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A phenomenological study of the proton charm and beauty content in the k_t-factorization formalism using KMR and MRW unintegrated parton distribution function

Naeimeh Olanj[a,*] **and Majid Modarres**[b]

[a] *Physics Department, Faculty of Science, Bu-Ali Sina University, Hamedan, Iran*
P. O. Box 65178

[b] *Department of Physics, University of Tehran, North Kargar Ave, Tehran, Iran,*
P. O. Box 1439955961

E-mail: n_olanj@basu.ac.ir, mmodares@ut.ac.ir

In this paper, we study the phenomenology of the charm and beauty content of the proton within the k_t-factorization formalism, using the integral version of the Kimber-Martin-Ryskin (KMR) and Martin-Ryskin-Watt (MRW) unintegrated parton distribution functions ($UPDF$) together with the angular ordering constraint (AOC). For this purpose, the reduced beauty cross section $\sigma_{\rm red}^{b\bar{b}}(x, Q^2)$ and the charm and beauty structure functions $F_2^{q\bar{q}}(x, Q^2)$ $(q = c, b)$ are calculated by the k_t-factorization formalism using KMR and MRW approaches and the $MMHT2014$-PDF set as input. Recently, Guiot and van Hameren demonstrated that the upper limit $k_{\rm max}$ of the transverse-momentum integration in the k_t-factorization formalism should be approximately equal to Q, the hard scale, to avoid overestimating the proton structure function. In the present work, we show that this condition ($k_{\rm max} = Q$) is not valid in the low and moderate energy regions. Furthermore, by considering both gluon and quark contributions at the same perturbative order and employing a physical gauge for the gluon, i.e., $A^\mu q'_\mu = 0$, in the calculation of $F_2^{q\bar{q}}$ within the k_t-factorization formalism, we do not encounter any overestimation of the theoretical predictions for different choices of $k_{\rm max} > Q$. Finally, the computed values of the reduced beauty cross section, $\sigma_{\rm red}^{b\bar{b}}$, and the charm and beauty structure functions, $F_2^{q\bar{q}}$, are compared with the experimental data of the $ZEUS$ and $H1$ collaborations in $HERA$ and theoretical predictions. In general, the results obtained from both the KMR and MRW methods are in perfect agreement with the experimental and theoretical predictions, especially at high energies, but the KMR framework is better consistent with the experimental data and theoretical predictions than the MRW framework at the low and moderate energies.

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1. Introduction

Heavy quark production plays a central role in deep inelastic scattering (DIS) at small Bjorken-x. It is known that up to 25% of the DIS cross section at small x arises from heavy-quark electroproduction [1]. Consequently, the proper determination of heavy-quark distributions is crucial for precision studies at the LHC, including Higgs, W, and Z boson production. Furthermore, the measurement of heavy-quark structure functions at $HERA$ provides stringent tests of $pQCD$ in the small x region. However, conventional collinear $pQCD$ faces theoretical challenges at small x, such as transverse-momentum effects and Reggeon dynamics. To overcome these limitations, the unintegrated parton distribution functions ($UPDF$) are introduced, which depend not only on x and the factorization scale μ, but also on the transverse momentum k_t. This makes them particularly suitable for high-energy phenomenology. In this study, we investigate the charm and beauty content of the proton using the Kimber-Martin-Ryskin (KMR) and the Martin-Ryskin-Watt (MRW) approaches to construct $UPDF$. These prescriptions extend the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi ($DGLAP$) equations in the LO and NLO approximation, respectively, along with a modification due to the angular ordering condition, which is the key dynamical property of the Ciafaloni-Catani-Fiorani-Marchesini ($CCFM$) formalism. We compute the reduced cross section and structure functions within the k_t-factorization formalism [2, 3], and benchmark the predictions against $HERA$ measurements using the $MMHT2014$-PDF [4] set at LO and NLO approximations, as input.

So, the paper is organized as follows: in the section II, a glimpse of the KMR [5] and MRW [6] approaches for the calculation of the $UPDF$ is presented. The formulation of $F_2^{q\bar{q}}$ and $\sigma_{red}^{b\bar{b}}$ based on the k_t-factorization formalism is given in the section III. Finally, the results as well as our discussions and conclusions are given in the section IV.

