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On the double counting subtraction at NLO* of the high-energy factorization approach

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We suggest improvements for double counting subtraction scheme, which is needed for the consistent treatment of real corrections in the high-energy limit, and apply it to the single photon production at the NLO* approximation of the high-energy factorization approach. The presented improvements allow us to avoid the oversubtraction problem.

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1. Introduction

The advantages in usage of high-energy factorization (HEF) approach [1] for description of various hard processes have been demonstrated, for example, in recent Refs. [2], so the motivation for extending this approach beyond the leading order (LO) approximation in α_s is clear; a short time ago, the scheme of the next-to-leading order (NLO) calculations has been developed [3]. In this context, the non-trivial problem of double counting between real NLO corrections (NLO⋆) and unintegrated PDFs (UPDFs), which are responsible for the resummation of initial state radiations in the HEF, appears and investigation of the corresponding subtraction scheme is necessary [4]. A similar procedure for localization of NLO⋆ corrections to impact factors (IF) to avoid double counting with Green's function has also been developed [5] within the BFKL formalism [6]. However, it turns out that in some cases this procedure can lead to oversubtraction, especially at low rapidity differences. In this paper, we provide a simple improvement of this procedure, thus eliminating oversubtraction, and apply it to the production of single photon [7], which is a convenient process for studying the subtraction terms due to the infrared (IR) finiteness of one of the NLO⋆ corrections.

2. HEF

For instance, we consider hadroproduction of the single photon $p\,(P_1)+p\,(P_2)\to\gamma\,(q_3)+X$[1], the cross section for this process within the HEF reads:

$$\sigma=\int\frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{d^2\mathbf{q}_{T1}}{\pi}\frac{d^2\mathbf{q}_{T2}}{\pi}\,\Phi_{i_1}(x_1,\mathbf{q}_{T1}^2,\mu^2)\,\hat{\sigma}_{i_1i_2}(x_{1,2},\mathbf{q}_{T1,2},\bar{\alpha}_s(\mu_R^2))\,\Phi_{i_2}(x_2,\mathbf{q}_{T2}^2,\mu^2);\quad(1)$$

adopting the parton Reggeization approach [8], hard scattering coefficient (HSC) $\hat{\sigma}_{i_1i_2}$ describing subprocess $i_1\,(q_1)+i_2\,(q_2)\to\gamma\,(q_3)$ is calculated with Reggeized partons $i_{1,2}$, i.e., Reggeized gluons R and quarks Q; $q_1=\left(q_1^+/2\right)n_-+q_{T1}$ and $q_2=\left(q_2^-/2\right)n_++q_{T2}$, where $q_1^+=x_1P^+$ and $q_2^-=x_2P^-$. In Eq. (1), the UPDFs are defined as follows:

$$\Phi_i(x,\mathbf{q}_T^2,\mu^2)=\sum_{\bar{i}}\int_0^1\frac{dz}{z}\,\theta\left(\Delta(|\mathbf{q}_T|,\mu)-z\right)\,C_{i\bar{i}}(z,\mathbf{q}_T^2,\mu^2)\,\tilde{f}_{\bar{i}}\left(\frac{x}{z},\mu^2\right),\quad(2)$$

where z is a fraction of the large light-cone component carried through resummation in $C_{i\bar{i}}$, $\tilde{f}_i\left(x,\mu^2\right)=xf_i\left(x,\mu^2\right)$, and μ is a typical hard scale; in Eq. (2), the cutoff $\Delta(|\mathbf{q}_T|,\mu)=\mu/(\mu+|\mathbf{q}_T|)$ ensures rapidity ordering between partons emitted in the UPDF and HSC in Eq. (1).

3. LO, NLO⋆, and double counting subtraction

The HSCs for the LO (Fig. 1 (a)) and NLO⋆ (Fig. 1 (b), (d)) subprocesses

$$Q\,(q_1)+\bar{Q}\,(q_2)\;\to\;\gamma\,(q_3),\quad(3)$$
$$Q\,(q_1)+R\,(q_2)\;\to\;\gamma\,(q_3)+q\,(q_4),\quad(4)$$
$$Q\,(q_1)+\bar{Q}\,(q_2)\;\to\;\gamma\,(q_3)+g\,(q_4),\quad(5)$$

[1]Light-cone vectors can be introduced such that $n_\mp=\left(2/\sqrt{S}\right)P_{1,2}$, so $q=q_L+q_T$, where $q_L=\left(q^+n_-+q^-n_+\right)/2$ with $q^\pm=(q,n_\pm)$, rapidity $y(q)=(1/2)\ln\left(q^+/q^-\right)$.

