PROCEEDINGS OF SCIENCE

Spin-flip gluon generalized transverse momentum dependent parton distribution at small-x

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The spin-flip processes in deep inelastic scatterings were believed to be suppressed at high energy until recently. A positive intercept for the spin-flip generalized transverse momentum-dependent parton distribution (GTMDs) $\mathrm{Re}(F_{1,2})$ was derived. This was done by analytically solving the integro-differential evolution equation for $\mathrm{Re}(F_{1,2})$. The surviving solution corresponds to conformal spin $n = 2$ and carries an explicit $\cos 3\phi_{k\Delta} + \cos\phi_{k\Delta}$ azimuthal dependence. As the imaginary part of $F_{1,2}$ is related to the spin-dependent odderon or gluon Sivers function and scales as $\mathrm{Im}(F_{1,2}) \sim x^0$, the positive intercept for $\mathrm{Re}(F_{1,2})$ implies that it is expected to dominate over the gluon Sivers function in the small-x limit and may directly impact the modeling of unpolarised GTMDs and associated spin-flip processes.

QCD at the Extremes (QCDEX2025)
1–5 Sept 2025
Online

*Speaker

PoS(QCDEX2025)012

1. Introduction

One of the central goals of the upcoming Electron-Ion Collider (EIC) is to map the internal structure of the proton and explore the high-density regime of Quantum Chromodynamics (QCD) at small Bjorken-x [1]. A major step toward this understanding involves studying the non-perturbative gluon-gluon correlators that describe the multi-dimensional dynamics of gluons inside hadrons. These correlators, expressed as off-forward matrix elements of color field strength tensors, can be parametrized in terms of the Generalized Transverse Momentum Dependent distributions (GTMDs). GTMDs depend on the longitudinal momentum fraction x, the gluon transverse momentum $k_\perp$, and the transverse momentum transfer $\Delta_\perp$, and they provide a unified description that connects Transverse Momentum Dependent distributions (TMDs), Generalized Parton Distributions (GPDs), and Parton Distribution Functions (PDFs).

While all GTMDs originate from the same non-perturbative correlator, their evolution with respect to the hard scale Q^2 and the longitudinal variable x can differ significantly. The helicity-conserving gluon GTMDs, such as those related to the H-type GPDs, are known to follow the Balitsky–Fadin–Kuraev–Lipatov (BFKL) evolution, corresponding to single $\alpha_s \ln(1/x)$ resummations [2, 3]. In contrast, much less is known about the helicity-flip, or spin-flip, gluon distributions E_g and their corresponding GTMDs. These have been assumed to be suppressed at high energies [4, 5]. However, recent work by Hatta and Zhou has shown that the spin-flip gluon distributions can exhibit Regge-like growth similar to the BFKL pomeron [6]. Building on this development, the present study focuses on the analytical solution of the small-x evolution equations for the spin-flip gluon GTMDs $f_{1,2}$ and $f_{1,3}$.

2. F-Type Spin-flip GTMDs

The gluon GTMD $F_{1,2}(x, \mathbf{k}_\perp, \mathbf{\Delta}_\perp)$ depends on the longitudinal momentum fraction x, the gluon transverse momentum $\mathbf{k}_\perp$, and the transverse momentum transfer $\mathbf{\Delta}_\perp$. It encodes information about the correlation between the gluon transverse polarization and the hadron spin. The gluon GTMDs can be defined through the parametrization of the off-forward bilocal correlator of the two gluon field strength tensors $W^{[i,j]}_{\lambda,\lambda'}$. Contraction of $W^{[i,j]}$ by symmetric δ^{ij} will project the four complex (or equivalently eight real) F-type unpolarized gluon GTMDs. In the off-forward limit, at the leading twist, we write [7–12], ,

$$\delta^{ij} W^{[i,j]}_{\lambda,\lambda'} = \frac{1}{2M}\bar{u}(p',\lambda')\left[F_{1,1} + i\frac{\sigma^{j+}k^j_\perp}{P^+}F_{1,2} + i\frac{\sigma^{j+}\Delta^j_\perp}{P^+}F_{1,3} + i\frac{\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F_{1,4}\right]u(p,\lambda). \quad (1)$$

In the eikonal limit, $\xi \ll 1$, one may write [11],

$$F_{1,n} = f_{1,n} + i\frac{k_\perp \cdot \Delta_\perp}{M^2}\tilde{f}_{1,n} \qquad \text{where n} = 1, 3, 4,$$

$$F_{1,2} = \frac{k_\perp . \Delta_\perp}{M^2} f_{1,2} + i\tilde{f}_{1,2} \qquad \text{for n} = 2. \quad (2)$$

Thus we see $F_{1,2}$ can be decomposed eight into real and imaginary components, where the imaginary component is closely related to C-odd spin-dependent exchanges (the spin-dependent odderon or the gluon Sivers-type contribution). Integrating the correlator over $k_\perp$ reproduces the gluon GPDs: the spin-nonflip H_g and the spin-flip E_g, with E_g receiving contributions from the spin-flip GTMDs, as shown,

$$xE_g \;=\; \int d^2k_\perp \left(-f_{1,1}(k_\perp) + \frac{k_\perp^2}{M^2} f_{1,2}(k_\perp) + 2f_{1,3}(k_\perp)\right). \tag{3}$$

Thus the real component $f_{1,2}$ carries the helicity-flip information that connects to E_g in the off-forward integrals.

