PROCEEDINGS OF SCIENCE

Dijets with a large rapidity separation as a probe for large extra dimension gravity in trans-Planckian eikonal regime

Anatolii Iu. Egorov,[a] **Victor T. Kim,**[a,*] **Viktor A. Murzin,**[a] **and Vadim A. Oreshkin**[a]

[a] *Petersburg Nuclear Physics Institute, NRC Kurchatov Institute, Gatchina, 188300, Russia*

E-mail: kim_vt@pnpi.nrcki.ru

Dijets with a large rapidity separation between jets serve as a probe for the gravity with large extra dimensions at proton colliders in the trans-Planckian eikonal regime, i. e., when $\sqrt{\hat{s}} \gg M_D \gg \sqrt{-\hat{t}}$. Here, $\hat{s}$ and $\hat{t}$ are the Mandelstam variables of the colliding parton-parton system, and M_D is the Planck mass scale in the space-time with n_D compactified extra dimensions. A relevant observable for this regime may be the cross section of high-mass ($M_{jj} \sim \sqrt{\hat{s}} \gg M_D$) dijet production with jets widely separated in rapidity. Then the standard model (SM) background should be calculated within the next-to-leading logarithmic (NLL) approximation of Lipatov-Fadin-Kuraev-Balitsky (BFKL) formalism of quantum chromodynamics (QCD) suitable for $\sqrt{\hat{s}} \gg \sqrt{-\hat{t}} \gg \Lambda_{\text{QCD}}$. The signal of the large extra dimension gravity alongside the NLL BFKL QCD background are estimated for the high-luminosity Large Hadron Collider (HL-LHC) and future colliders such as FCCpp and CEPC-SppC.

Open Workshop "QCD at the Extremes", 1-11 September 2025

*Speaker

ISSN 1824-8039. Published by SISSA Medialab.

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1. Introduction

Searches for large extra dimension gravity proposed by Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1] at collider energies can be promising in the trans-Planckian eikonal regime $\sqrt{\hat{s}} \gg M_D \gg \sqrt{-\hat{t}}$ [2, 3], where $\hat{s}$ and $\hat{t}$ are the Mandelstam variables of colliding parton-parton system and M_D is the Planck mass scale in the space-time with n_D compactified extra dimensions.

A relevant observable for this regime may be the cross section of high-mass ($M_{jj} \sim \sqrt{\hat{s}} \gg M_D$) dijet production with a large rapidity separation between jets. Then the standard model (SM) background should be calculated within the next-to-leading logarithmic (NLL) approximation of Lipatov-Fadin-Kuraev-Balitsky (BFKL) evolution formalism [4–8] of quantum chromodynamics (QCD) suitable for semi-hard scattering regime $\sqrt{\hat{s}} \gg \sqrt{-\hat{t}} \gg \Lambda_{\text{QCD}}$ within the next-to-leading logarithmic (NLL) approximation [9, 10].

Recent measurements of the cross sections of production of dijets with a large rapidity separation between jets [11, 12] at the Large Hadron Collider showed that a good description of the data was provided by the NLL BFKL approach [13, 14]. At the same time, predictions based on the well-established Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (DGLAP) equations [15–19] are quite different from the data for pairs of jets widely separated in rapidity. This is consistent with the idea that the DGLAP equations are suitable primarily for hard scattering kinematics regime ($\sqrt{s} \simeq Q \gg \Lambda_{\text{QCD}}$), where Q is the characteristic hard scale of the collision, and rapidity intervals between jets are not large.

Therefore, searches for new physics beyond the SM, e. g., the ADD gravity [1], at the high-luminosity Large Hadron Collider (HL-LHC) [20] and future colliders, such as FCCpp [21] and CEPC-SppC [22], by using dijets with a large rapidity separation should take into account new BFKL evolution dynamics for background estimates.

The ADD gravity model [1] was investigated at the LHC in proton-proton (pp) collisions with $\sqrt{s} = 8$ and 13 TeV in dilepton, diphoton, and dijet final states, focusing on the regime where the parton collision energy, $\sqrt{\hat{s}} \sim \sqrt{-\hat{t}}$, is below the assumed Planck scale M_D in the presence of large extra dimensions [23–29]. A lower bound on the gravity scale in the ADD model was established by the measurements at approximately $M_D \sim 7 \div 10$ TeV, depending on the specific model convention. These searches are based on the effective linearized gravity framework, formulated by Giudice, Rattazzi and Wells (GRW) [2], which leads to divergent tree-level diagrams due to integrating over Kaluza-Klein (KK) modes in the compactified extra dimensions. The ultraviolet cutoff is taken to be M_D. Naive dimensional analysis and unitarity considerations suggest that the effective ADD gravity theory may remain valid up to energies near or somewhat above the M_D scale [2]. In general, effective theories are expected to apply at energies much lower than the new physics scale, i.e., $\sqrt{\hat{s}} \ll M_D$. Therefore, when $\sqrt{\hat{s}} \sim \sqrt{-\hat{t}} \sim M_D$, as is the case in the LHC searches, interpreting the results requires an extra caution.

The trans-Planckian eikonal regime, proposed by GRW in Ref. [3], is an alternative kinematic scenario for probing ADD gravity. In this regime $\sqrt{\hat{s}} \gg M_D$, while the momentum transferred by the exchanged t-channel gravitons satisfies $\sqrt{-\hat{t}} \ll M_D$. The eikonalization process removes ultraviolet sensitivity [3], because it favors gravitons with large wavelengths, which are insensitive to ultraviolet local counterterms. Partons scattered in this regime produce dijets where the jets are widely separated in rapidity. Consequently, the dominant SM background for ADD gravity searches

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in the trans-Planckian eikonal kinematics comes from QCD interactions in the semi-hard regime.

