alpha_{eff} \equiv \alpha_{eff} \left[\frac{q^2}{q^2+M_V^2}\right]^2$.
- MassCutScheme=2: Finally, this scheme is based on angular ordering. This introduces the boson mass M_V through a limit in the z integrals (and therefore also in α_{eff}) [4]: $\alpha_{eff} \equiv \alpha_{eff} \cdot \Theta\left(q^2(1-z)^2 - M_V^2\right)$ and $z_M = 1 - \frac{M_V}{q}$

All scenarios consider the splitting functions in the massless limit. However, in the first two cases, $z_M \to 1$, while in MassCutScheme=2 the z-integration is limited. These three distinct schemes result in different collinear and notably, transverse momentum dependent distributions.

3. QCD fits of initial parton distributions

The initial collinear and TMD distributions are determined in the PB-method through fits to precision DIS cross-section data from HERA [5], as detailed in Ref. [6]. Having said that, including photons and electroweak bosons requires a new determination of the initial parameters, since additional radiation of photons and heavy bosons off quarks will imply a reduction in the estimation of quark densities. Therefore, new fits of the initial parameters are performed via the xFitter package [7] to obtain a complete new set of QCD and EW parton densities following the same methodology as in Ref. [6]. The same parametrisation form is implemented for the QCD starting distributions, same starting scale and heavy flavour masses. Furthermore, including photon and heavy-boson evolution slightly increases χ^2 compared to the original pure-QCD fit.

Uncertainties of the distributions are obtained, as described in Ref. [6]: the Pumplin method [8] is applied to estimate both statistical and systematic uncertainties. Model uncertainties are obtained by new fits with changed heavy quark masses, starting energy scale and the minimum propagating parton virtuality. In the case of the electroweak densities, uncertainties with the highest impact are those from the quark distributions at large x along with the statistical and systematic contributions from both the measurements and the modelling of QCD partons.

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3.1 Collinear parton densities

The collinear densities for the up quark, the gluon and the photon are shown in Fig. 1 at $\mu = 100$ GeV. Here, the experimental (red) and model (blue) uncertainties are shown separately. In the photon density, the prediction from MSHT20 [9] (total and inelastic) is included for comparison.

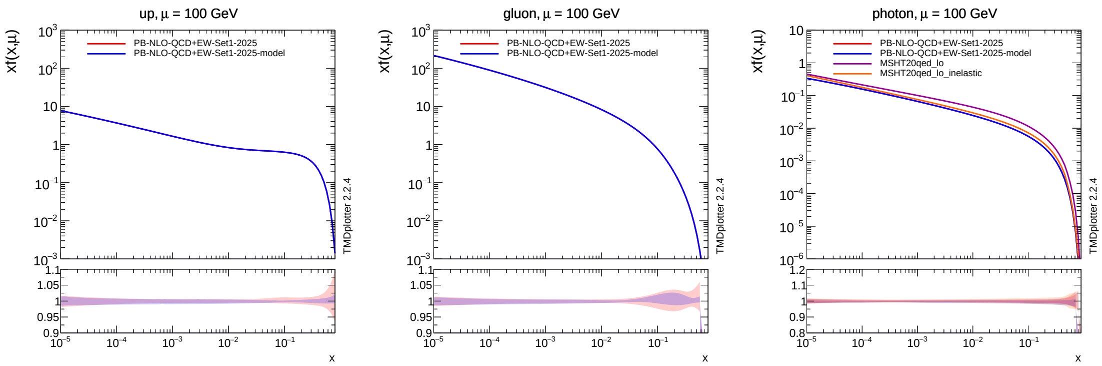

Figure 1: The collinear up quark (left), gluon (centre) and photon density (right) for PB Set1 at $\mu = 100$ GeV as a function of x. For comparison, in the photon distribution the prediction from MSHT20 [9] (total and inelastic) are shown and experimental and model uncertainties are included.

The collinear densities for Z^0 and $W^{\pm}$ are presented in Fig. 2 using MassCutScheme=1, where the lower panels include the uncertainties similarly to the QCD partons and photon distributions in Fig. 1. When comparing the different MassCutSchemes, only MassCutScheme=1 results in a non-zero heavy boson density at low energy scales μ, while in the other schemes there is either a mass cut or even a rather sharp threshold in k_T around the heavy boson mass, as is the case for MassCutScheme=2.

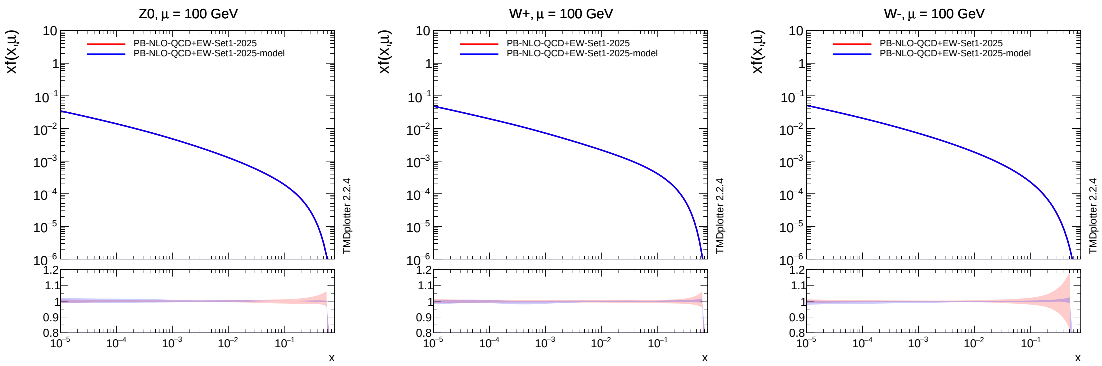

Figure 2: The collinear heavy boson densities for PB Set1 at $\mu = 100$ GeV as a function of x including experimental and model uncertainties.

4. Conclusion

An unified determination of QCD and electroweak parton densities is presented solving an extended DGLAP evolution via the PB method. This enables the simultaneous determination of PDF and transverse momentum dependent densities for both QCD partons and EW bosons.

NLO collinear splitting functions are implemented for QCD partons and LO splitting functions and couplings are employed for EW bosons. The initial distributions for quarks and gluons are fitted to DIS data from HERA. Including EW boson contributions neither dramatically modifies the fit parameters nor its quality. Additionally, there are ongoing validation efforts oriented to match the consistency of the EW boson densities with respect to DIS measurements. This kind of studies clearly favours the treatment of the heavy boson mass as an additional term, as described in MassCutScheme=1, rather than a sharp mass cut or a limit the z-integration. Both the PDF and TMD densities for QCD partons as well as for EW bosons, are the first determinations directly fitted to DIS precision measurements and such densities are available in LHAPDF and TMDlib formats.

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