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A Glimpse into the Double Unintegrated Parton Distribution Functions

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We analyze inclusive jet and dijet production in positron(electron)–proton collisions at HERA energies. Using UPDFs with the KaTie parton-level event generator and DUPDFs through direct evaluation, we examine different ordering constraints and virtuality choices across the KMR and MRW approaches. The (z, k_t)-factorization framework is found to give better agreement with ZEUS data, particularly at high Q^2. We also study Z boson production in proton–proton collisions at $\sqrt{s} = 13$ TeV within the k_t and (z, k_t)-factorization frameworks using MRW parton distributions. By including subprocesses up to next-to-leading order and according for final-state leptons, we extend earlier analyses. Comparisons with ATLAS, CMS, and LHCb data show that while both approaches perform similarly at central rapidities, the (z, k_t)-factorization framework provides closer agreement with data at forward rapidities. In both cases, differential cross sections are obtained with the KaTie parton-level event generator for k_t-factorization, while for (z, k_t)-factorization they are computed directly.

Together, these studies highlight the importance of DUPDFs, which preserve full parton kinematics and improve predictions across different processes and energy scales. They provide a consistent framework for describing both electroweak boson production at the LHC and jet observables at HERA.

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1. Introduction

Accurate theoretical predictions for observables in high-energy proton–proton collisions are essential for testing the framework of Quantum Chromodynamics (QCD) and exploring possible physics beyond the Standard Model. These predictions are particularly demanding because the proton is a composite system of quarks and gluons, confined by the strong interaction. Unlike elementry point-like particles, hadronic cross sections cannot be calculated directly. Instead, the factorization theorem provides a rigorous approach, expressing the hadronic differential cross section (DCS) as a convolution of perturbatively calculable partonic cross sections with non-perturbative parton distribution functions (PDFs). Over the years, several factorization schemes have been developed to describe the internal dynamics of partons within the proton, including the collinear, k_t, and (z, k_t)-factorization frameworks. In the collinear factorization approach, partons are assumed to move strictly parallel to the proton's momentum, carrying only a longitudinal momentum fraction x. Their distributions are represented by collinear PDFs, $f_a(x, \mu^2)$, where μ denotes the factorization scale. The evolution of these distributions with respect to μ is governed by the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [1], which describe sequential parton emissions along the evolution ladder. At small x, however, the transverse momentum k_t of the partons becomes comparable to their longitudinal momentum component xP (with P being the proton momentum). In this region, the collinear approximation becomes inadequate, as it neglects the intrinsic transverse motion of partons. This limitation gave rise to the concept of unintegrated parton distribution functions (UPDFs), $f_a(x, k_t^2, \mu^2)$, which depend explicitly on both x and k_t. Originally formulated for gluons –dominant at small x– these distributions evolve according to the Balitsky–Fadin–Kuraev–Lipatov (BFKL) [2] and Catani–Ciafaloni–Fiorani–Marchesini (CCFM) [3] equations. The CCFM approach provides a smooth interpolation between the small-x (BFKL) and large-x (DGLAP) limits by introducing angular ordering along the parton cascade. Although, CCFM offers a consistent framework for gluon evolution, it does not naturally extend to quark distributions, which become increasingly significant at larger x. To overcome this limitation, several phenomenological models have been developed, including the Kimber–Martin–Ryskin (KMR) [4], Martin–Ryskin–Watt (MRW) [5], and Parton Branching (PB) [6] approaches. These methods generate UPDFs for both quarks and gluons across a broad kinematic range, enabling the application of k_t-factorization to a wide class of high-energy processes.

Beyond these schemes, a more general formalism –known as the (z, k_t)-factorization– has been introduced to extend the traditional k_t-factorization by incorporating the full kinematic dependence of both the hard incoming parton and the last emitted parton in the evolution chain. Proposed by Martin, Ryskin, and Watt (MRW), this framework employs Double Unintegrated Parton Distribution Functions (DUPDFs) [7].