In this talk based on Ref. [30], we present the results of the dijet production cross section calculation due to large extra dimension gravity in the ADD model in trans-Planckian eikonal regime and compare them with QCD background estimates performed using two approaches, accounting separately for DGLAP or BFKL evolution. The calculations are conducted under conditions expected at the HL-LHC and future colliders, such as FCCpp and CEPC-SppC, with collision energies up to 100 TeV. The calculations indicate that applying DGLAP dynamics at large rapidities in the semi-hard regime can cause an overestimation of the QCD background by several orders of magnitude, potentially leading to misinterpretation of experimental data and a missed chance to identify new physics.

2. Large extra dimension gravity in trans-Planckian eikonal regime and dijets with a large rapidity separation

The work by GRW Ref. [3] explores the possibility of parton scattering through multiple KK graviton exchanges in the trans-Planckian regime using the eikonal approximation, where $\sqrt{\hat{s}} \gg M_D \gg \sqrt{-\hat{t}}$. In this kinematic regime, the gravity signal is expected to manifest itself as an excess in the production of dijets with jets that are widely separated in rapidity. This approach resums an infinite series of graviton exchanges, valid when the momentum transfer is small compared to the center-of-mass energy. The summation of an infinite series of successive graviton exchanges results in the following expression for the parton-parton cross section (see for details formulas (18), (20) and (60) in Ref. [3]):

$$\frac{d\hat{\sigma}_{\text{eik}}}{d\Delta y} = \frac{\pi b_c^4 \hat{s} e^{\Delta y}}{(1+e^{\Delta y})^2} |F_D(x)|^2 \tag{1}$$

where b_c is a novel length parameter in case of extra dimensions, function $F_D(x)$ defines scattering amplitude in eikonal regime, $x = b_c\sqrt{\hat{s}}/\sqrt{1+e^{\Delta y}}$, and Δy is the separation in rapidity between the two jets of a dijet. In trans-Planckian ($\sqrt{\hat{s}}/M_D \gg 1$) eikonal ($-\hat{t}/\hat{s} \ll 1$) regime $\sqrt{\hat{s}}$ is equal to jet-jet invariant mass $M_{jj} = \sqrt{\hat{s}}$, and $-\hat{t}/\hat{s} = 1/(1+e^{\Delta y})$. Using the parton-parton cross section (1), one can convolute it with parton distribution functions (PDFs) to obtain the dijet production cross section in pp collisions due to multiple graviton exchanges. It is important to note that the cross section (1) is valid only in the regime where $-\hat{t}/\hat{s} \ll 1$, corresponding to large rapidity intervals ($\Delta y \gg 1$). How this expression matches the region of small Δy remains basically unclear.

From the signature of the expected signal (high-mass dijets with a large rapidity separation) it is clear that the dominant SM background arises from QCD dijet production. Accurate modeling and estimation QCD background is essential for identifying any excess due to new physics.

In this talk based on Ref. [30], we provide an estimation of the QCD background based on both DGLAP and BFKL evolution. The DGLAP-based estimation is performed as a convolution of PDFs evolving according to the leading logarithmic (LL) DGLAP equations with the leading order (LO) QCD matrix elements. Additionally, a DGLAP-based prediction is generated using the Monte Carlo (MC) event generator package PYTHIA8 [31]. The BFKL-based predictions are supplied at both LL and NLL accuracy. The BFKL calculations follow the methodology described in Refs. [13, 32–34]. An approach that systematically eliminates scale-setting ambiguities in NLL BFKL calculations by resummation of running coupling constant contributions was developed by

Brodsky, Fadin, Kim, Lipatov and Pivovarov (BFKLP) [35]. This method essentially generalizes the Brodsky, Lepage and Mackenzie (BLM) optimal scale-setting procedure [36] to the non-Abelian case. The BFKLP scales were precalculated exactly using the equation (40) from Ref. [32] as a function of $\sqrt{\hat{s}}$ and transverse momenta of jets forming a dijet system $p_{\perp 1}$ and $p_{\perp 2}$.

To search for ADD gravity in the trans-Planckian eikonal regime using dijet final states, one must select the most forward and most backward in rapidity jets, known as the Mueller-Navelet (MN) dijets [37], and apply the high-mass cut on the dijet mass, $M_{jj} > M_{jj\mathrm{min}}$. The selection procedure is as follows. At first, select all jets in an event with transverse momentum above $p_{\perp\mathrm{min}}$. Next, identify a single pair of jets with the largest rapidity separation in each event, the MN dijet. Finally, from these MN dijets, choose those with $M_{jj} > M_{jj\mathrm{min}}$. In this work we set $p_{\perp\mathrm{min}} = 20$ GeV. We select only MN dijets formed of jets with rapidity $|y| < y_{\mathrm{max}} = 4.7$, to match the geometry of the detectors, in particularly the CMS, at the LHC.