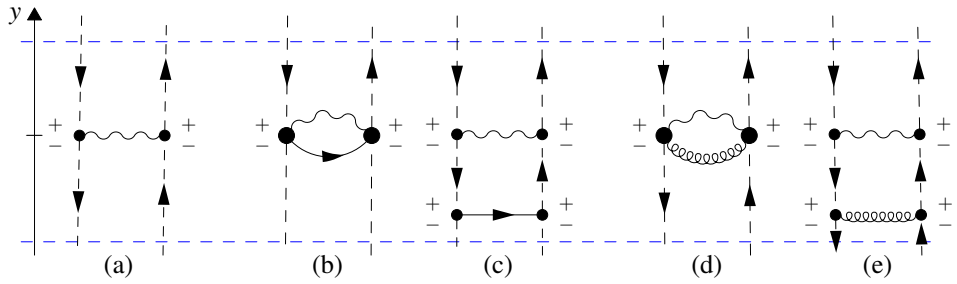

Figure 1: LO, NLO⋆ and corresponding subtraction terms plotted on the rapidity axis. Blue lines denotes boundaries of forward / backward regions.

already were calculated in Refs. [7]. The subprocesses (4), (5) have double counting with the subprocess (3) when integrating over the entire $dy_4 = d\hat{z}/(1-\hat{z})$, where $y_4 = \ln\left(|\mathbf{q}_{T4}|/\left((1-\hat{z})\,q_2^-\right)\right)$ and $\hat{z} = q_3^-/q_2^-$, such that the additional parton $i = q, g$ goes deeply forward / backward in rapidity space, so it should be included in the UPDF (2). This contributions can be subtracted as it shown in Fig. 1 (c), (e) (the permutations $+ \leftrightarrow -$ are assumed), the gauge-invariant amplitudes reads:

$$A_a^{\mu}(\text{Fig. 1 (c)}) = egT_a\,\Theta(\hat{z})\left(\frac{q_2^-}{2|\mathbf{q}_{T2}|}\right)\bar{u}(q_4)\,\gamma_+(q,-q_4)\,\frac{1}{\not{q}}\,\hat{P}^+\,\Gamma^{\mu}(q_1,-q)\,u(q_{L1}), \quad (6)$$

$$A_a^{\mu\nu}(\text{Fig. 1 (e)}) = egT_a\,\Theta(\hat{z})\,\bar{u}(q_{L2})\,\Gamma^{\nu}(q,q_2)\,\frac{1}{\not{q}}\,\hat{P}^+\,\Gamma^{\mu}(q_1,-q)\,u(q_{L1}), \quad (7)$$

where $q = q_1 - q_3$, propagator of the Reggeized quark is $\theta(Y)\,\left(i/\not{q}\right)\,\hat{P}^+$, $\hat{P}^+ = (1/4)\,\not{n}_-\not{n}_+$, and

$$\gamma_+(q,-q_4) = \not{n}_+ + \not{q}\,\frac{2}{(q-q_4)^-},$$
$$\Gamma^{\mu}(q_1,-q) = \gamma^{\mu} - \not{q}_1\,\frac{n_-^{\mu}}{(q_1-q)^-} + \not{q}\,\frac{n_+^{\mu}}{(q_1-q)^+};$$

in Eqs. (6), (7) cutoff function

$$\Theta(\hat{z}) = \theta\left(\frac{|\mathbf{q}_{T3}|}{|\mathbf{q}_{T4}|} - \hat{z}\right)\theta\left(\frac{|\mathbf{q}_{T2}||\mathbf{q}_{T3}|}{\Delta(|\mathbf{q}_{T2}|,\mu)\,\mathbf{q}_{T4}^2} - \hat{z}\right) \quad (8)$$

has the following origin. The propagator contain θ–function of

$$Y = y(q_3) - y(q_4) = \ln\left(\frac{1-\hat{z}}{\hat{z}}\,\frac{|\mathbf{q}_{T3}|}{|\mathbf{q}_{T4}|}\right),$$

which divide photon and parton i by rapidities, this generates the first factor in Eq. (8). However, the real correction still can be far from the UPDF in the rapidity space due to the Reggeized parton i_2, so it seems natural at least to require $Y > Y_2$ also, where rapidity difference between parton i and last emitted parton in $C_{i_2\bar{i}_2}$ (2) reads:

$$Y_2 = \ln\left(\frac{z_2/(1-z_2)}{1-\hat{z}}\,\frac{|\mathbf{q}_{T4}|}{|\mathbf{q}_{T2}|}\right),$$

