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Magnetic field-induced anisotropic interaction in heavy quark bound states

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We have investigated how a strong magnetic field (**B**) could decipher the anisotropic interaction in heavy quark (Q) and antiquark ($\bar{Q}$) bound states through the perturbative thermal QCD in real-time formalism. So we thermalize the Schwinger propagator for quarks in the lowest Landau level and the Feynman propagator for gluons to calculate the gluon self-energy up to one loop for massless flavours. For the quark-loop contribution to the self-energy, the medium does not have any temperature correction and the vacuum term gives rise an anisotropic term whereas the gluon-loop yields the temperature correction. This finding in quark-loop contribution corroborates the equivalence of a massless QED in (1+1)-dimension with the massless thermal QCD in strong magnetic field, which (quark sector) is reduced to (1+1)-dimension (longitudinal). This anisotropy in the self-energy is then being translated into the permittivity of the medium, which now behaves like a tensor. Thus the permittivity of the medium in the momentum space makes the $Q\bar{Q}$ potential in the coordinate space anisotropic in strong **B**. As a matter of fact, the potential for $Q\bar{Q}$-pairs aligned transverse to **B** is more attractive than the parallel alignment. However, the potential is always more attractive due to the softening of the electric screening mass whereas the (magnitude) imaginary-part of the potential becomes smaller, compared to $B = 0$. As a consequence, the binding energies (B.E.) of the ground states of $c\bar{c}$ and $b\bar{b}$ get increased and the widths (Γ) get decreased compared to $B = 0$. The above medium modifications to the properties of $Q\bar{Q}$ bound states then facilitate to study their quasi-free dissociation in the medium in a strong magnetic field. The dissociation temperatures are estimated for J/ψ and Υ states quantitatively as 1.59T$_\mathrm{c}$ and 2.22T$_\mathrm{c}$, respectively, which are found higher than the estimate in the absence of strong magnetic field. Thus strong B impedes the early dissolution of $Q\bar{Q}$ bound states in the medium.

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1. Introduction

Lattice Quantum Chromodynamics predicts that at extreme conditions of high temperatures and/or high densities, quarks confined inside the hadrons get deconfined and roam in an extended region of space (much bigger than the size of a hadron), known as quark-gluon plasma (QGP). This novel phenomenon can be seen as a generic property of nonabelian gauge theories at high energies, celebrated as Asymptotic Freedom. It is believed that such state of matter also existed in our present universe around one microsecond after the big bang, in the core of the dense stars, in the terrestrial laboratory of ultra-relativistic heavy ion collision (URHIC) experiments etc. As we know from the ongoing URHIC experiments at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC), a very strong magnetic field, perpendicular to the reaction plane, is produced in the very early stages of the collisions due to the very large relative velocity of spectator quarks in non-central events[1, 2], ranging from m_π^2 (10^{18} Gauss) at RHIC [3] to $15m_\pi^2$ at LHC [4]. Initially, it was believed that the magnetic field decays very fast just after it is produced, but the life time of the magnetic field is elongated if the medium have finite value of the electrical conductivity. In the recent years various research activities are going on in which the effects of the background magnetic field on the various properties of QGP have been incorporated, which in turn lead to many novel phenomena related to QCD. Out of many signatures of the QGP, the suppression of the heavy quarkonia is a very promising one. The heavy quark and antiquark pairs are produced in URHICs on a very short time-scale $\sim 1/2m_Q$ (m_Q is the mass of the heavy quark) and the pair develops into the physical resonances (heavy quarkonia) over a formation time. So by studying the properties of the heavy quarkonia, we can get some understanding about the medium and *vice versa*. Since the masses of the heavy quarks m_Q are much larger than the intrinsic QCD scale (Λ_{QCD}), the velocity of the heavy quarks (v) is very small in the bound states. The $Q\bar{Q}$ pair could then be treated like a nonrelativistic system. Thus the similarity in the time scales for the production of strong magnetic field and the formation of heavy quarkonia motivates us to study the effect of the strong magnetic field on $Q\bar{Q}$ interaction. In the recent years the effect of the magnetic field have been studied on the production of the heavy quarkonia in[5, 6] and magnetic conversion of η_c to the J/ψ in the presence of strong magnetic field in[7, 8]. Moreover, the static properties of quarkonia [9, 10] were studied in the presence of magnetic field. There have been lattice results on the heavy-quark potential and screening masses, both of which show novel anisotropic behaviors between transverse and longitudinal directions with respect to the magnetic field direction [11]. The anisotropic behaviours in the $Q\bar{Q}$ potential can be viewed as a manifestation of the breaking of rotational invariance in the presence of magnetic field. In nonrelativistic quantum mechanics, assuming the electron possessing the spin, the orientational term in the potential energy arises due to the interaction of spin magnetic moment with the external magnetic field. In relativistic quantum mechanics, the Dirac equation in the nonrelativistic limit manifests the aforesaid orientational term in Pauli-Schrodinger equation. However, in abovementioned potential studies at finite temperature [12, 13] based on the pertubative thermal QCD, the magnetic field did not reveal any anisotropic nature in $Q\bar{Q}$ interaction, *like* in aforesaid lattice studies. Therefore, our aim is to uncover the tensorial (anisotropic) part in $Q\bar{Q}$ interaction by an external magnetic field. The anisotropic interaction may influence meson spectra, string breaking, and thermalization, with possible observable effects such as modifications in elliptic flow.