A central difference between this framework and the standard k_t-factorization using KMR or MRW UPDFs lies in the treatment of the propagator virtuality. While the UPDF-based formalism assumes $k^2 = -k_t^2$, the (z, k_t)-factorization generalizes it to $k^2 = -k_t^2/(1 - z)$, where z is the momentum fraction carried by the hard parton relative to its parent. consequently, this framework extends the description beyond the small-x and small-z regime by introducing explicit z-dependence in the cross section. Therefore, one must use DUPDFs, $f_a(x, z, k_t^2, \mu^2)$, instead of the simpler UPDFs, providing a more comprehensive representation of parton dynamics.

2. Cross Sections in Different Factorization Frameworks

In the collinear factorization approach, it is assumed that each parton moves collinearly with the parent proton and directly participates in the hard collision. Within this framework, the total proton-proton collision cross section can be expressed as:

$$\sigma = \sum_{i,j\in q,g} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} f_i(x_1,\mu^2) f_j(x_2,\mu^2)\hat{\sigma}_{ij}. \tag{1}$$

The k_t-factorization framework provides a more comprehensive formalism for describing cross sections in high-energy hadronic collisions–particularly in processes such as Drell-Yan lepton pair production– where transverse momentum cannot be neglected. In this study, we employ the KaTie parton-level event generator [8] to perform calculations within this framework. The general proton-proton collision cross section is then given by:

$$\sigma = \sum_{i,j\in q,g} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{dk_{1t}^2}{k_{1t}^2}\frac{dk_{2t}^2}{k_{2t}^2} f_i(x_1,k_{1t}^2,\mu^2) f_j(x_2,k_{2t}^2,\mu^2)\hat{\sigma}^*_{ij}. \tag{2}$$

In the new formalism the UPDFs become z-dependent, (these are called DUPDFs), and hence one needs to modify hadronic factorization formula for calculating the cross section, compared to the k_t-factorization approach. Therefore, by generalizing the k_t-factorization framework, one can write the general p-p cross section formula in the (z, k_t)-factorization as:

$$\sigma = \sum_{i,j\in q,g} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2 \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \int_0^\infty \frac{dk_{1t}^2}{k_{1t}^2} \int_0^\infty \frac{dk_{2t}^2}{k_{2t}^2} f_i(x_1,z_1,k_{1t}^2,\mu^2) f_j(x_2,z_2,k_{2t}^2,\mu^2)$$
$$\times\hat{\sigma}^*_{ij}(x_1,x_2,z_1,z_2,k_{1t}^2,k_{2t}^2,\mu^2). \tag{3}$$

In this framework due to considering the z, fractional momenta of parent parton in the last step, one can have the full kinematics of the last step. Therefore, the last step emitted parton enters directly into the calculation.

3. Approaches to DUPDFs

The (z, k_t)-factorization framework were first introduced by Martin, Ryskin, and Watt with the aim of generalizing the UPDFs of the MRW approach. The first approach, which is based on the same KMR method, is called DKMR, in which the $z \to 1$ cut is imposed on the quark (antiquark) and soft gluon last step emissions. The second approach, which is based on the MRW method, is called DMRW, in which the cut is applied only to the soft gluon emissions. The third approach is similar to the DMRW method, which we call DMRW' in our results, but k_t is replaced by k and $\theta(\mu^2 - k^2)$ is added to prevent k_t^2 from becoming larger than μ^2. Three models for calculating DUPDFs are presented here. Initially, we will analyze diagrams pertaining to inclusive jet and dijet production [9], followed by a discussion of the diagrams related to Z boson production [10].