This study examines pp collisions at energies of $\sqrt{s} = 13$, 40 and 100 TeV, spanning from current LHC energies up to those expected for FCCpp and CEPC-SppC. Although the ADD signal to QCD background ratio is expected to improve with increasing $M_{jj\mathrm{min}}$, the cross sections diminish significantly at large $M_{jj\mathrm{min}}$. Thus, the choice of the high dijet mass cuts is based on the projected integrated luminosities of the HL-LHC, FCCpp and CEPC-SppC. Specifically, for $\sqrt{s} = 13$ TeV we consider $M_{jj\mathrm{min}} = 6$ and 9 TeV; for $\sqrt{s} = 40$ TeV, $M_{jj\mathrm{min}} = 9$ and 30 TeV; and for $\sqrt{s} = 100$ TeV, $M_{jj\mathrm{min}} = 30$, and 70 TeV. One can see the impact of these $M_{jj} > M_{jj\mathrm{min}}$ cuts in Sec. 4, which contains the results of our computations.

3. Numerical estimates

The ADD gravity signal cross section is computed by convolving the partonic cross section given in Eq. (1) with PDFs from NNPDF31_lo_as0130 [38], accessed through the LHAPDF framework [39]. The ADD model depends on two parameters: M_D and the number of extra dimensions n_D. This study considers two values, $n_D = 2$ and 6, covering the typical range. The values of Planck scale with extra dimensions, M_D, are chosen so that the signal cross section is roughly comparable to the QCD background at $\Delta y = 8.7$. The cross sections vanish near the kinematic limit $\Delta y = 9.4$, imposed by the rapidity cut $|y| < y_{\mathrm{max}} = 4.7$. Therefore, $\Delta y = 8.7$ is chosen as the largest Δy unaffected by the $|y| < y_{\mathrm{max}} = 4.7$ selection. Since the partonic cross section in Eq. (1) relies on the eikonal approximation with $\hat{s} \gg |\hat{t}|$, signal calculations are performed for $\Delta y > 3.25$ ($|\hat{t}|/\hat{s} < 0.04$). At smaller Δy values, the eikonal approximation may become less reliable. Therefore, the most accurate results using this approximation occur at the largest $\Delta y = 8.7$ ($|\hat{t}|/\hat{s} \approx 1.7 \cdot 10^{-4}$).

The QCD background within the DGLAP approximation is evaluated by either convolving LO QCD partonic cross sections with NNPDF31_lo_as0130 PDFs evolved through LL DGLAP evolution[1] or using the PYTHIA8 MC generator with the CP5 tune [41]. Originally PYTHIA8

[1]Note that a misleading notation is commonly used elsewhere. In that notation, "leading order (LO)" denotes calculations with convolution of LO partonic subprocess and leading logarithmic (LL) PDF. We are using here a more relevant notation LO+LL. This notation is clear when using an approximation, e. g., next-to-leading order (NLO) partonic subprocess convoluted with LL PDF: NLO+LL DGLAP in case of using MC-generator POWHEG BOX [40] to describe dijet production at CMS [11].

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performed at LO+LL DGLAP accuracy as well. However, the CP5 tune updates it by using a NLO strong coupling constant and NNLO PDFs, along with rapidity ordering in the initial-state radiation. This rapidity ordering makes behavior of PYTHIA8 predictions closer to the BFKL kinematics, improving the modeling of gluon emissions and parton evolution for high-energy collisions [42]. The validity of these adjustments require careful theoretical consistency checks. Jets in the PYTHIA8 calculations are reconstructed using the anti-k_T algorithm [43] implemented in the FASTJET package [44], with a jet size parameter of 0.4, consistent with standard practice in LHC measurements at $\sqrt{s} = 13$ TeV.

The QCD background under the BFKL approximation is computed as outlined in Sec. 2, providing both LL and NLL BFKL predictions. For the LL BFKL approximation, NNPDF31_lo_as0130 PDFs are employed, while the NLL BFKL calculations use MSTW2008nlo68cl [45] PDFs. In the NLL BFKL corrections, the k_T algorithm of jet reconstruction is applied with a jet size of 0.4, following the procedure detailed in Ref. [46].

4. Results and discussion

The results for the cross section of dijets with a large rapidity separation between jets (MN dijets) in the NLL BFKL approximation [13, 14, 30] in pp collisions at $\sqrt{s} = 13$, 40 and 100 TeV are shown in Fig. 1. The calculations are presented with and without dijet mass selection $M_{jj} > M_{jj\text{min}}$ in order to illustrate the influence of the high dijet mass, M_{jj}, selection.

The ADD large extra dimension gravity cross section alongside the MN dijet production with the high dijet mass selections $M_{jj} > M_{jj\text{min}}$ in pp collisions at $\sqrt{s} = 13$, 40 and 100 TeV are shown in Figs. 2, 3, and 4, respectively.

Calculation of the signal in the trans-Planckian eikonal regime [2, 3, 30] for the ADD gravity with large extra dimensions [1] is presented for several parameter choices. The number of extra dimensions considered are $n_D = 2$ and 6, reflecting typical values studied in this theoretical framework. This allows for exploration of how the cross sections and phenomenology depend on the dimensionality of the extra spatial dimensions. The examined values of the Planck scale in the presence of extra dimensions are as follows:

- $M_D = 1.5$ and 3 TeV for $\sqrt{s} = 13$ TeV with $M_{jj} > 6$ and 9 TeV (Fig. 2);
- $M_D = 1.5$ and 3 TeV for $\sqrt{s} = 40$ TeV with $M_{jj} > 9$ TeV (Fig. 3a);
  $M_D = 10$ and 15 TeV for $\sqrt{s} = 40$ TeV with $M_{jj} > 30$ TeV (Fig. 3b);
- $M_D = 10$ and 15 TeV for $\sqrt{s} = 100$ TeV with $M_{jj} > 30$ TeV (Fig. 4a);
  $M_D = 30$ and 40 TeV for $\sqrt{s} = 100$ TeV with $M_{jj} > 70$ TeV (Fig. 4b).