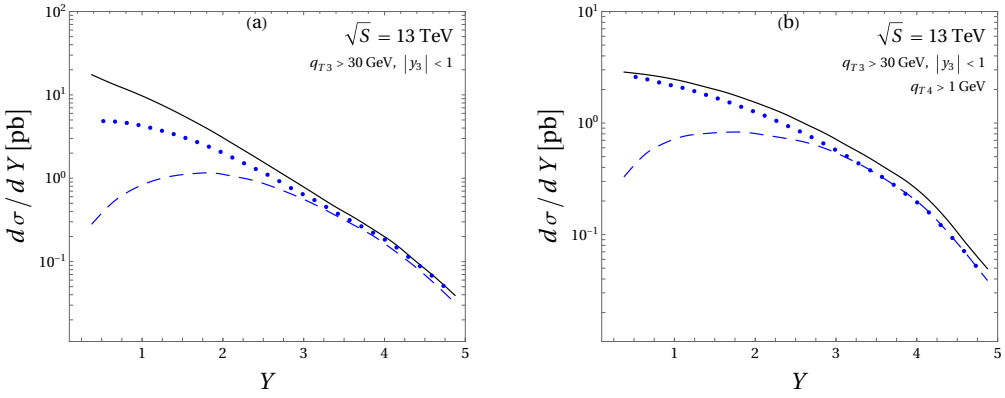

Figure 2: Cross section as function of Y (a) for (4) and (b) for (5). Different contributions are shown: NLO⋆ (black lines) and subtraction terms without / with second factor in Eq. (8) (dotted / dashed blue lines).

here z_2 is the same as z in Eq. (2) for UPDF Φ_{i_2} in Eq. (1). Probably, the most accurate way is to amend the HEF factorization (1) for subtraction terms by keeping the z_2–dependence in HSC through $\theta(Y - Y_2)$. Another way is to keep Eq. (1) by estimating $z_2 \sim \Delta(|\mathbf{q}_{T2}|, \mu)$, see Eq. (2), which leads to the second factor in Eq. (8).

The HSCs corresponding to Eqs. (6), (7) can be written as follows:

$$\frac{d\hat{\sigma}\left(Q\left(q_1\right) + i_2\left(q_2\right) \to \gamma\left(q_3\right) + i\left(q_4\right)\right)}{d\mathbf{q}_{T3}^2 dy_3 d\mathbf{q}_{T4}^2 d\Delta\phi_{34}} = \int_0^1 \frac{d\hat{z}\,\Theta(\hat{z})}{1-\hat{z}} \left(\frac{1}{\hat{z}} + \frac{\mathbf{q}_{T2}^2}{q^2}\right) \frac{3\delta^{(4)}(q_1 - q_3 - q)}{32\pi q_3^+ q_2^- (-q^2)} \times \frac{d\Psi_\gamma(\mathbf{q}_{T1}^2, \mathbf{q}_T^2)}{d\mathbf{q}_{T3}^2 dy_3} \frac{d\Psi_i(\mathbf{q}_T^2, \mathbf{q}_{T2}^2)}{\mathbf{q}_{T4}^2 d\mathbf{q}_{T4}^2}, \tag{9}$$

where photon IF describes $Q\left(q_1\right) + \bar{Q}\left(q\right) \to \gamma\left(q_3\right)$:

$$\frac{d\Psi_\gamma(\mathbf{q}_{T1}^2, \mathbf{q}_T^2)}{d\mathbf{q}_{T3}^2 dy_3} = 4\pi\alpha \frac{C_A}{N_c^2} \left(\mathbf{q}_{T1}^2 + \mathbf{q}_T^2\right), \tag{10}$$

quark IF stands for $Q\left(q\right) + R\left(q_2\right) \to q\left(q_4\right)$:

$$\frac{d\Psi_q(\mathbf{q}_T^2, \mathbf{q}_{T2}^2)}{d\mathbf{q}_{T4}^2} = 4\pi\alpha_s \frac{C_A C_F}{N_c(N_c^2 - 1)} \left(\mathbf{q}_T + \mathbf{q}_{T2}\right)^2, \tag{11}$$

gluon IF $\Psi_g(\mathbf{q}_T^2, \mathbf{q}_{T2}^2)$ for $Q\left(q\right) + \bar{Q}\left(q_2\right) \to g\left(q_4\right)$ can be easily obtained from Eq. (10). In the Regge limit $\hat{z} \to 0$, the integral in Eq. (9) generates $\ln\left(1/\hat{z}\right)$ and subtracts one iteration of rapidity evolution. Using Eq. (11), one can note that in the case $i = q$, Eq. (9) is IR finite, contrary to $i = g$.

For numerical calculations, we used improved Kimber-Martin-Ryskin-Watt UPDFs [8]. Cross sections as functions of the rapidity difference Y are shown in Fig. 2. For the subprocess (5) we set $|\mathbf{q}_{T4}| > 1$ GeV to eliminate IR divergences. In both cases, subtraction terms (9) without the second factor in Eq. (8) oversubtracts the corresponding NLO⋆ terms at low Y; the latter is cured by taking into account the second factor in Eq. (8), which suppresses subtraction terms at low Y and doesn't affect at large Y, where NLO⋆ terms get subtracted as it should be.

Conclusions

We have considered the double counting subtraction procedure in the application to the single photon production at NLO⋆ approximation of the HEF approach. This procedure can be improved by introducing second θ-function in Eq. (8), which eliminates the oversubtraction at low Y.

Acknowledgments

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