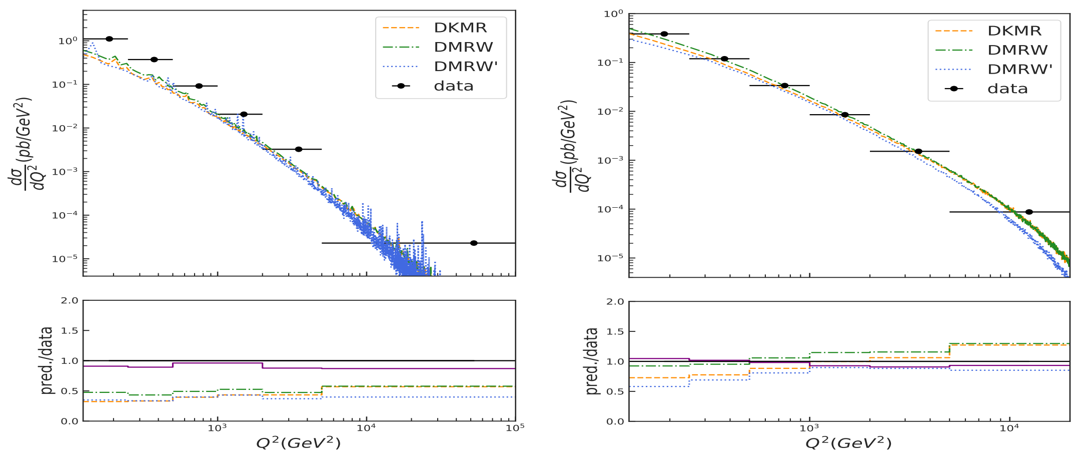

Figure 1: The left panel show a comparison of the inclusive jet production cross section, for the (z, k_t)-factorization with the DUPDF models, and right panel for the inclusive dijet production [9]

4. Results and Discussion

In Fig. 1, all DUPDFs predictions undershoot the DCS data with respect to Q^2. We also see that the result of DMRW' is smaller with respect to the predictions of DKMR and DMRW due to the cutoff and choice of virtuality. In contrast to the corresponding data of inclusive jet production, it can be observed that the results of the dijet subprocesses using DUPDFs are in excellent agreement, in the right panel [9]. The predictions in Fig. 2 of the DKMR and DMRW are in relatively good agreement with the data of differential cross section with respect to the transverse energy channels of the inclusive jet and dijet prediction. We also see the result of the DMRW' is smaller with respect to the predictions of DKMR and DMRW due to the cutoff and virtuality. Because we are working at relatively small center-of-mass energy, the virtuality k^2 becomes large. A small energy corresponds to a large momentum fraction z. When a significant value of z is subtracted from one in the denominator k^2, the denominator decreases, leading to an increase in the k^2 value, or virtuality. This increase in virtuality occurs in comparison to other DUPDFs that have a k_t^2 scale at the same center-of-mass energy. Ultimately, this rise in virtuality leads to a suppression of the results due to the strong ordering constraint. As a result of this, DMRW' predictions become smaller than other DUPDFs [9].

In Fig. 3, the left panel, one observes the variation of KMR, MRW and DMRW approaches with respect to the y_z . It is evident from this panel that, their results are close to each other in the Z boson rapidity region of $y_z < 4$, while they become separate from each other in the $y_z > 4$, wherein the MRW fails to describe the data well in that region. Additionally, although the ResBos result describes the data well across all rapidity regions, it tends to overestimate at large rapidities. [10]. In Fig. 3, the middle panel, a comparison between the contributions of the higher order sub-processes, denoted by σ_2 , and lower order sub-processes, denoted by σ_1, for the MRW and the DMRW are compared. As it is obvious from this Figure, the role of higher order sub-processes, is negligible relative to the lower order sub-processes. Therefore one can safely ignore their contributions into our calculation [10]. In Fig. 3, the right panel, it can be observed the double DCS with respect to P_t^{ll} in various rapidity regions of the produced Z boson. Similar to our previous results for the cross