The predictions of the ADD model exhibit less sensitivity to the number of extra dimensions n_D and are more strongly impacted by the value of the scale M_D. The reduced sensitivity of ADD model predictions to the number of extra dimensions n_D can be understood as follows. The change of n_D at a fixed scale M_D alters the size of the extra dimensions, which remain large enough for the particles to be insensitive to this change thanks to the vast difference between particle interaction scales and the scale corresponding to the size of extra dimensions. In contrast, varying M_D directly

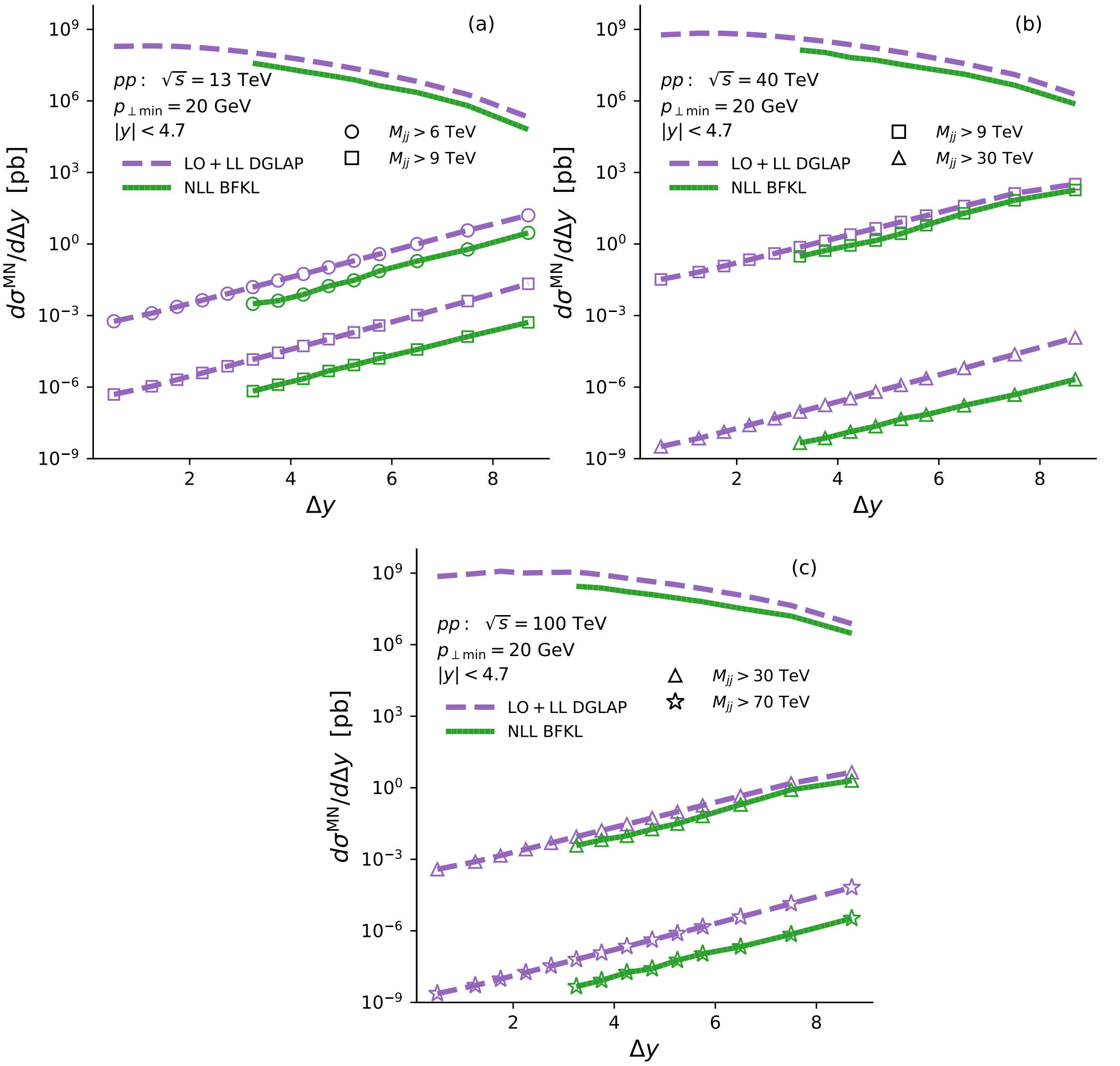

Figure 1: An impact of the imposed cut $M_{jj} > M_{jj\text{min}}$ on the MN dijet cross section of QCD background in pp collisions at $\sqrt{s}$ =13 (a), 40 (b) and 100 (c) TeV. The QCD background includes contributions calculated using the LO+LL DGLAP (violet dashed) and NLL BFKL (green solid) approximations. Markers indicate different $M_{jj\text{min}}$ values: circles correspond to $M_{jj\text{min}}$ = 6 TeV, squares to $M_{jj\text{min}}$ = 9 TeV, triangles to $M_{jj\text{min}}$ = 30 TeV, and stars to $M_{jj\text{min}}$ = 70 TeV.

modifies the strength of the gravitational coupling to matter, thus having a more pronounced effect on observable signals.