2. Medium modification to Q-$\bar{Q}$ complex potential in a strong magnetic field

The real and imaginary parts of the dielectric permittivity are used to find the medium modification to complex $Q\bar{Q}$ potential, respectively. The real-part of the medium modified potential consists of central and noncentral components: $\mathrm{Re}\, V(r,\theta_r;T,B) = \mathrm{Re}\, V_{\mathrm{cent}}(r;T,B) + \mathrm{Re}\, V_{\mathrm{noncent}}(r,\theta_r;T,B)$

$$\begin{aligned}\mathrm{Re}\, V_{\mathrm{cent}}(r;T,B) &= -\frac{\alpha {m_D^q}^2}{\mu_D}\left[\frac{e^{-\hat{r}}}{\hat{r}}\left(\frac{\hat{r}}{4}+\frac{1}{\hat{r}}+\frac{1}{\hat{r}^2}+\frac{1}{2}+\frac{\mu_D^2}{{m_D^q}^2}\right)-\frac{1}{\hat{r}^3}-\frac{1}{12}+\frac{\mu_D^2}{{m_D^q}^2}\right]\\ &+\frac{4{m_D^q}^2\sigma\hat{r}}{{\mu_D}^3}\left[\frac{e^{-\hat{r}}}{\hat{r}}\left(\frac{1}{2\hat{r}}+\frac{1}{\hat{r}^2}+\frac{1}{\hat{r}^3}+\frac{1}{8}+\frac{\mu_D^2}{2\hat{r}{m_D^q}^2}\right)+\frac{1}{24\hat{r}}-\frac{1}{\hat{r}^4}-\frac{\mu_D^2}{2\hat{r}^2{m_D^q}^2}+\frac{\mu_D^2}{2\hat{r}{m_D^q}^2}\right]\end{aligned} \quad (1)$$

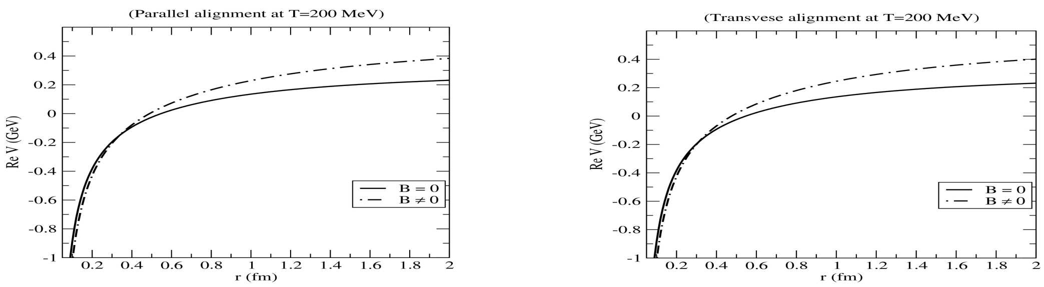
$$\begin{aligned}\mathrm{Re}\, V_{\mathrm{noncent}}(r,\theta_r;T,B) &= \cos^2\theta_r\left[\frac{\alpha {m_D^q}^2}{\mu_D}\left\{\frac{3e^{-\hat{r}}}{\hat{r}}\left(\frac{\hat{r}}{6}+\frac{1}{\hat{r}}+\frac{1}{\hat{r}^2}+\frac{1}{2}\right)-\frac{1}{\hat{r}^3}\right\}\right.\\ &\left.-\frac{4{m_D^q}^2}{{\mu_D}^3}\sigma\hat{r}\left\{\frac{3e^{-\hat{r}}}{\hat{r}}\left(\frac{5}{12\hat{r}}+\frac{1}{\hat{r}^2}+\frac{1}{\hat{r}^3}+\frac{1}{12}\right)+\frac{1}{12\hat{r}^2}-\frac{1}{\hat{r}^4}\right\}\right]. \quad (2)\end{aligned}$$