2. A glimpse of the KMR and the MRW approaches

The KMR-$UPDF$ approach at LO approximation are derived based on the $DGLAP$ evolution equation, incorporating the angular ordering constraint (AOC), which is the consequence of coherent gluon emissions. They are obtained through the following relation:

$$f_a(x, k_t^2, \mu^2) = T_a(k_t, \mu)\frac{\alpha_s({k_t}^2)}{2\pi}\int_x^{1-\Delta} dz P_{aa'}(z)\frac{x}{z}\,a'\left(\frac{x}{z}, {k_t}^2\right), \tag{1}$$

where $P_{aa'}(x)$ are the corresponding splitting functions, and the survival probability factors, T_a, are evaluated from:

$$T_a(k_t, \mu) = \exp\left[-\int_{k_t^2}^{\mu^2}\frac{\alpha_s(k_t'^2)}{2\pi}\frac{dk_t'^2}{k_t'^2}\sum_{a'}\int_0^{1-\Delta} dz' P_{a'a}(z')\right]. \tag{2}$$

Where $\Delta = \frac{k_t}{\mu+k_t}$ and T_a is considered to be unity for $k_t > \mu$. In the MRW approach, the $UPDF$ is calculated up to the NLO level. The procedure is largely similar to the KMR approach, with the key difference that the angular ordering is imposed only to the terms in which the splitting functions

are singular, i.e., the terms which include P_{qq} and P_{gg}. Also, this approach use both the NLO contributions of the splitting functions and PDF, as following:

$$f_a(x, k_t^2, \mu^2) = \int_x^1 dz T_a(k^2, \mu^2) \frac{\alpha_s(k^2)}{2\pi} \sum_{b=q,g} P_{ab}^{(0+1)}(z)\, b\left(\frac{x}{z}, k^2\right) \Theta(\mu^2 - k^2), \tag{3}$$

where

$$T_a(k^2, \mu^2) = \exp\left(- \int_{k^2}^{\mu^2} \frac{\alpha_s(\kappa^2)}{2\pi} \frac{d\kappa^2}{\kappa^2} \sum_{b=q,g} \int_0^1 d\zeta \zeta P_{ba}^{(0+1)}(\zeta) \right). \tag{4}$$

According to condition $k^2 < \mu^2$, it is clear that the MRW approach exhibits a closer consistency with the requirements of the $DGLAP$ evolution equations, in contrast to the KMR approach.

3. The formulation of $F_2^{q\bar{q}}$ and $\sigma_{red}^{q\bar{q}}$ in the k_t-factorization approach

The reduced heavy quark cross section is defined as

$$\sigma_{\rm red}^{Q\bar{Q}} = \frac{d^2\sigma^{Q\bar{Q}}}{dx_{\rm Bj}\, dQ^2} \cdot \frac{x_{\rm Bj} Q^4}{2\pi\alpha^2(Q^2)\left[1 + (1-y)^2\right]} = F_2^{Q\bar{Q}} - \frac{y^2}{1 + (1-y)^2} F_L^{Q\bar{Q}}, \tag{5}$$

where Q^2 is the virtuality of the exchanged photon γ^*, $y = \frac{P\cdot q}{P\cdot k}$ is the lepton inelasticity and the variable $x_{\rm Bj} = \frac{Q^2}{2P\cdot q}$ denotes the Bjorken scaling variable. At $HERA$, heavy quarks are mainly produced via photon-gluon fusion. The corresponding cross sections are directly sensitive to the gluon density in the proton. Therefore, it is appropriate to use the k_t-factorization formalism to calculate the heavy quark structure functions $F_2^{q\bar{q}}$ and $F_L^{q\bar{q}}$, as following:

$$\begin{aligned} F_2^{q\bar{q}}(x, Q^2) = e_q^2 \frac{Q^2}{4\pi} \int_{k_0^2}^{k_{t,\max}^2} \frac{dk_t^2}{k_t^4} \int_0^1 d\beta \int_{k_0^2}^{k_{t,\max}^2} d\kappa_t^2\, \alpha_s(\mu^2)\, f_g\left(\frac{x}{z}, k_t^2, \mu^2\right) \Theta(1 - \tfrac{x}{z}) \\ \times \Bigg\{ \left[\beta^2 + (1-\beta)^2\right] \left(\frac{\kappa_t}{D_1} - \frac{\kappa_t - k_t}{D_2}\right)^2 \\ + \left[m_q^2 + 4Q^2\beta^2(1-\beta)^2\right] \left(\frac{1}{D_1} - \frac{1}{D_2}\right)^2 \Bigg\} \\ + e_q^2 \frac{Q^2}{\kappa_t^2} \frac{\alpha_s(k_t^2)}{2\pi} \int_{k_0^2}^{k_{t,\max}^2} \frac{dk_t^2}{k_t^4} \int_x^1 dz\, P_{qg}(z) \left[f_q\left(\frac{x}{z}, k_t^2, Q^2\right) + f_{\bar{q}}\left(\frac{x}{z}, k_t^2, Q^2\right) \right], \end{aligned} \tag{6}$$

where $D_1 = \kappa_t^2 + \beta(1-\beta)Q^2 + m_q^2$, $D_2 = (\kappa_t - k_t)^2 + \beta(1-\beta)Q^2 + m_q^2$ and β denotes the light-cone momentum fraction of the photon that is carried by the internal quark. For more details, see the reference [3].