The QCD background predictions in various approximations are provided with: LO+LL DGLAP approximation; DGLAP-based MC generator PYTHIA8 with the CP5 tune; calculations including LL and NLL BFKL corrections. All of them are presented for all considered collision energies and dijet mass selections. This provides a comprehensive comparison of different theoretical approaches and modeling techniques for the QCD background across a variety of kinematic conditions. The NLL BFKL predictions consistently yield lower estimates for the QCD background compared to other calculations (up to two orders of magnitude). Additionally, the high dijet mass

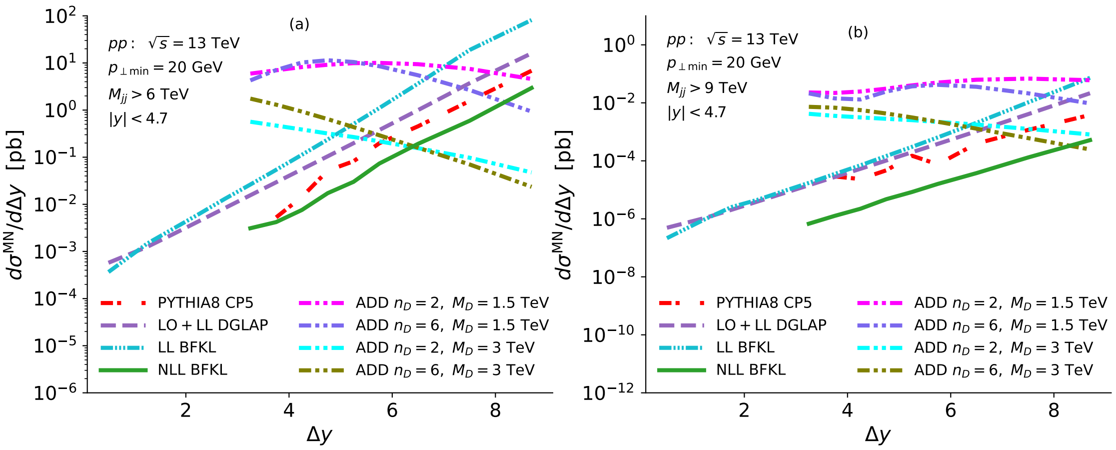

Figure 2: The MN dijet cross section with the imposed dijet mass selection $M_{jj} > M_{jj\mathrm{min}}$ in pp collisions at $\sqrt{s} = 13$ TeV. The ADD gravity signal is shown for various parameter choices, including the number of extra dimensions n_D, and the Plank scale M_D in the presence of n_D extra dimensions. The QCD background includes either contributions calculated with the LO+LL DGLAP or LL/NLL BFKL corrections. Panel (a) corresponds to $M_{jj\mathrm{min}} = 6$ TeV, and panel (b) to $M_{jj\mathrm{min}} = 9$ TeV.

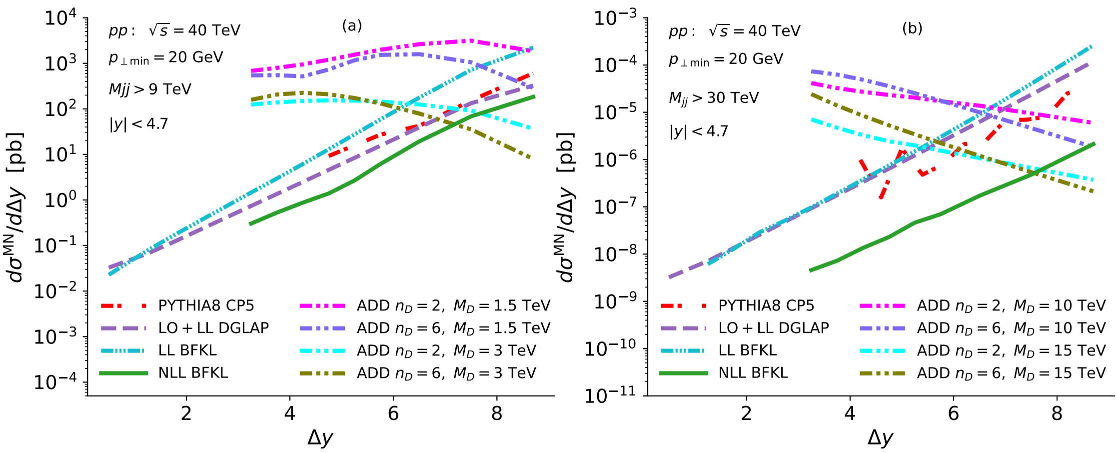

Figure 3: The MN dijet cross section with the imposed dijet mass selection $M_{jj} > M_{jj\mathrm{min}}$ in pp collisions at $\sqrt{s} = 40$ TeV. The ADD gravity signal is shown for various parameter choices, including the number of extra dimensions n_D, and the Plank scale M_D in the presence of n_D extra dimensions. The QCD background includes either contributions calculated with the LO+LL DGLAP or LL/NLL BFKL corrections. Panel (a) corresponds to $M_{jj\mathrm{min}} = 9$ TeV, and panel (b) to $M_{jj\mathrm{min}} = 30$ TeV.

selection $M_{jj} > M_{jj\mathrm{min}}$ has the most pronounced effect on the NLL BFKL predictions among the various QCD approaches, leading to a stronger suppression of the background in this regime.