with $\hat{r} = r\mu_D$ and $\mu_D^2 = ({m_D^g}^2 + \frac{{m_D^q}^2}{2})$. The imaginary-part of the potential also consists of central and noncentral components: Im V(r, $\theta_r;T,B$) = Im $V_{\mathrm{cent}}(r;T,B)$ + Im $V_{\mathrm{noncent}}(r,\theta_r;T,B)$

$$\begin{aligned}\mathrm{Im}\, V_{\mathrm{cent}}(r;T,B) &= \frac{\alpha {m_D^g}^2 T\hat{r}^2}{9{\mu_D}^2}(-4+3\gamma_E+3\log\hat{r})\\ &-\frac{2\sigma {m_D^g}^2 T}{{\mu_D}^4}\left(\frac{\hat{r}^2}{6}+\frac{(-107+60\gamma_E+60\log\hat{r})\hat{r}^4}{3600}\right)\\ &-\frac{2\alpha {m_D^g}^2{m_D^q}^2 T}{{\mu_D}^4}\left[\int_0^\infty\frac{zdz}{(z^2+1)^3}\left(-\frac{\sin(z\hat{r})}{(z\hat{r})}-\frac{2\cos(z\hat{r})}{(z\hat{r})^2}+\frac{2\sin(z\hat{r})}{(z\hat{r})^3}+\frac{1}{3}\right)\right]\\ &-\frac{4\sigma {m_D^g}^2{m_D^q}^2 T}{{\mu_D}^6}\left[\int_0^\infty\frac{dz}{z(z^2+1)^3}\left(-\frac{\sin(z\hat{r})}{(z\hat{r})}-\frac{2\cos(z\hat{r})}{(z\hat{r})^2}+\frac{2\sin(z\hat{r})}{(z\hat{r})^3}+\frac{1}{3}\right)\right], \quad (3)\end{aligned}$$

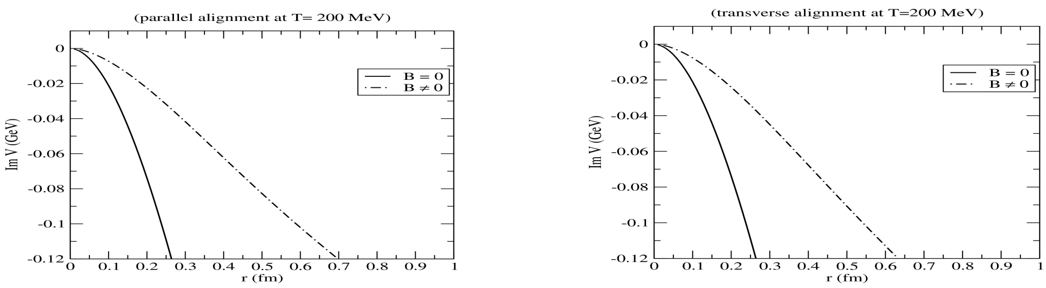
$$\begin{aligned}\mathrm{Im}\, V_{\mathrm{noncent}}(r,\theta_r;T,B) &= \left[-\frac{2\alpha {m_D^g}^2{m_D^q}^2 T}{{\mu_D}^4}\left\{\int_0^\infty\frac{zdz}{(z^2+1)^3}\left(\frac{2\sin(z\hat{r})}{(z\hat{r})}+\frac{6\cos(z\hat{r})}{(z\hat{r})^2}-\frac{6\sin(z\hat{r})}{(z\hat{r})^3}\right)\right\}\right.\\ &\left.-\frac{4\sigma {m_D^g}^2{m_D^q}^2 T}{{\mu_D}^6}\left\{\int_0^\infty\frac{dz}{z(z^2+1)^3}\left(\frac{2\sin(z\hat{r})}{(z\hat{r})}+\frac{6\cos(z\hat{r})}{(z\hat{r})^2}-\frac{6\sin(z\hat{r})}{(z\hat{r})^3}\right)\right\}\right]\cos^2\theta_r.\end{aligned}$$