4. Results, discussions and conclusions

In this study, we investigated the beauty and charm content of the proton within the framework of k_t-factorization by calculating the reduced cross section $\sigma_{\rm red}^{b\bar{b}}(x, Q^2)$ and the structure function

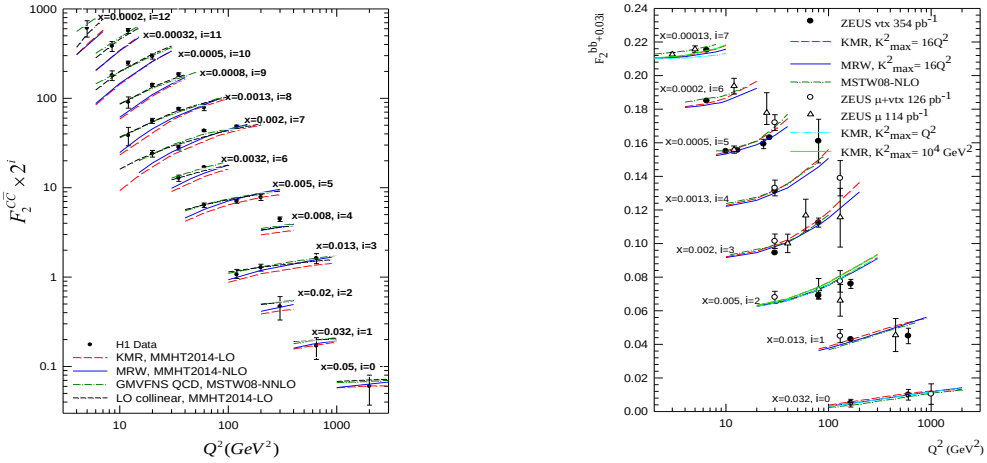

Figure 1: The charm and beauty structure functions as a function of Q^2 for various x values [2, 3].

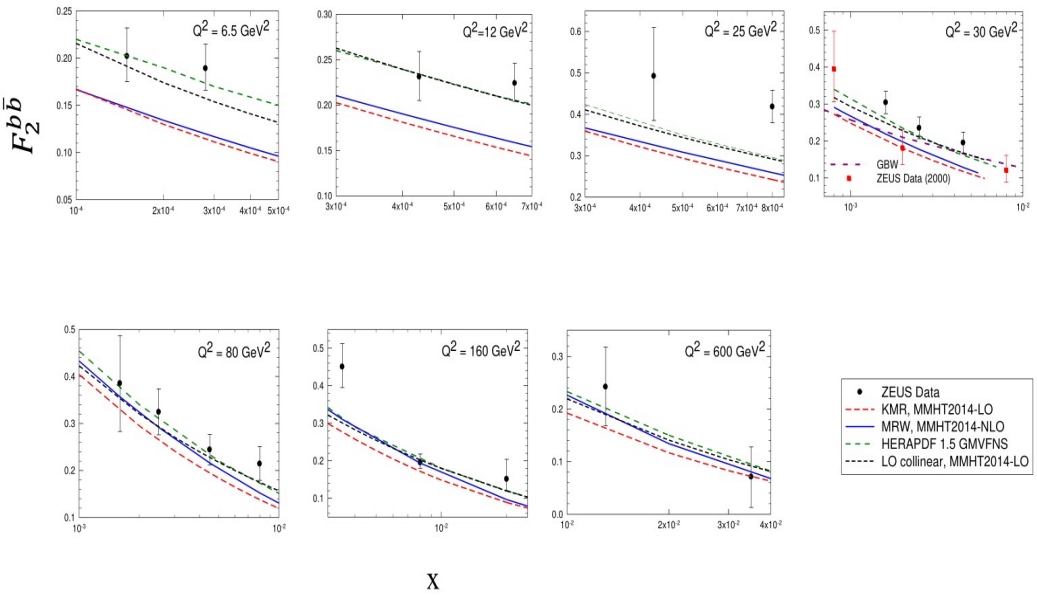

Figure 2: The charm structure functions as a function of Q^2 for various x values [2].