Calculations incorporating the full NLL BFKL resummation provide a better description of dijet measurements at large rapidity separations Δy in the LHC experiments at TeV scales [11–14, 47–53]. Given that, the NLL BFKL predictions are regarded as the most reliable estimations for the QCD background in this kinematic regime.

Considering the largest rapidity separation currently accessible in experiments at the LHC

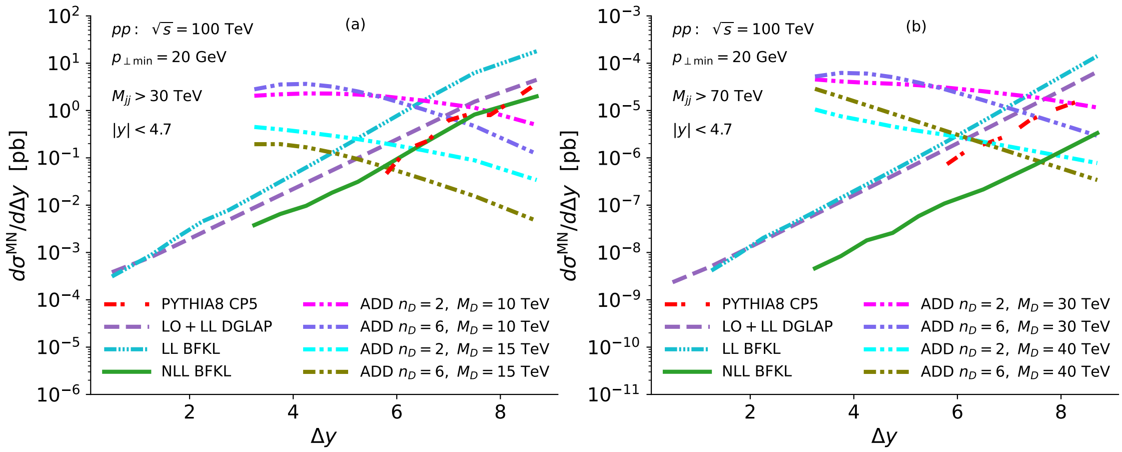

Figure 4: The MN dijet cross section with the imposed dijet mass selection $M_{jj} > M_{jj\text{min}}$ in pp collisions at $\sqrt{s} = 100$ TeV. The ADD gravity signal is shown for various parameter choices, including the number of extra dimensions n_D, and the Plank scale M_D in the presence of n_D extra dimensions. The QCD background includes either contributions calculated with the LO+LL DGLAP or LL/NLL BFKL corrections. Panel (a) corresponds to $M_{jj\text{min}} = 30$ TeV, and panel (b) to $M_{jj\text{min}} = 70$ TeV.

$\Delta y = 8.7$ and the background calculated including NLL BFKL corrections, it can be concluded that measurements of MN dijet production with the high dijet mass selection in pp collisions are sensitive to ADD gravity for $M_D < 3$ TeV at $\sqrt{s} = 13$ TeV; for $M_D < 10$ TeV at $\sqrt{s} = 40$ TeV; and for $M_D < 30$ TeV at $\sqrt{s} = 100$ TeV. The sensitivity may improve further if the transition to eikonal regime occurs at lower values of Δy. The estimated sensitivity of the proposed approach appears to be lower compared to existing searches conducted at the LHC [23–29]. However, it is important to remember that the LHC searches rely on divergent behavior of effective theory near the scale of new physics. The complementary approach discussed here explores a distinct kinematic regime, thereby it will help to reinforce and validate the conclusions.

To measure a cross section of the order of $\sim 10^{-6}$ pb, an integrated luminosity at least ~ 1 ab^{-1} is required, which is expected to be available at future facilities such as the HL-LHC, FCC-hh, and CEPC-SppC. However, at high luminosities and large rapidities, it becomes necessary to disentangle overlapping pp collisions occurring within the same or nearby bunch crossings (pile-up). This imposes stringent requirements for high granularity in the next generation of detectors to effectively resolve and identify large rapidity jets from individual pp collision events and correct for possible overlap effects.

5. Summary

The signal of gravity in the presence of large extra dimensions, as formulated by Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1], is studied for events with dijets, where the jets are widely separated in rapidity, within the trans-Planckian eikonal regime defined by $\sqrt{\hat{s}} \gg M_D \gg \sqrt{-\hat{t}}$ in proton-proton (pp) collisions. The calculations are performed for experimental conditions projected for future collider facilities such as the HL-LHC, FCCpp and CEPC-SppC, covering pp collision energies ranging from $\sqrt{s} = 13$ TeV up to 100 TeV. Parameter values such as the number of extra

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dimensions $n_D = 2$ and 6, and various Planck scales in presence of extra dimensions, M_D, across collision energies are explored.

The QCD background is modeled using several approaches including LO+LL DGLAP, PYTHIA8 with CP5 tune, and LL/NLL BFKL calculations, with the NLL BFKL calculation providing the most reliable background estimates. Sensitivity to ADD gravity is established for specific lower bounds of M_D increasing with collision energy. The approach complements the existing LHC searches by probing a different kinematic regime and requires high integrated luminosities and high detector granularity to resolve rare events at large rapidities.

The numerical estimates also show that using DGLAP dynamics at large rapidities in the semi-hard regime may significantly overestimate the QCD background which could result in misinterpreting experimental data and overlooking potential signals of new physics.