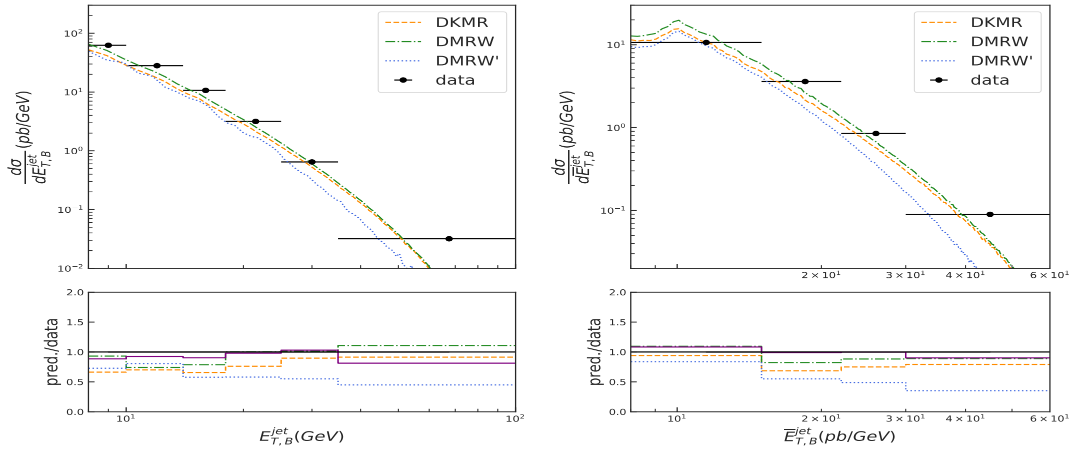

Figure 2: The left panel show a comparison of the inclusive jet production cross section, for the (z, k_t)-factorization with the DUPDF models, and right panel for the inclusive dijet production [9]

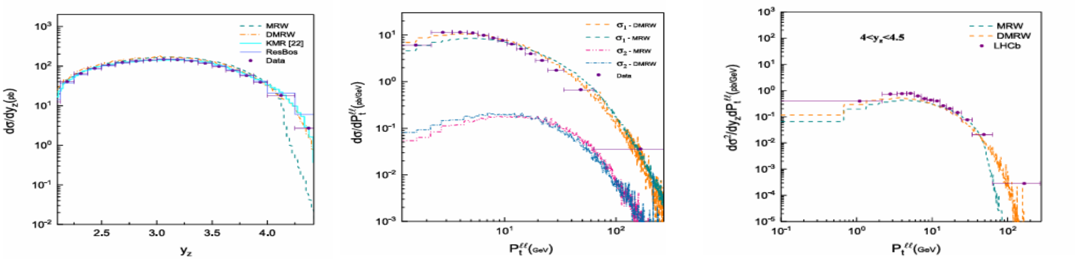

Figure 3: The left (middle) panel shows the differential cross section with respect to the y (P_t^{ll}), while the right panel presents the double differential cross section with respect to the P_t^{ll} [10]

section with respect to P_t^{ll}, it can be seen relatively the same behavior in all of the regions except where $4 < y_z < 4.5$. In fact as we move toward large rapidity regions, the DMRW becomes much better relative to the MRW, especially in small and large dilepton transverse momentum regions [10].

5. Conclusions

In summary, incorporating the final-step parton emission within the (z, k_t)-factorization framework significantly improves the theoretical description of inclusive jet and dijet production cross sections. This enhanced framework serves as a robust alternative to the traditional k_t-factorization framework, offering better consistency with experimental data [9].

Moreover, the comparative analysis of Z boson production demonstrates that the k_t and (z, k_t)-factorization schemes exhibit similar performance across most kinematic regions. However, notable differences emerge at large rapidities, where the (z, k_t)-factorization yields superior agreement with experimental observations [10]. Overall, these finding highlight the theoretical importance of Double Unintegrated Parton Distribution Functions (DUPDFs) in capturing complete parton kinematics. The formalism not only improves the precision of QCD-based predictions but also establishes a unified framework for describing both electroweak boson production at the LHC and jet observables at HERA.

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