$F_2^{q\bar{q}}(x, Q^2), q = c, b$ using the KMR and MRW $UPDF$. In accordance with the objectives of this study, we have calculated and plotted the charm and beauty structure functions in the figure 1, as a function of x for different values of Q^2 and in the figure 2, the charm structure functions as a function of Q^2 for various x values, by using the KMR (dash curves) and MRW (full curves) approaches, with the input LO and NLO-$MMHT2014$ set of PDF, respectively.

In the figure 3, the reduced beauty cross section are displayed in the k_t-factorization formalism using the KMR and MRW approaches as a function of x for different values of Q^2 with $k^2_{max} =$

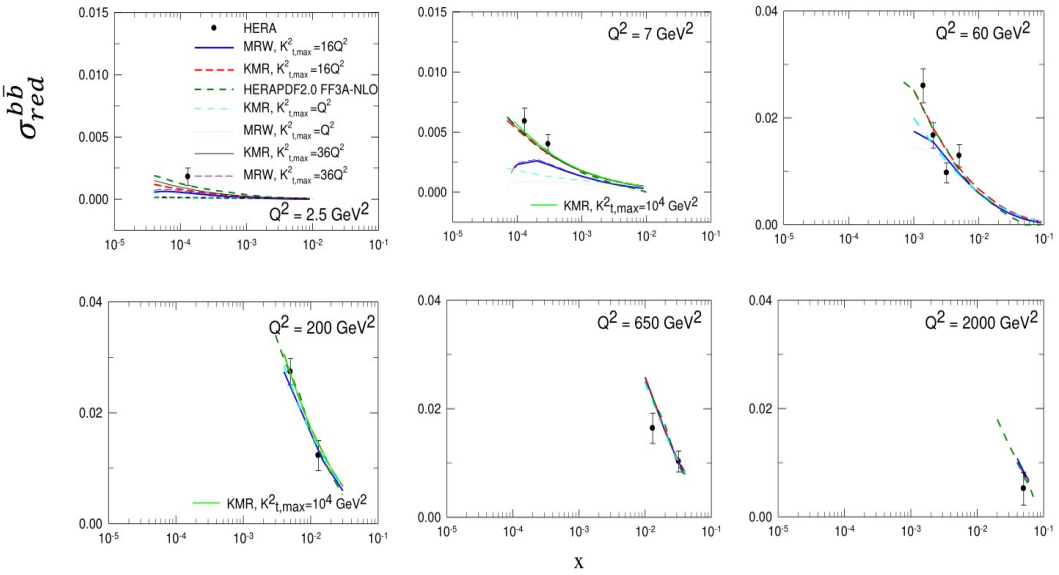

Figure 3: The reduced beauty cross section as a function of x for various Q^2 values [3].

$Q^2, 16Q^2$ in all panels, $k^2_{max} = 36Q^2$ and $k^2_{max} = 10^4 GeV^2$, in some panels. It should be noted that in the coupling constant calculations, we have used $\Lambda_{\rm QCD} = 300$ MeV for calculations related to charm quark, while for calculations related to beauty quark, we have considered $\alpha_{s,\rm LO}(M_Z^2) = 0.135$ and $\alpha_{s,\rm NLO}(M_Z^2) = 0.118$.

These theoretical results are compared with the experimental data obtained by the $H1$ and $ZEUS$ collaborations at the $HERA$ accelerator. As can be seen, at high energies, the KMR and MRW approaches yield similar results but they become separated as the hard scale and x decrease, confirming the robustness of the k_t-factorization approach in this region. However, at low and moderate energies, the KMR approach demonstrates better consistency with experimental observations, indicating its suitability for this kinematic range. It should be noted that this decrease in difference with increasing hard scale Q and x is due to the use of the scale $k^2 = \frac{k_t^2}{1-z}$ and $MMHT2014\text{-}NLO\ PDF$ set instead of the scale k_t^2 and $MMHT2014\text{-}LO\ PDF$ set in the MRW approach. Note that this decrease cannot be a result of different use of cut-off and splitting functions. We also examined the effect of varying the upper limit of transverse momentum integration $k_{\rm max}$. Our findings suggest that increasing $k_{\rm max}$ does not lead to an overestimation of the proton structure function. For computational efficiency, the optimal choice is to set $k_{\rm max} = Q$ at high energies and $k_{\rm max} = 4Q$ at low energies, which significantly reduces computation time without compromising accuracy. Overall, the k_t-factorization formalism provides a computationally simpler and reliable framework compared to full $pQCD$ calculations. It is especially useful for initial studies of heavy-quark production and exclusive processes at small x.

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