Acknowledgments

The authors thank the supercomputing system "Konstantinov" of the PIK Data Centre at NRC KI–PNPI for providing with computing resources. V.T.K. thanks the Organizing Committee of the first Open Workshop "QCD at the Extremes", especially Hannes Jung, for an opportunity to present the talk.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The Hierarchy problem and new dimensions at a millimeter*, *Phys. Lett. B* **429** (1998) 263 [hep-ph/9803315].

[2] G.F. Giudice, R. Rattazzi and J.D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, *Nucl. Phys. B* **544** (1999) 3 [hep-ph/9811291].

[3] G.F. Giudice, R. Rattazzi and J.D. Wells, *Transplanckian collisions at the LHC and beyond*, *Nucl. Phys. B* **630** (2002) 293 [hep-ph/0112161].

[4] L.N. Lipatov, *Reggeization of the Vector Meson and the Vacuum Singularity in Nonabelian Gauge Theories*, *Sov. J. Nucl. Phys.* **23** (1976) 338.

[5] V.S. Fadin, E.A. Kuraev and L.N. Lipatov, *On the Pomeranchuk Singularity in Asymptotically Free Theories*, *Phys. Lett. B* **60** (1975) 50.

[6] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Multi-Reggeon processes in the Yang-Mills theory*, *Sov. Phys. JETP* **44** (1976) 443.

[7] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *The Pomeranchuk singularity in nonabelian gauge theories*, *Sov. Phys. JETP* **45** (1977) 199.

[8] I.I. Balitsky and L.N. Lipatov, *The Pomeranchuk singularity in quantum chromodynamics*, *Sov. J. Nucl. Phys.* **28** (1978) 822.

[9] V.S. Fadin and L.N. Lipatov, *BFKL pomeron in the next-to-leading approximation*, *Phys. Lett. B* **429** (1998) 127 [hep-ph/9802290].

[10] M. Ciafaloni and G. Camici, *Energy scale(s) and next-to-leading BFKL equation*, *Phys. Lett. B* **430** (1998) 349 [hep-ph/9803389].

[11] CMS collaboration, *Study of dijet events with large rapidity separation in proton-proton collisions at $\sqrt{s} = 2.76$ TeV*, *JHEP* **03** (2022) 189 (2022) [2111.04605].

[12] CMS collaboration, *Dijet Events with Large Rapidity Separation in Proton–Proton Collisions at $\sqrt{s}$= 2.76 TeV with CMS Detector*, *Phys. Atom. Nucl.* **85** (2022) 951.

[13] A.I. Egorov and V.T. Kim, *Next-to-leading BFKL evolution for dijets with large rapidity separation at different LHC energies*, *Phys. Rev. D* **108** (2023) 014010 [2305.19854].

[14] A.I. Egorov and V.T. Kim, *Manifestation of Asymptotic QCD Effects in Dijet Production at Large Hadron Collider with 2.76 TeV*, *Phys. Part. Nucl. Lett.* **22** (2025) 224.

[15] V.N. Gribov and L.N. Lipatov, *Deep inelastic e p scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 438.

[16] V.N. Gribov and L.N. Lipatov, *e+e- pair annihilation and deep inelastic ep scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 675.

[17] L.N. Lipatov, *The parton model and perturbation theory*, *Sov. J. Nucl. Phys.* **20** (1975) 94.

[18] G. Altarelli and G. Parisi, *Asymptotic freedom in parton language*, *Nucl. Phys. B* **126** (1977) 298.

[19] Y.L. Dokshitzer, *Calculation of the structure functions for deep inelastic scattering and e+e- annihilation by perturbation theory in quantum chromodynamics.*, *Sov. Phys. JETP* **46** (1977) 641.

[20] I. Béjar Alonso, O. Brüning, P. Fessia, L. Rossi, L. Tavian and M. Zerlauth, eds., *High-Luminosity Large Hadron Collider (HL-LHC): Technical design report*, CERN Yellow Reports: Monographs, CERN, Geneva (2020), 10.23731/CYRM-2020-0010.

[21] FCC collaboration, *FCC-hh: The Hadron Collider: Future Circular Collider Conceptual Design Report Volume 3*, *Eur. Phys. J. ST* **228** (2019) 755.

[22] J. Gao, *CEPC and SppC Status — From the completion of CDR towards TDR*, *Int. J. Mod. Phys. A* **36** (2021) 2142005.

[23] ATLAS collaboration, *Search for contact interactions and large extra dimensions in the dilepton channel using proton-proton collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, *Eur. Phys. J. C* **74** (2014) 3134 [1407.2410].

[24] CMS collaboration, *Search for physics beyond the standard model in dilepton mass spectra in proton-proton collisions at $\sqrt{s} = 8$ TeV*, *JHEP* **04** (2015) 025 (2015) [1412.6302].

[25] CMS collaboration, *Search for high-mass resonances in dilepton final states in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *JHEP* **06** (2018) 120 (2018) [1803.06292].

[26] CMS collaboration, *Search for physics beyond the standard model in high-mass diphoton events from proton-proton collisions at $\sqrt{s}$ = 13 TeV*, Phys. Rev. D **98** (2018) 092001 [1809.00327].

[27] CMS collaboration, *Search for new physics in dijet angular distributions using proton–proton collisions at $\sqrt{s}$ = 13 TeV and constraints on dark matter and other models*, Eur. Phys. J. C **78** (2018) 789 [1803.08030].