3. Evolution of $F_{1,2}$

The small-x evolution equation for $f_{1,2}$ as derived by Hatta and Zhou in [6], is a non-linear integro-differential equation. In the dilute regime, where the non-linear term can be dropped, the equation can be written as

$$\frac{\partial}{\partial Y}\mathcal{F}_{1,2}(x,k_\perp) \;=\; \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left\{\mathcal{F}_{1,2}(x,k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp}\mathcal{F}_{1,2}(x,k_\perp) + \frac{2(k_\perp . k'_\perp)^2 - k_\perp^2 k'^2_\perp - k_\perp^4}{k_\perp^4}\mathcal{F}_{1,2}(x,k'_\perp)\right\}. \tag{4}$$

where $Y = \ln(1/x)$ is the rapidity and the function $\mathcal{F}_{1,2}$ is defined for convenience and is related to $f_{1,2}$ as,

$$f_{1,2} = k_\perp^2 \frac{\partial^2}{\partial k_\perp^i \partial k_\perp^i}\mathcal{F}_{1,2}. \tag{5}$$

The first two terms, on the right-hand side of Eq.(4), essentially constitute the BFKL kernel, and we get the corresponding BFKL eigenvalue $\chi_{BFKL}(n,\gamma)$. However, the full eigenvalue $\chi_{1,2}(n,\gamma)$ for the above evolution equation is

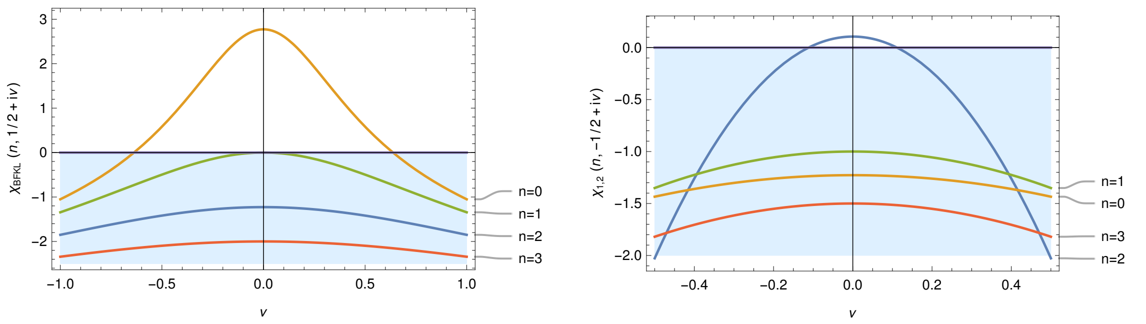
$$\chi_{1,2}(n,\gamma) = 2\psi(1) - \frac{1}{2}\psi\left(\gamma + \frac{|n|}{2}\right) - \frac{1}{2}\psi\left(\gamma + \frac{|n|}{2} + 2\right) - \frac{1}{2}\psi\left(-\gamma + \frac{|n|}{2} - 1\right) - \frac{1}{2}\psi\left(-\gamma + \frac{|n|}{2} + 1\right). \tag{6}$$

All IR divergences are mutually canceled, leading to an IR finite evolution equation. Interestingly, unlike the BFKL eigenvalue, for which the saddle point is located at $Re\,(\gamma) = 1/2$, the saddle point of $\chi_{1,2}(n,\gamma)$ is at $Re\,(\gamma) = -1/2$ for all n (Fig.1). The surviving solution corresponds to the conformal spin $n = 2$, leading to an explicit $2\cos 2\phi_{k\Delta}$ dependence in the GTMDs. Finally, we have

$$\mathrm{Re}(F_{1,2}) \sim \left(\frac{1}{x}\right)^{\alpha_s(4\ln 2 - 8/3)} (\cos 3\phi_{k\Delta} + \cos\phi_{k\Delta}).$$

4. Summary

In this work, we have presented the analytical study of the small-x evolution of the spin-flip gluon GTMDs $F_{1,2}$. These distributions describe helicity-flip processes and are probed in off-forward parton dynamics. Our analysis demonstrates that the evolution equation for $\mathrm{Re}(F_{1,2})$ is



(a) $\chi_{BFKL}(n, \gamma)$ at the saddle point $\gamma = 1/2 + i0$. **(b)** $\chi_{1,2}(n, \gamma)$ at the saddle point $\gamma = -1/2 + i0$.

Figure 1: Eigen values of the two kernels, at their respective saddle points, as a function of ν.

infrared safe and produces a closed self-consistent form. The solution exhibits a positive intercept,

$$\mathrm{Re}(F_{1,2}) \sim x^{-\bar{\alpha}_s(4\ln 2 - 8/3)},$$

implying that it grows as x decreases and is expected to dominate over the gluon Sivers function in the small-x regime. The corresponding solution carries conformal spin $n = 2$, leading to a distinct azimuthal dependence $\cos(3\phi_{k\Delta}) + \cos(\phi_{k\Delta})$, which manifests itself as angular correlations in future experiments. From a phenomenological perspective, the study of spin-flip GTMDs has recently gained attention due to the realization that GTMDs can be accessed through diffractive dijet production in deep inelastic ep and eA scattering, as well as via exclusive pion or vector meson production [13–15]. While extensive progress has been made in understanding quark GTMDs and their small-x evolution, the gluon sector—particularly the helicity and spin-flip distributions—remains comparatively less explored. The present work provides a theoretical foundation for studying such effects and highlights new possibilities for probing gluon spin-orbit dynamics at small x.

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