It is thus inferred that the strong magnetic field introduces angular dependence into the $Q\bar{Q}$ interaction. which becomes more attractive when the $Q\bar{Q}$ pair is aligned transverse to the magnetic field than when the pair is aligned (parallel alignment) along the magnetic field, which is reflected in Figure 1. In the presence of the strong **B**, the $Q\bar{Q}$ potential gets less screened compared its counterpart in the absence of magnetic field, which is due to softening of the screening/Debye masses. The effects of a background magnetic field on the screening of both electric and magnetic fields in the deconfined medium were much earlier studied by computing the electric and magnetic electric screening masses, respectively, by measuring the Polyakov loop correlators on the lattice [11]. They found that the magnetic field enhances an increase of both screening masses and in addition, induces an anisotropy in Polyakov loop correlators, which in turn is translated into an anisotropy in $Q\bar{Q}$ interaction. However, the lattice estimates for the electric screening masses are somehow much larger than our results. We have displayed the effect of strong magnetic field on the ImV in Figure 2 as a function of (r) with respect to the direction of B for two orientations of $\mathbf{r}$. It is found that the magnitude of imaginary-part in general gets reduced in strong B compared to $B = 0$, which can again be attributed due to the softening of the Debye mass. However, the decrease (in magnitude) is lesser in the transverse direction than in the direction of magnetic field.



(a) (b)

Figure 1: (a) ReV (at a temperature T=200 MeV) as a function of r along the direction of strong magnetic field i) at $(eB = 15m_\pi^2)$, ii) for $B = 0$. (b) Same as in (a) but the orientation becomes transverse.

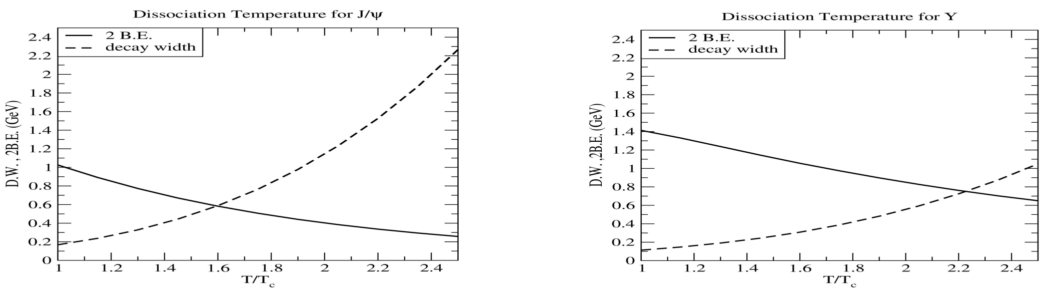


(a) (b)

Figure 2: Variation of the ImV as a function of r at T=200 MeV with the identical of Figure 1

3. Properties of Quarkonia in strong B and its dissociation

We have solved the Schrodinger equation numerically with the potential thus obtained in (1) and (2). The real- and imaginary-parts of the potential yield the binding energies and in-medium widths of the bound states, respectively. Since real-part has both spherical and nonspherical component and the nonspherical (angular) component is very small compared to the spherical component, so we have treated the nonspherical component as a perturbation and calculated the binding energies for the J/Ψ and Υ states in a first-order perturbation theory. The binding energies thus obtained numerically decreases with the temperature. Finally we have studied the quasi-free dissociation of quarkonium states by the competition of binding energies and medium widths in Figure 3, which originate from the real- and imaginary-parts of the potential in a medium, respectively. The quantitative study of the dissociation is made by the above competition between screening and Landau damping, in particular, when the binding energy of a particular quarkonium state (i) is equal to the half of its width, *i.e.* $\text{B.E.}|_i = \frac{\Gamma_i}{2}$. Since both quantities (B.E. and Γ) depend on the temperature and the strength of the strong magnetic field, so the above relation gives the temperature of the hot medium (T_d) in a strong B at which the $Q\bar{Q}$ state gets excited and moves to the continuum. We have thus obtained the T_d's for J/Ψ and Υ as 1.59 $\mathrm{T_C}$ and 2.22 $\mathrm{T_C}$, respectively.



(a) (b)

Figure 3: The decay width vs 2× binding energy for J/Ψ and Υ in a strong magnetic field at $eB = 15m_\pi^2$.

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