[28] CMS collaboration, *Search for contact interactions and large extra dimensions in the dilepton mass spectra from proton-proton collisions at $\sqrt{s}$ = 13 TeV*, JHEP **04** (2019) 114 (2019) [1812.10443].

[29] CMS collaboration, *Search for resonant and nonresonant new phenomena in high-mass dilepton final states at $\sqrt{s}$ = 13 TeV*, JHEP **07** (2021) 208 (2021) [2103.02708].

[30] A.I. Egorov, V.T. Kim, V.A. Murzin and V.A. Oreshkin, *Dijets with large rapidity separation at the next-to-leading BFKL for search of large extra dimension gravity at colliders*, 2510.24625.

[31] T. Sjostrand, S. Mrenna and P.Z. Skands, *A Brief Introduction to PYTHIA 8.1*, Comput. Phys. Commun. **178** (2008) 852 [0710.3820].

[32] F. Caporale, D.Y. Ivanov, B. Murdaca and A. Papa, *Brodsky-Lepage-Mackenzie optimal renormalization scale setting for semihard processes*, Phys. Rev. D **91** (2015) 114009 [1504.06471].

[33] F. Caporale, D.Y. Ivanov, B. Murdaca and A. Papa, *Mueller-Navelet small-cone jets at LHC in next-to-leading BFKL*, Nucl. Phys. B **877** (2013) 73 [1211.7225].

[34] F.G. Celiberto, D.Y. Ivanov, B. Murdaca and A. Papa, *Mueller–Navelet jets at 13 TeV LHC: dependence on dynamic constraints in the central rapidity region*, Eur. Phys. J. C **76** (2016) 224 [1601.07847].

[35] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, *The QCD pomeron with optimal renormalization*, JETP Lett. **70** (1999) 155 [hep-ph/9901229].

[36] S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, *On the Elimination of Scale Ambiguities in Perturbative Quantum Chromodynamics*, Phys. Rev. D **28** (1983) 228.

[37] A.H. Mueller and H. Navelet, *An inclusive minijet cross section and the bare pomeron in QCD*, Nucl. Phys. B **282** (1987) 727.

[38] NNPDF collaboration, *Parton distributions from high-precision collider data*, Eur. Phys. J. C **77** (2017) 663 [1706.00428].

[39] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht et al., *LHAPDF6: parton density access in the LHC precision era*, Eur. Phys. J. C **75** (2015) 132 [1412.7420].

[40] S. Alioli, P. Nason, C. Oleari and E. Re, *A general framework for implementing NLO calculations in shower Monte Carlo programs: the POWHEG BOX*, JHEP **06** (2010) 043 [1002.2581].

[41] CMS collaboration, *Extraction and validation of a new set of CMS PYTHIA8 tunes from underlying-event measurements*, Eur. Phys. J. C **80** (2020) 4 [1903.12179].

[42] H. Jung and G.P. Salam, *Hadronic final state predictions from CCFM: The Hadron level Monte Carlo generator CASCADE*, Eur. Phys. J. C **19** (2001) 351 [hep-ph/0012143].

[43] M. Cacciari, G.P. Salam and G. Soyez, *The Anti-k(t) jet clustering algorithm*, JHEP **04** (2008) 063 (2008) [0802.1189].

[44] M. Cacciari, G.P. Salam and G. Soyez, *FastJet User Manual*, Eur. Phys. J. C **72** (2012) 1896 [1111.6097].

[45] A.D. Martin, W.J. Stirling, R.S. Thorne and G. Watt, *Parton distributions for the LHC*, Eur. Phys. J. C **63** (2009) 189 [0901.0002].

[46] D. Colferai and A. Niccoli, *The NLO jet vertex in the small-cone approximation for kt and cone algorithms*, JHEP **04** (2015) 071 (2015) [1501.07442].

[47] ATLAS collaboration, *Measurement of dijet production with a veto on additional central jet activity in pp collisions at $\sqrt{s} = 7$ TeV using the ATLAS detector*, JHEP **09** (2011) 053 (2011) [1107.1641].

[48] ATLAS collaboration, *Measurements of jet vetoes and azimuthal decorrelations in dijet events produced in pp collisions at $\sqrt{s} = 7$ TeV using the ATLAS detector*, Eur. Phys. J. C **74** (2014) 3117 [1407.5756].

[49] CMS collaboration, *Measurement of the inclusive production cross sections for forward jets and for dijet events with one forward and one central jet in pp collisions at $\sqrt{s} = 7$ TeV*, JHEP **06** (2012) 036 (2012) [1202.0704].

[50] CMS collaboration, *Ratios of dijet production cross sections as a function of the absolute difference in rapidity between jets in proton-proton collisions at $\sqrt{s} = 7$ TeV*, Eur. Phys. J. C **72** (2012) 2216 [1204.0696].

[51] CMS collaboration, *Azimuthal decorrelation of jets widely separated in rapidity in pp collisions at $\sqrt{s} = 7$ TeV*, JHEP **08** (2016) 139 (2016) [1601.06713].

[52] CMS collaboration, *Study of dijet events with a large rapidity gap between the two leading jets in pp collisions at $\sqrt{s} = 7$ TeV*, Eur. Phys. J. C **78** (2018) 242 [1710.02586].

[53] TOTEM, CMS collaboration, *Hard color-singlet exchange in dijet events in proton-proton collisions at $\sqrt{s} = 13$ TeV*, Phys. Rev. D **104** (2021) 032009 [2102.06945].

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